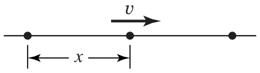
Chapter 1

Problems 1-1 through 1-4 are for student research.

1-5

(a) Point vehicles



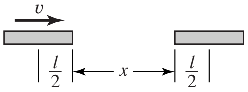
$$Q = \frac{\text{cars}}{\text{hour}} = \frac{v}{x} = \frac{42.1v - v^2}{0.324}$$

Seek stationary point maximum

$$\frac{dQ}{dv} = 0 = \frac{42.1 - 2v}{0.324} \quad \therefore v^* = 21.05\,\text{mph}$$

$$Q^* = \frac{42.1(21.05) - 21.05^2}{0.324} = 1368\,\text{cars/h} \quad \textit{Ans.}$$

(b)



$$Q = \frac{v}{x + l} = \left(\frac{0.324}{v(42.1) - v^2} + \frac{l}{v}\right)^{-1}$$

Maximize Q with $l = 10/5280$ mi

v	Q	
22.18	1221.431	
22.19	1221.433	
22.20	1221.435	←
22.21	1221.435	
22.22	1221.434	

$$\%\text{ loss of throughput} = \frac{1368 - 1221}{1221} = 12\% \quad \textit{Ans.}$$

(c) $\%\text{ increase in speed } \dfrac{22.2 - 21.05}{21.05} = 5.5\%$

Modest change in optimal speed *Ans.*

1-6 This and the following problem may be the student's first experience with a figure of merit.

- Formulate fom to reflect larger figure of merit for larger merit.
- Use a maximization optimization algorithm. When one gets into computer implementation and answers are not known, minimizing instead of maximizing is the largest error one can make.

$$\sum F_V = F_1 \sin\theta - W = 0$$

$$\sum F_H = -F_1 \cos\theta - F_2 = 0$$

From which

$$F_1 = W/\sin\theta$$

$$F_2 = -W\cos\theta/\sin\theta$$

$$\begin{aligned}
\text{fom} &= -\$ = -¢\gamma \text{ (volume)} \\
&\doteq -¢\gamma(l_1 A_1 + l_2 A_2) \\
A_1 &= \frac{F_1}{S} = \frac{W}{S\sin\theta}, \quad l_2 = \frac{l_1}{\cos\theta} \\
A_2 &= \left|\frac{F_2}{S}\right| = \frac{W\cos\theta}{S\sin\theta} \\
\text{fom} &= -¢\gamma\left(\frac{l_2}{\cos\theta}\,\frac{W}{S\sin\theta} + \frac{l_2 W\cos\theta}{S\sin\theta}\right) \\
&= \frac{-¢\gamma W l_2}{S}\left(\frac{1+\cos^2\theta}{\cos\theta\sin\theta}\right)
\end{aligned}$$

Set leading constant to unity

$\theta°$	fom
0	$-\infty$
20	−5.86
30	−4.04
40	−3.22
45	−3.00
50	−2.87
54.736	−2.828
60	−2.886

$\theta^* = 54.736°$ *Ans.*

$\text{fom}^* = -2.828$

Alternative:

$$\frac{d}{d\theta}\left(\frac{1+\cos^2\theta}{\cos\theta\sin\theta}\right) = 0$$

And solve resulting transcendental for θ^*.

Check second derivative to see if a maximum, minimum, or point of inflection has been found. Or, evaluate fom on either side of θ^*.

1-7

(a) $x_1 + x_2 = X_1 + e_1 + X_2 + e_2$

$$\text{error} = e = (x_1 + x_2) - (X_1 + X_2)$$
$$= e_1 + e_2 \quad \textit{Ans.}$$

(b) $x_1 - x_2 = X_1 + e_1 - (X_2 + e_2)$

$$e = (x_1 - x_2) - (X_1 - X_2) = e_1 - e_2 \quad \textit{Ans.}$$

(c) $x_1 x_2 = (X_1 + e_1)(X_2 + e_2)$

$$e = x_1x_2 - X_1X_2 = X_1e_2 + X_2e_1 + e_1e_2$$
$$\doteq X_1e_2 + X_2e_1 = X_1X_2\left(\frac{e_1}{X_1} + \frac{e_2}{X_2}\right) \quad \textit{Ans.}$$

(d) $\dfrac{x_1}{x_2} = \dfrac{X_1 + e_1}{X_2 + e_2} = \dfrac{X_1}{X_2}\left(\dfrac{1 + e_1/X_1}{1 + e_2/X_2}\right)$

$$\left(1 + \frac{e_2}{X_2}\right)^{-1} \doteq 1 - \frac{e_2}{X_2} \quad \text{and} \quad \left(1 + \frac{e_1}{X_1}\right)\left(1 - \frac{e_2}{X_2}\right) \doteq 1 + \frac{e_1}{X_1} - \frac{e_2}{X_2}$$

$$e = \frac{x_1}{x_2} - \frac{X_1}{X_2} \doteq \frac{X_1}{X_2}\left(\frac{e_1}{X_1} - \frac{e_2}{X_2}\right) \quad \textit{Ans.}$$

1-8

(a) $x_1 = \sqrt{5} = 2.236\,067\,977\,5$

$$X_1 = 2.23 \quad \text{3-correct digits}$$
$$x_2 = \sqrt{6} = 2.449\,487\,742\,78$$
$$X_2 = 2.44 \quad \text{3-correct digits}$$
$$x_1 + x_2 = \sqrt{5} + \sqrt{6} = 4.685\,557\,720\,28$$
$$e_1 = x_1 - X_1 = \sqrt{5} - 2.23 = 0.006\,067\,977\,5$$
$$e_2 = x_2 - X_2 = \sqrt{6} - 2.44 = 0.009\,489\,742\,78$$
$$e = e_1 + e_2 = \sqrt{5} - 2.23 + \sqrt{6} - 2.44 = 0.015\,557\,720\,28$$

$$\text{Sum} = x_1 + x_2 = X_1 + X_2 + e$$
$$= 2.23 + 2.44 + 0.015\,557\,720\,28$$
$$= 4.685\,557\,720\,28 \text{ (Checks)} \quad \textit{Ans.}$$

(b) $X_1 = 2.24, \quad X_2 = 2.45$

$$e_1 = \sqrt{5} - 2.24 = -0.003\,932\,022\,50$$
$$e_2 = \sqrt{6} - 2.45 = -0.000\,510\,257\,22$$
$$e = e_1 + e_2 = -0.004\,442\,279\,72$$
$$\text{Sum} = X_1 + X_2 + e$$
$$= 2.24 + 2.45 + (-0.004\,442\,279\,72)$$
$$= 4.685\,557\,720\,28 \quad \textit{Ans.}$$

1-9

(a) $\sigma = 20(6.89) = 137.8$ MPa

(b) $F = 350(4.45) = 1558$ N $= 1.558$ kN

(c) $M = 1200$ lbf $\cdot$ in $(0.113) = 135.6$ N $\cdot$ m

(d) $A = 2.4(645) = 1548$ mm^2

(e) $I = 17.4$ in^4 $(2.54)^4 = 724.2$ cm^4

(f) $A = 3.6(1.610)^2 = 9.332$ km^2

(g) $E = 21(1000)(6.89) = 144.69(10^3)$ MPa $= 144.7$ GPa

(h) $v = 45$ mi/h $(1.61) = 72.45$ km/h

(i) $V = 60$ in^3 $(2.54)^3 = 983.2$ cm^3 $= 0.983$ liter

1-10

(a) $l = 1.5/0.305 = 4.918$ ft $= 59.02$ in

(b) $\sigma = 600/6.89 = 86.96$ kpsi

(c) $p = 160/6.89 = 23.22$ psi

(d) $Z = 1.84(10^5)/(25.4)^3 = 11.23$ in^3

(e) $w = 38.1/175 = 0.218$ lbf/in

(f) $\delta = 0.05/25.4 = 0.00197$ in

(g) $v = 6.12/0.0051 = 1200$ ft/min

(h) $\epsilon = 0.0021$ in/in

(i) $V = 30/(0.254)^3 = 1831$ in^3

1-11

(a) $$\sigma = \frac{200}{15.3} = 13.1\ \text{MPa}$$

(b) $$\sigma = \frac{42(10^3)}{6(10^{-2})^2} = 70(10^6)\ \text{N/m}^2 = 70\ \text{MPa}$$

(c) $$y = \frac{1200(800)^3(10^{-3})^3}{3(207)10^9(64)10^3(10^{-3})^4} = 1.546(10^{-2})\ \text{m} = 15.5\ \text{mm}$$

(d) $$\theta = \frac{1100(250)(10^{-3})}{79.3(10^9)(\pi/32)(25)^4(10^{-3})^4} = 9.043(10^{-2})\ \text{rad} = 5.18^\circ$$

1-12

(a) $$\sigma = \frac{600}{20(6)} = 5\ \text{MPa}$$

(b) $$I = \frac{1}{12}8(24)^3 = 9216\ \text{mm}^4$$

(c) $$I = \frac{\pi}{64}32^4(10^{-1})^4 = 5.147\ \text{cm}^4$$

(d) $$\tau = \frac{16(16)}{\pi(25^3)(10^{-3})^3} = 5.215(10^6)\ \text{N/m}^2 = 5.215\ \text{MPa}$$

1-13

(a) $$\tau = \frac{120(10^3)}{(\pi/4)(20^2)} = 382\ \text{MPa}$$

(b) $$\sigma = \frac{32(800)(800)(10^{-3})}{\pi(32)^3(10^{-3})^3} = 198.9(10^6)\ \text{N/m}^2 = 198.9\,\text{MPa}$$

(c) $$Z = \frac{\pi}{32(36)}(36^4 - 26^4) = 3334\ \text{mm}^3$$

(d) $$k = \frac{(1.6)^4\,(10^{-3})^4(79.3)(10^9)}{8(19.2)^3(10^{-3})^3(32)} = 286.8\,\text{N/m}$$

Chapter 2

2-1 From Table A-20

$$S_{ut} = 470\,\text{MPa}\ (68\,\text{kpsi}), \quad S_y = 390\,\text{MPa}\ (57\,\text{kpsi}) \quad \textit{Ans.}$$

2-2 From Table A-20

$$S_{ut} = 620\,\text{MPa}\ (90\,\text{kpsi}), \quad S_y = 340\,\text{MPa}\ (49.5\,\text{kpsi}) \quad \textit{Ans.}$$

2-3 Comparison of yield strengths:

$$S_{ut} \text{ of G10 500 HR is } \frac{620}{470} = 1.32 \text{ times larger than SAE1020 CD} \quad \textit{Ans.}$$

$$S_{yt} \text{ of SAE1020 CD is } \frac{390}{340} = 1.15 \text{ times larger than G10500 HR} \quad \textit{Ans.}$$

From Table A-20, the ductilities (reduction in areas) show,

$$\text{SAE1020 CD is } \frac{40}{35} = 1.14 \text{ times larger than G10500} \quad \textit{Ans.}$$

The stiffness values of these materials are identical *Ans.*

	S_{ut} MPa (kpsi)	S_y MPa (kpsi)	Table A-20 Ductility $R\%$	Table A-5 Stiffness GPa (Mpsi)
SAE1020 CD	470(68)	390 (57)	40	207(30)
UNS10500 HR	620(90)	340(495)	35	207(30)

2-4 From Table A-21

$$1040\text{ Q\&T} \quad \bar{S}_y = 593\,(86)\ \text{MPa (kpsi)} \quad \text{at } 205^\circ\text{C } (400^\circ\text{F}) \quad \textit{Ans.}$$

2-5 From Table A-21

$$1040\text{ Q\&T} \quad R = 65\% \quad \text{at } 650^\circ\text{C } (1200^\circ\text{F}) \quad \textit{Ans.}$$

2-6 Using Table A-5, the specific strengths are:

$$\text{UNS G10350 HR steel:} \quad \frac{S_y}{W} = \frac{39.5(10^3)}{0.282} = 1.40(10^5)\text{ in} \quad \textit{Ans.}$$

$$\text{2024 T4 aluminum:} \quad \frac{S_y}{W} = \frac{43(10^3)}{0.098} = 4.39(10^5)\text{ in} \quad \textit{Ans.}$$

$$\text{Ti-6Al-4V titanium:} \quad \frac{S_y}{W} = \frac{140(10^3)}{0.16} = 8.75(10^5)\text{ in} \quad \textit{Ans.}$$

ASTM 30 gray cast iron has no yield strength. *Ans.*

2-7 The specific moduli are:

$$\text{UNS G10350 HR steel:} \quad \frac{E}{W} = \frac{30(10^6)}{0.282} = 1.06(10^8)\text{ in} \quad Ans.$$

$$\text{2024 T4 aluminum:} \quad \frac{E}{W} = \frac{10.3(10^6)}{0.098} = 1.05(10^8)\text{ in} \quad Ans.$$

$$\text{Ti-6Al-4V titanium:} \quad \frac{E}{W} = \frac{16.5(10^6)}{0.16} = 1.03(10^8)\text{ in} \quad Ans.$$

$$\text{Gray cast iron:} \quad \frac{E}{W} = \frac{14.5(10^6)}{0.26} = 5.58(10^7)\text{ in} \quad Ans.$$

2-8

$$2G(1+\nu) = E \quad \Rightarrow \quad \nu = \frac{E - 2G}{2G}$$

From Table A-5

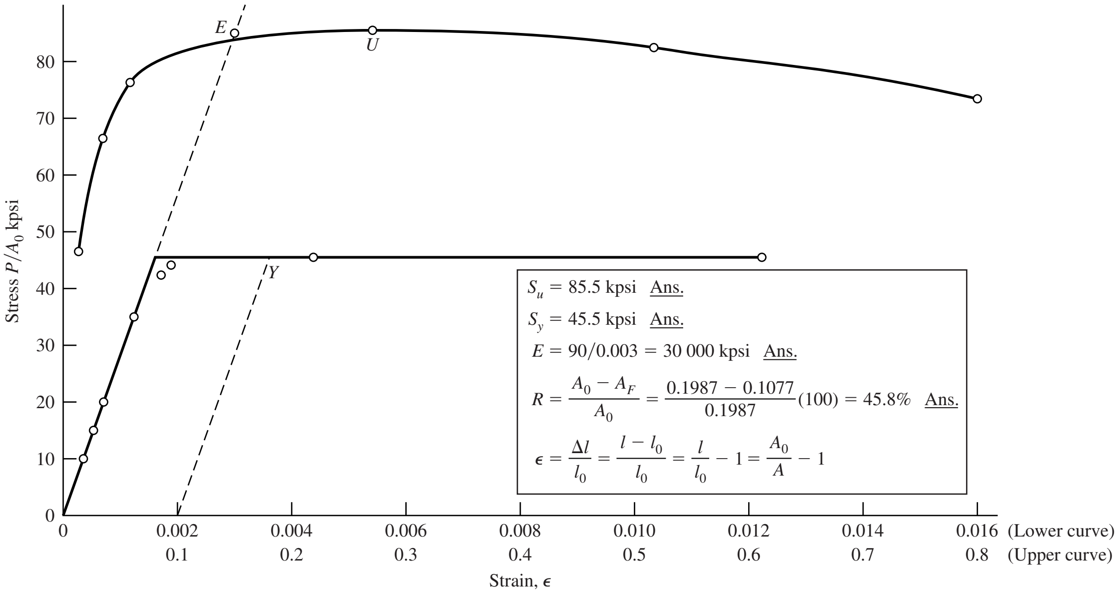
$$\text{Steel:} \quad \nu = \frac{30 - 2(11.5)}{2(11.5)} = 0.304 \quad Ans.$$

$$\text{Aluminum:} \quad \nu = \frac{10.4 - 2(3.90)}{2(3.90)} = 0.333 \quad Ans.$$

$$\text{Beryllium copper:} \quad \nu = \frac{18 - 2(7)}{2(7)} = 0.286 \quad Ans.$$

$$\text{Gray cast iron:} \quad \nu = \frac{14.5 - 2(6)}{2(6)} = 0.208 \quad Ans.$$

2-9



$S_u = 85.5$ kpsi <u>Ans.</u>

$S_y = 45.5$ kpsi <u>Ans.</u>

$E = 90/0.003 = 30\,000$ kpsi <u>Ans.</u>

$$R = \frac{A_0 - A_F}{A_0} = \frac{0.1987 - 0.1077}{0.1987}(100) = 45.8\% \quad \underline{\text{Ans.}}$$

$$\epsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1 = \frac{A_0}{A} - 1$$

2-10 To plot σ_{true} vs. ε, the following equations are applied to the data.

$$A_0 = \frac{\pi(0.503)^2}{4} = 0.1987\,\text{in}^2$$

Eq. (2-4)

$$\varepsilon = \ln\frac{l}{l_0} \quad \text{for} \quad 0 \le \Delta L \le 0.0028\,\text{in}$$

$$\varepsilon = \ln\frac{A_0}{A} \quad \text{for} \quad \Delta L > 0.0028\,\text{in}$$

$$\sigma_{\text{true}} = \frac{P}{A}$$

The results are summarized in the table below and plotted on the next page.
The last 5 points of data are used to plot $\log\sigma$ vs $\log\varepsilon$

The curve fit gives $\quad m = 0.2306$

$$\log\sigma_0 = 5.1852 \quad \Rightarrow \quad \sigma_0 = 153.2\ \text{kpsi} \qquad \textit{Ans.}$$

For 20% cold work, Eq. (2-10) and Eq. (2-13) give,

$$A = A_0(1 - W) = 0.1987(1 - 0.2) = 0.1590\,\text{in}^2$$

$$\varepsilon = \ln\frac{A_0}{A} = \ln\frac{0.1987}{0.1590} = 0.2231$$

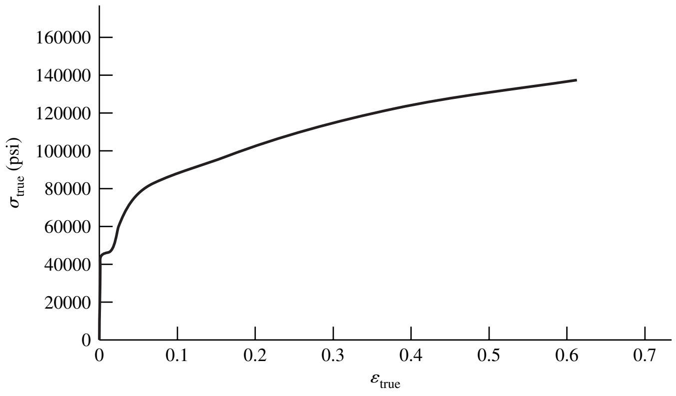
Eq. (2-14):

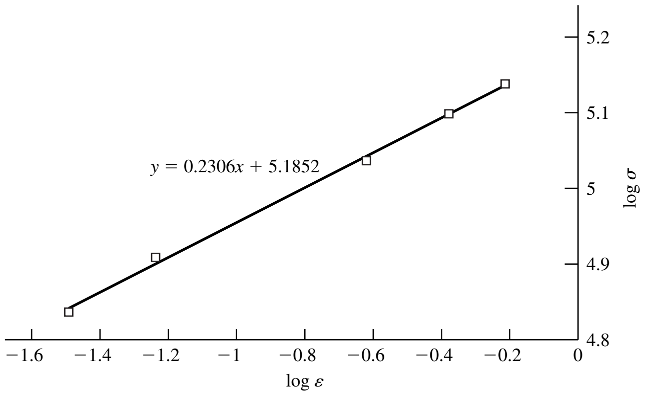
$$S'_y = \sigma_0\varepsilon^m = 153.2(0.2231)^{0.2306} = 108.4\,\text{kpsi} \quad \textit{Ans.}$$

Eq. (2-15), with $S_u = 85.5$ kpsi from Prob. 2-9,

$$S'_u = \frac{S_u}{1 - W} = \frac{85.5}{1 - 0.2} = 106.9\,\text{kpsi} \quad \textit{Ans.}$$

P	ΔL	A	ε	σ_{true}	$\log\varepsilon$	$\log\sigma_{\text{true}}$
0	0	0.198 713	0	0		
1 000	0.0004	0.198 713	0.000 2	5032.388	−3.699 01	3.701 774
2 000	0.0006	0.198 713	0.000 3	10 064.78	−3.522 94	4.002 804
3 000	0.0010	0.198 713	0.000 5	15 097.17	−3.301 14	4.178 895
4 000	0.0013	0.198 713	0.000 65	20 129.55	−3.187 23	4.303 834
7 000	0.0023	0.198 713	0.001 149	35 226.72	−2.939 55	4.546 872
8 400	0.0028	0.198 713	0.001 399	42 272.06	−2.854 18	4.626 053
8 800	0.0036	0.198 4	0.001 575	44 354.84	−2.802 61	4.646 941
9 200	0.0089	0.197 8	0.004 604	46 511.63	−2.336 85	4.667 562
9 100		0.196 3	0.012 216	46 357.62	−1.913 05	4.666 121
13 200		0.192 4	0.032 284	68 607.07	−1.491 01	4.836 369
15 200		0.187 5	0.058 082	81 066.67	−1.235 96	4.908 842
17 000		0.156 3	0.240 083	108 765.2	−0.619 64	5.036 49
16 400		0.130 7	0.418 956	125 478.2	−0.377 83	5.098 568
14 800		0.107 7	0.612 511	137 418.8	−0.212 89	5.138 046





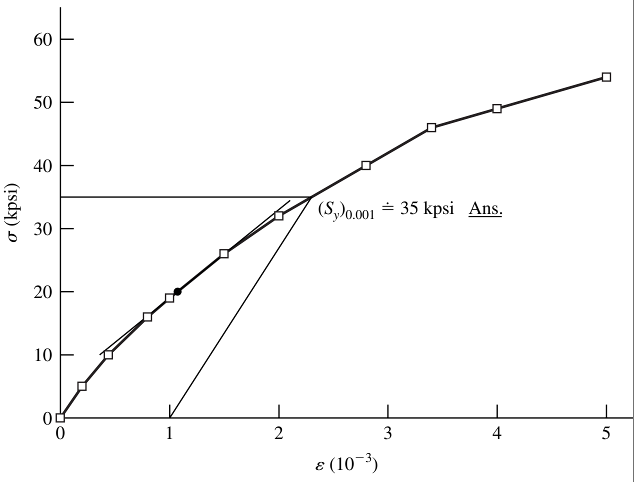
2-11 Tangent modulus at $\sigma = 0$ is

$$E_0 = \frac{\Delta\sigma}{\Delta\varepsilon} \doteq \frac{5000 - 0}{0.2(10^{-3}) - 0} = 25(10^6)\text{ psi}$$

At $\sigma = 20$ kpsi

$$E_{20} \doteq \frac{(26-19)(10^3)}{(1.5-1)(10^{-3})} = 14.0(10^6)\ \text{psi} \quad \textit{Ans.}$$

$\varepsilon(10^{-3})$	σ (kpsi)
0	0
0.20	5
0.44	10
0.80	16
1.0	19
1.5	26
2.0	32
2.8	40
3.4	46
4.0	49
5.0	54



2-12 Since $|\varepsilon_o| = |\varepsilon_i|$

$$\left|\ln \frac{R+h}{R+N}\right| = \left|\ln \frac{R}{R+N}\right| = \left|-\ln \frac{R+N}{R}\right|$$

$$\frac{R+h}{R+N} = \frac{R+N}{R}$$

$$(R+N)^2 = R(R+h)$$

From which, $$N^2 + 2RN - Rh = 0$$

The roots are: $$N = R\left[-1 \pm \left(1 + \frac{h}{R}\right)^{1/2}\right]$$

The + sign being significant,

$$N = R\left[\left(1 + \frac{h}{R}\right)^{1/2} - 1\right] \quad Ans.$$

Substitute for N in

$$\varepsilon_o = \ln \frac{R+h}{R+N}$$

Gives $$\varepsilon_0 = \ln\left[\frac{R+h}{R + R\left(1 + \frac{h}{R}\right)^{1/2} - R}\right] = \ln\left(1 + \frac{h}{R}\right)^{1/2} \quad Ans.$$

These constitute a useful pair of equations in cold-forming situations, allowing the surface strains to be found so that cold-working strength enhancement can be estimated.

2-13 From Table A-22

AISI 1212 $$S_y = 28.0 \text{ kpsi}, \quad \sigma_f = 106 \text{ kpsi}, \quad S_{ut} = 61.5 \text{ kpsi}$$

$$\sigma_0 = 110 \text{ kpsi}, \quad m = 0.24, \quad \varepsilon_f = 0.85$$

From Eq. (2-12) $$\varepsilon_u = m = 0.24$$

Eq. (2-10) $$\frac{A_0}{A_i'} = \frac{1}{1-W} = \frac{1}{1-0.2} = 1.25$$

Eq. (2-13) $$\varepsilon_i = \ln 1.25 = 0.2231 \quad \Rightarrow \quad \varepsilon_i < \varepsilon_u$$

Eq. (2-14) $$S_y' = \sigma_0 \varepsilon_i^m = 110(0.2231)^{0.24} = 76.7 \text{ kpsi} \quad Ans.$$

Eq. (2-15) $$S_u' = \frac{S_u}{1-W} = \frac{61.5}{1-0.2} = 76.9 \text{ kpsi} \quad Ans.$$

2-14 For $H_B = 250$,

Eq. (2-17) $$\begin{aligned} S_u &= 0.495\,(250) = 124 \text{ kpsi} \\ &= 3.41\,(250) = 853 \text{ MPa} \end{aligned} \quad Ans.$$

2-15 For the data given,

$$\sum H_B = 2530 \quad \sum H_B^2 = 640\,226$$

$$\bar{H}_B = \frac{2530}{10} = 253 \quad \hat{\sigma}_{HB} = \sqrt{\frac{640\,226 - (2530)^2/10}{9}} = 3.887$$

Eq. (2-17)

$$\bar{S}_u = 0.495(253) = 125.2 \text{ kpsi} \quad \textit{Ans.}$$

$$\bar{\sigma}_{su} = 0.495(3.887) = 1.92 \text{ kpsi} \quad \textit{Ans.}$$

2-16 From Prob. 2-15, $\bar{H}_B = 253$ and $\hat{\sigma}_{HB} = 3.887$

Eq. (2-18)

$$\bar{S}_u = 0.23(253) - 12.5 = 45.7 \text{ kpsi} \quad \textit{Ans.}$$

$$\hat{\sigma}_{su} = 0.23(3.887) = 0.894 \text{ kpsi} \quad \textit{Ans.}$$

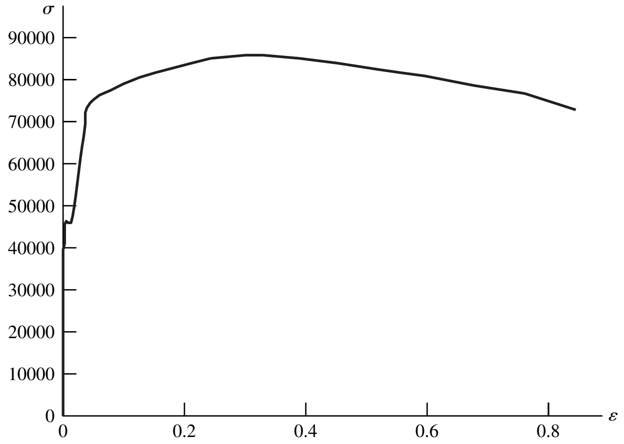
2-17

(a)

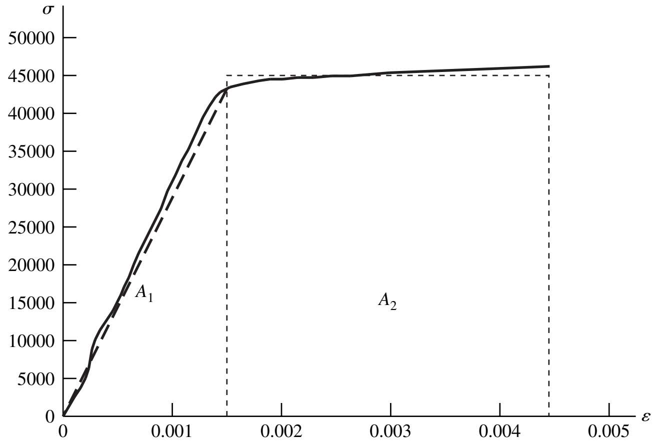
$$u_R \doteq \frac{45.5^2}{2(30)} = 34.5 \text{ in} \cdot \text{lbf/in}^3 \quad \textit{Ans.}$$

(b)

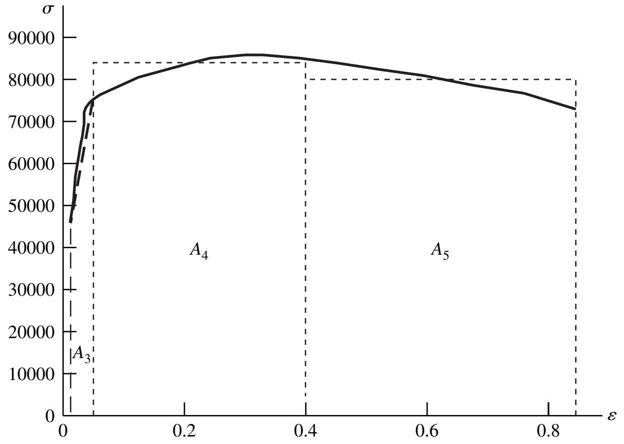
P	ΔL	A	$A_0/A - 1$	ε	$\sigma = P/A_0$
0	0			0	0
1 000	0.0004			0.0002	5 032.39
2 000	0.0006			0.0003	10 064.78
3 000	0.0010			0.0005	15 097.17
4 000	0.0013			0.000 65	20 129.55
7 000	0.0023			0.001 15	35 226.72
8 400	0.0028			0.0014	42 272.06
8 800	0.0036			0.0018	44 285.02
9 200	0.0089			0.004 45	46 297.97
9 100		0.1963	0.012 291	0.012 291	45 794.73
13 200		0.1924	0.032 811	0.032 811	66 427.53
15 200		0.1875	0.059 802	0.059 802	76 492.30
17 000		0.1563	0.271 355	0.271 355	85 550.60
16 400		0.1307	0.520 373	0.520 373	82 531.17
14 800		0.1077	0.845 059	0.845 059	74 479.35



All data points



First 9 data points



Last 6 data points

$$u_T \doteq \sum_{i=1}^{5} A_i = \frac{1}{2}(43\,000)(0.001\,5) + 45\,000(0.004\,45 - 0.001\,5)$$

$$+ \frac{1}{2}(45\,000 + 76\,500)(0.059\,8 - 0.004\,45)$$

$$+ 81\,000(0.4 - 0.059\,8) + 80\,000(0.845 - 0.4)$$

$$\doteq 66.7(10^3)\text{in} \cdot \text{lbf/in}^3 \quad \textit{Ans.}$$

2-18 $m = Al\rho$

<u>For stiffness</u>, $k = AE/l$, or, $A = kl/E$.

Thus, $m = kl^2\rho/E$, and, $M = E/\rho$. Therefore, $\beta = 1$

From Fig. 2-16, ductile materials include Steel, Titanium, Molybdenum, Aluminum, and Composites.

<u>For strength</u>, $S = F/A$, or, $A = F/S$.

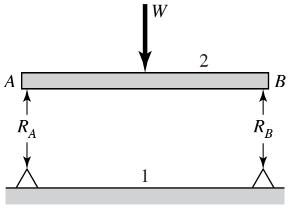
Thus, $m = Fl\ \rho/S$, and, $M = S/\rho$.

From Fig. 2-19, lines parallel to S/ρ give for ductile materials, Steel, Nickel, Titanium, and composites.

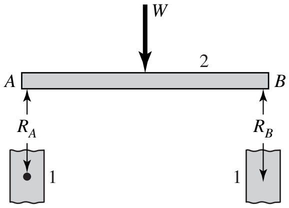
Common to both stiffness and strength are Steel, Titanium, Aluminum, and Composites. *Ans.*

Chapter 3

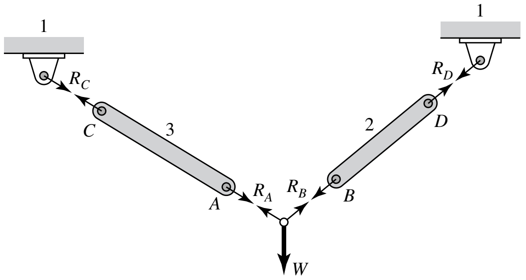
3-1



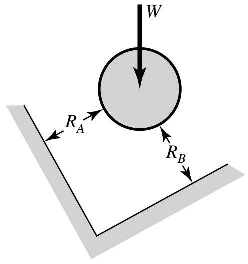
(a)



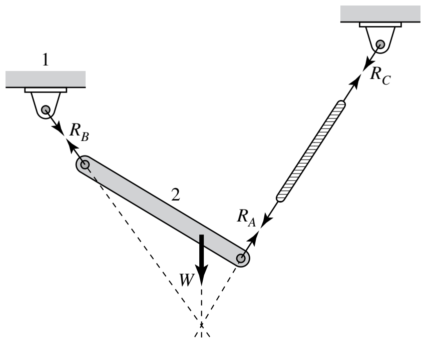
(b)



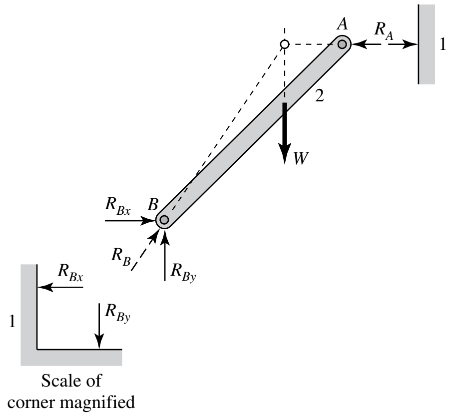
(c)



(d)



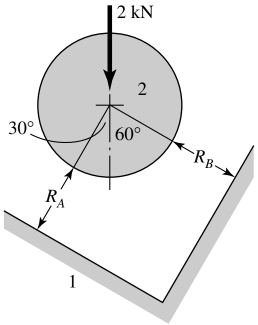
(e)

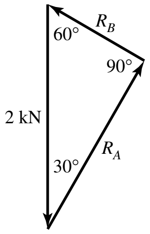


(f)

3-2

(a)

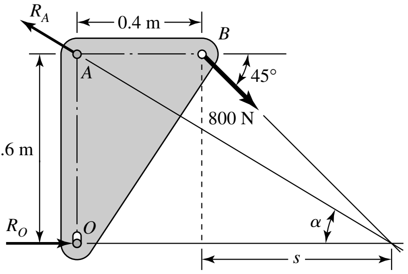




$$R_A = 2\sin 60 = 1.732\ \text{kN} \quad \textit{Ans.}$$

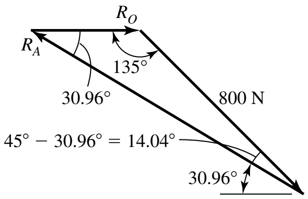
$$R_B = 2\sin 30 = 1\ \text{kN} \quad \textit{Ans.}$$

(b)



$$S = 0.6\ \text{m}$$

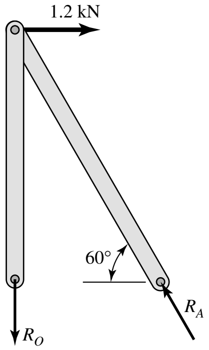
$$\alpha = \tan^{-1}\frac{0.6}{0.4+0.6} = 30.96^\circ$$

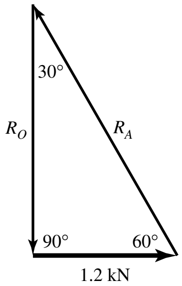


$$\frac{R_A}{\sin 135} = \frac{800}{\sin 30.96} \quad \Rightarrow \quad R_A = 1100\ \text{N} \quad \textit{Ans.}$$

$$\frac{R_O}{\sin 14.04} = \frac{800}{\sin 30.96} \quad \Rightarrow \quad R_O = 377\ \text{N} \quad \textit{Ans.}$$

(c)

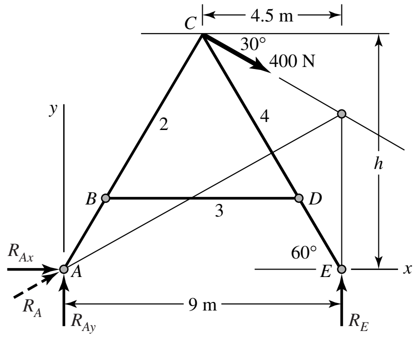




$$R_O = \frac{1.2}{\tan 30} = 2.078\ \text{kN} \quad \textit{Ans.}$$

$$R_A = \frac{1.2}{\sin 30} = 2.4\ \text{kN} \quad \textit{Ans.}$$

(d) Step 1: Find R_A and R_E



$$h = \frac{4.5}{\tan 30} = 7.794\,\text{m}$$

$$\circlearrowleft + \sum M_A = 0$$

$$9R_E - 7.794(400\cos 30) - 4.5(400\sin 30) = 0$$

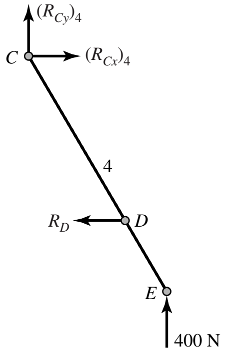
$$R_E = 400\ \text{N} \quad \textit{Ans.}$$

$$\sum F_x = 0 \quad R_{Ax} + 400\cos 30 = 0 \quad \Rightarrow \quad R_{Ax} = -346.4\,\text{N}$$

$$\sum F_y = 0 \quad R_{Ay} + 400 - 400\sin 30 = 0 \quad \Rightarrow \quad R_{Ay} = -200\,\text{N}$$

$$R_A = \sqrt{346.4^2 + 200^2} = 400\,\text{N} \quad \textit{Ans.}$$

Step 2: Find components of R_C on link 4 and R_D



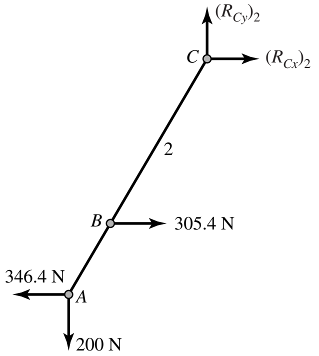
$$\curvearrowleft + \sum M_C = 0$$

$$400(4.5) - (7.794 - 1.9)R_D = 0 \quad \Rightarrow \quad R_D = 305.4\,\text{N} \quad \textit{Ans.}$$

$$\sum F_x = 0 \quad \Rightarrow \quad (R_{Cx})_4 = 305.4\,\text{N}$$

$$\sum F_y = 0 \quad \Rightarrow \quad (R_{Cy})_4 = -400\,\text{N}$$

Step 3: Find components of R_C on link 2

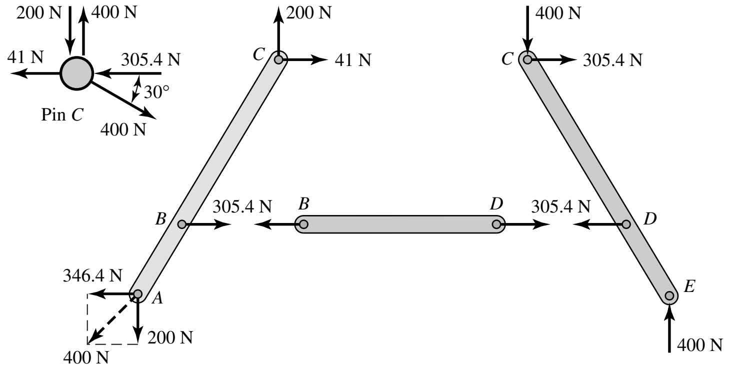


$$\sum F_x = 0$$

$$(R_{Cx})_2 + 305.4 - 346.4 = 0 \quad \Rightarrow \quad (R_{Cx})_2 = 41\,\text{N}$$

$$\sum F_y = 0$$

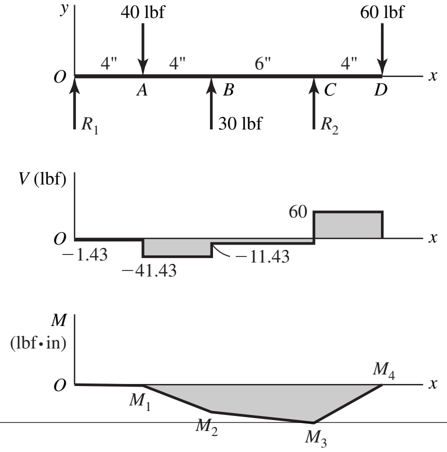
$$(R_{Cy})_2 = 200\,\text{N}$$



Ans.

3-3

(a)



$$\curvearrowleft + \sum M_0 = 0$$

$$-18(60) + 14R_2 + 8(30) - 4(40) = 0$$

$$R_2 = 71.43\text{ lbf}$$

$$\sum F_y = 0\text{: } R_1 - 40 + 30 + 71.43 - 60 = 0$$

$$R_1 = -1.43\text{ lbf}$$

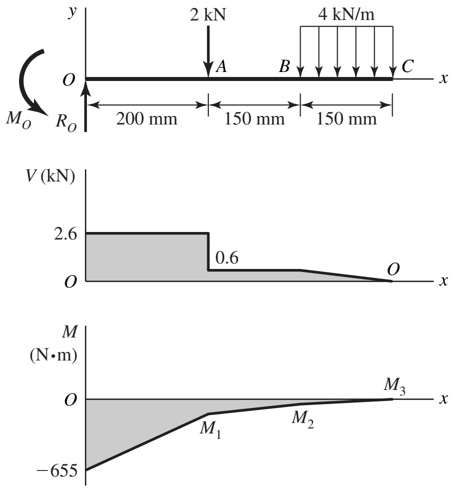
$$M_1 = -1.43(4) = -5.72\text{ lbf}\cdot\text{in}$$

$$M_2 = -5.72 - 41.43(4) = -171.44\text{ lbf}\cdot\text{in}$$

$$M_3 = -171.44 - 11.43(6) = -240\text{ lbf}\cdot\text{in}$$

$$M_4 = -240 + 60(4) = 0 \quad \text{checks!}$$

(b)



$$\sum F_y = 0$$

$$R_0 = 2 + 4(0.150) = 2.6\,\text{kN}$$

$$\sum M_0 = 0$$

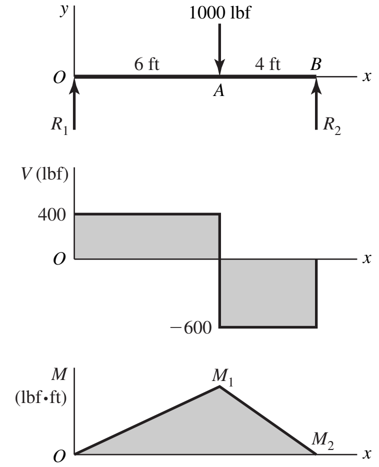
$$\begin{aligned} M_0 &= 2000(0.2) + 4000(0.150)(0.425) \\ &= 655\ \text{N} \cdot \text{m} \end{aligned}$$

$$M_1 = -655 + 2600(0.2) = -135\ \text{N} \cdot \text{m}$$

$$M_2 = -135 + 600(0.150) = -45\ \text{N} \cdot \text{m}$$

$$M_3 = -45 + \frac{1}{2}600(0.150) = 0 \quad \text{checks!}$$

(c)



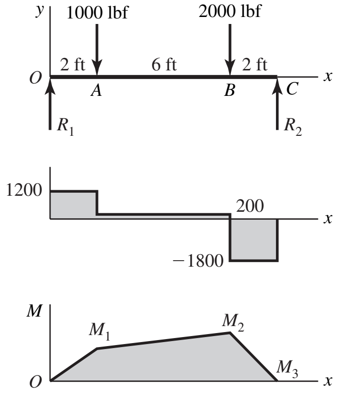
$$\sum M_0 = 0\text{: } 10R_2 - 6(1000) = 0 \quad \Rightarrow \quad R_2 = 600\ \text{lbf}$$

$$\sum F_y = 0\text{: } R_1 - 1000 + 600 = 0 \quad \Rightarrow \quad R_1 = 400\ \text{lbf}$$

$$M_1 = 400(6) = 2400\ \text{lbf} \cdot \text{ft}$$

$$M_2 = 2400 - 600(4) = 0 \quad \text{checks!}$$

(d)



$$\circlearrowleft + \sum M_C = 0$$

$$-10R_1 + 2(2000) + 8(1000) = 0$$

$$R_1 = 1200\ \text{lbf}$$

$$\sum F_y = 0\text{: } 1200 - 1000 - 2000 + R_2 = 0$$

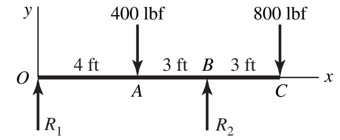
$$R_2 = 1800\ \text{lbf}$$

$$M_1 = 1200(2) = 2400\ \text{lbf} \cdot \text{ft}$$

$$M_2 = 2400 + 200(6) = 3600\ \text{lbf} \cdot \text{ft}$$

$$M_3 = 3600 - 1800(2) = 0 \quad \text{checks!}$$

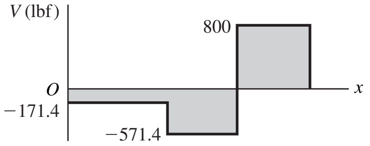
(e)



$$\circlearrowleft + \sum M_B = 0$$

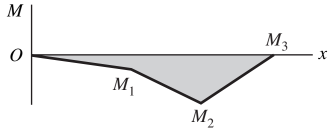
$$-7R_1 + 3(400) - 3(800) = 0$$

$$R_1 = -171.4 \text{ lbf}$$



$$\sum F_y = 0\text{: } -171.4 - 400 + R_2 - 800 = 0$$

$$R_2 = 1371.4 \text{ lbf}$$

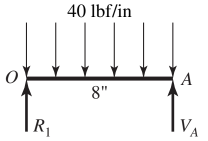


$$M_1 = -171.4(4) = -685.7 \text{ lbf} \cdot \text{ft}$$

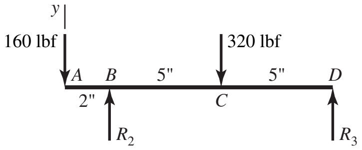
$$M_2 = -685.7 - 571.4(3) = -2400 \text{ lbf} \cdot \text{ft}$$

$$M_3 = -2400 + 800(3) = 0 \quad \text{checks!}$$

(f) Break at *A*



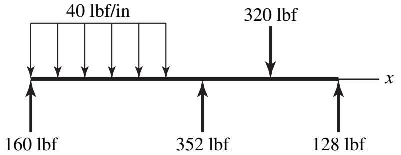
$$R_1 = V_A = \frac{1}{2}40(8) = 160 \text{ lbf}$$



$$\circlearrowleft + \sum M_D = 0$$

$$12(160) - 10R_2 + 320(5) = 0$$

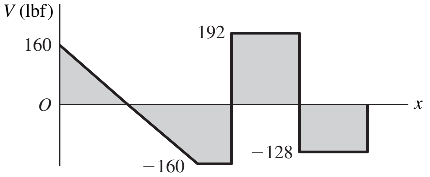
$$R_2 = 352 \text{ lbf}$$

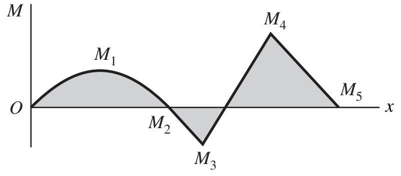


$$\sum F_y = 0$$

$$-160 + 352 - 320 + R_3 = 0$$

$$R_3 = 128 \text{ lbf}$$





$$M_1 = \frac{1}{2}160(4) = 320 \text{ lbf} \cdot \text{in}$$

$$M_2 = 320 - \frac{1}{2}160(4) = 0 \quad \text{checks! (hinge)}$$

$$M_3 = 0 - 160(2) = -320 \text{ lbf} \cdot \text{in}$$

$$M_4 = -320 + 192(5) = 640 \text{ lbf} \cdot \text{in}$$

$$M_5 = 640 - 128(5) = 0 \quad \text{checks!}$$

3-4

(a)
$$q = R_1\langle x\rangle^{-1} - 40\langle x-4\rangle^{-1} + 30\langle x-8\rangle^{-1} + R_2\langle x-14\rangle^{-1} - 60\langle x-18\rangle^{-1}$$
$$V = R_1 - 40\langle x-4\rangle^{0} + 30\langle x-8\rangle^{0} + R_2\langle x-14\rangle^{0} - 60\langle x-18\rangle^{0} \tag{1}$$
$$M = R_1 x - 40\langle x-4\rangle^{1} + 30\langle x-8\rangle^{1} + R_2\langle x-14\rangle^{1} - 60\langle x-18\rangle^{1} \tag{2}$$

for $x = 18^+ \quad V = 0 \quad$ and $\quad M = 0 \quad$ Eqs. (1) and (2) give

$$0 = R_1 - 40 + 30 + R_2 - 60 \quad \Rightarrow \quad R_1 + R_2 = 70 \tag{3}$$
$$0 = R_1(18) - 40(14) + 30(10) + 4R_2 \quad \Rightarrow \quad 9R_1 + 2R_2 = 130 \tag{4}$$

Solve (3) and (4) simultaneously to get $R_1 = -1.43$ lbf, $R_2 = 71.43$ lbf. *Ans.*

From Eqs. (1) and (2), at $x = 0^+$, $V = R_1 = -1.43$ lbf, $M = 0$

$x = 4^+$: $\quad V = -1.43 - 40 = -41.43,\ M = -1.43x$

$x = 8^+$: $\quad V = -1.43 - 40 + 30 = -11.43$

$\quad M = -1.43(8) - 40(8-4)^1 = -171.44$

$x = 14^+$: $\quad V = -1.43 - 40 + 30 + 71.43 = 60$

$\quad M = -1.43(14) - 40(14-4) + 30(14-8) = -240.$

$x = 18^+$: $\quad V = 0,\ M = 0 \quad$ See curves of V and M in Prob. 3-3 solution.

(b) $q = R_0\langle x\rangle^{-1} - M_0\langle x\rangle^{-2} - 2000\langle x-0.2\rangle^{-1} - 4000\langle x-0.35\rangle^{0} + 4000\langle x-0.5\rangle^{0}$

$$V = R_0 - M_0\langle x\rangle^{-1} - 2000\langle x-0.2\rangle^{0} - 4000\langle x-0.35\rangle^{1} + 4000\langle x-0.5\rangle^{1} \tag{1}$$
$$M = R_0 x - M_0 - 2000\langle x-0.2\rangle^{1} - 2000\langle x-0.35\rangle^{2} + 2000\langle x-0.5\rangle^{2} \tag{2}$$

at $x = 0.5^+$ m, $V = M = 0$, Eqs. (1) and (2) give

$$R_0 - 2000 - 4000(0.5 - 0.35) = 0 \quad \Rightarrow \quad R_1 = 2600\,\text{N} = 2.6\ \text{kN} \quad \textit{Ans.}$$
$$R_0(0.5) - M_0 - 2000(0.5 - 0.2) - 2000(0.5 - 0.35)^2 = 0$$

with $R_0 = 2600$ N, $M_0 = 655$ N · m *Ans.*

With R_0 and M_0, Eqs. (1) and (2) give the same V and M curves as Prob. 3-3 (note for V, $M_0\langle x\rangle^{-1}$ has no physical meaning).

(c)
$$q = R_1\langle x\rangle^{-1} - 1000\langle x-6\rangle^{-1} + R_2\langle x-10\rangle^{-1}$$
$$V = R_1 - 1000\langle x-6\rangle^{0} + R_2\langle x-10\rangle^{0} \tag{1}$$
$$M = R_1 x - 1000\langle x-6\rangle^{1} + R_2\langle x-10\rangle^{1} \tag{2}$$

at $x = 10^+$ ft, $\quad V = M = 0$, Eqs. (1) and (2) give

$$R_1 - 1000 + R_2 = 0 \quad \Rightarrow \quad R_1 + R_2 = 1000$$
$$10R_1 - 1000(10 - 6) = 0 \quad \Rightarrow \quad R_1 = 400\,\text{lbf}, \quad R_2 = 1000 - 400 = 600\,\text{lbf}$$

$0 \le x \le 6$: $\quad V = 400\,\text{lbf}, \quad M = 400x$

$6 \le x \le 10$: $\quad V = 400 - 1000(x-6)^0 = 600$ lbf

$\quad M = 400x - 1000(x-6) = 6000 - 600x$

See curves of Prob. 3-3 solution.

(d)
$$q = R_1\langle x\rangle^{-1} - 1000\langle x-2\rangle^{-1} - 2000\langle x-8\rangle^{-1} + R_2\langle x-10\rangle^{-1}$$
$$V = R_1 - 1000\langle x-2\rangle^{0} - 2000\langle x-8\rangle^{0} + R_2\langle x-10\rangle^{0} \tag{1}$$
$$M = R_1 x - 1000\langle x-2\rangle^{1} - 2000\langle x-8\rangle^{1} + R_2\langle x-10\rangle^{1} \tag{2}$$

At $x = 10^+$, $\quad V = M = 0$ from Eqs. (1) and (2)

$$R_1 - 1000 - 2000 + R_2 = 0 \quad \Rightarrow \quad R_1 + R_2 = 3000$$

$$10R_1 - 1000(10-2) - 2000(10-8) = 0 \quad \Rightarrow \quad R_1 = 1200\,\text{lbf},$$

$$R_2 = 3000 - 1200 = 1800\,\text{lbf}$$

$0 \le x \le 2$: $\quad V = 1200\,\text{lbf}, \quad M = 1200x\ \text{lbf}\cdot\text{ft}$

$2 \le x \le 8$: $\quad V = 1200 - 1000 = 200\,\text{lbf}$

$$M = 1200x - 1000(x-2) = 200x + 2000\ \text{lbf}\cdot\text{ft}$$

$8 \le x \le 10$: $\quad V = 1200 - 1000 - 2000 = -1800\,\text{lbf}$

$$M = 1200x - 1000(x-2) - 2000(x-8) = -1800x + 18\,000\ \text{lbf}\cdot\text{ft}$$

Plots are the same as in Prob. 3-3.

(e)

$$q = R_1\langle x\rangle^{-1} - 400\langle x-4\rangle^{-1} + R_2\langle x-7\rangle^{-1} - 800\langle x-10\rangle^{-1}$$

$$V = R_1 - 400\langle x-4\rangle^{0} + R_2\langle x-7\rangle^{0} - 800\langle x-10\rangle^{0} \qquad (1)$$

$$M = R_1 x - 400\langle x-4\rangle^{1} + R_2\langle x-7\rangle^{1} - 800\langle x-10\rangle^{1} \qquad (2)$$

at $x = 10^+$, $\quad V = M = 0$

$$R_1 - 400 + R_2 - 800 = 0 \quad \Rightarrow \quad R_1 + R_2 = 1200 \qquad (3)$$

$$10R_1 - 400(6) + R_2(3) = 0 \quad \Rightarrow \quad 10R_1 + 3R_2 = 2400 \qquad (4)$$

Solve Eqs. (3) and (4) simultaneously: $R_1 = -171.4$ lbf, $R_2 = 1371.4$ lbf

$0 \le x \le 4$: $\quad V = -171.4\,\text{lbf}, \quad M = -171.4x\ \text{lbf}\cdot\text{ft}$

$4 \le x \le 7$: $\quad V = -171.4 - 400 = -571.4\ \text{lbf}$

$$M = -171.4x - 400(x-4)\ \text{lbf}\cdot\text{ft} = -571.4x + 1600$$

$7 \le x \le 10$: $\quad V = -171.4 - 400 + 1371.4 = 800\,\text{lbf}$

$$M = -171.4x - 400(x-4) + 1371.4(x-7) = 800x - 8000\,\text{lbf}\cdot\text{ft}$$

Plots are the same as in Prob. 3-3.

(f)

$$q = R_1\langle x\rangle^{-1} - 40\langle x\rangle^{0} + 40\langle x-8\rangle^{0} + R_2\langle x-10\rangle^{-1} - 320\langle x-15\rangle^{-1} + R_3\langle x-20\rangle$$

$$V = R_1 - 40x + 40\langle x-8\rangle^{1} + R_2\langle x-10\rangle^{0} - 320\langle x-15\rangle^{0} + R_3\langle x-20\rangle^{0} \qquad (1)$$

$$M = R_1 x - 20x^2 + 20\langle x-8\rangle^{2} + R_2\langle x-10\rangle^{1} - 320\langle x-15\rangle^{1} + R_3\langle x-20\rangle^{1} \qquad (2)$$

$M = 0$ at $x = 8$ in $\quad \therefore 8R_1 - 20(8)^2 = 0 \quad \Rightarrow \quad R_1 = 160\,\text{lbf}$

at $x = 20^+$, $\quad V$ and $M = 0$

$$160 - 40(20) + 40(12) + R_2 - 320 + R_3 = 0 \quad \Rightarrow \quad R_2 + R_3 = 480$$

$$160(20) - 20(20)^2 + 20(12)^2 + 10R_2 - 320(5) = 0 \quad \Rightarrow \quad R_2 = 352\ \text{lbf}$$

$$R_3 = 480 - 352 = 128\ \text{lbf}$$

$0 \le x \le 8$: $\quad V = 160 - 40x\ \text{lbf}, \quad M = 160x - 20x^2\ \text{lbf}\cdot\text{in}$

$8 \le x \le 10$: $\quad V = 160 - 40x + 40(x-8) = -160\,\text{lbf},$

$$M = 160x - 20x^2 + 20(x-8)^2 = 1280 - 160x\ \text{lbf}\cdot\text{in}$$

$10 \le x \le 15$: $\quad V = 160 - 40x + 40(x-8) + 352 = 192\,\text{lbf}$

$$M = 160x - 20x^2 + 20(x-8) + 352(x-10) = 192x - 2240$$

$$15 \le x \le 20: \quad V = 160 - 40x + 40(x-8) + 352 - 320 = -128\,\text{lbf}$$

$$M = 160x - 20x^2 - 20(x-8) + 352(x-10) - 320(x-15)$$

$$= -128x + 2560$$

Plots of V and M are the same as in Prob. 3-3.

3-5 Solution depends upon the beam selected.

3-6

(a) Moment at center, $x_c = (l - 2a)/2$

$$M_c = \frac{w}{2}\left[\frac{l}{2}(l-2a) - \left(\frac{l}{2}\right)^2\right] = \frac{wl}{2}\left(\frac{l}{4} - a\right)$$

At reaction, $|M_r| = wa^2/2$

$a = 2.25$, $l = 10$ in, $w = 100$ lbf/in

$$M_c = \frac{100(10)}{2}\left(\frac{10}{4} - 2.25\right) = 125 \text{ lbf} \cdot \text{in}$$

$$M_r = \frac{100(2.25^2)}{2} = 253.1 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

(b) Minimum occurs when $M_c = |M_r|$

$$\frac{wl}{2}\left(\frac{l}{4} - a\right) = \frac{wa^2}{2} \quad \Rightarrow \quad a^2 + al - 0.25l^2 = 0$$

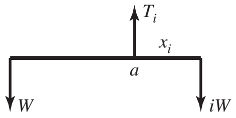
Taking the positive root

$$a = \frac{1}{2}\left[-l + \sqrt{l^2 + 4(0.25l^2)}\right] = \frac{l}{2}\left(\sqrt{2} - 1\right) = 0.2071l \quad \textit{Ans.}$$

for $l = 10$ in and $w = 100$ lbf, $\quad M_{\min} = (100/2)[(0.2071)(10)]^2 = 214 \text{ lbf} \cdot \text{in}$

3-7 For the ith wire from bottom, from summing forces vertically

(a)



$$T_i = (i+1)W$$

From summing moments about point a,

$$\sum M_a = W(l - x_i) - iWx_i = 0$$

Giving,

$$x_i = \frac{l}{i+1}$$

So

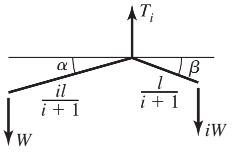
$$W = \frac{l}{1+1} = \frac{l}{2}$$

$$x = \frac{l}{2+1} = \frac{l}{3}$$

$$y = \frac{l}{3+1} = \frac{l}{4}$$

$$z = \frac{l}{4+1} = \frac{l}{5}$$

(b) With straight rigid wires, the mobile is not stable. Any perturbation can lead to all wires becoming collinear. Consider a wire of length l bent at its string support:



$$\sum M_a = 0$$

$$\sum M_a = \frac{iWl}{i+1}\cos\alpha - \frac{ilW}{i+1}\cos\beta = 0$$

$$\frac{iWl}{i+1}(\cos\alpha - \cos\beta) = 0$$

Moment vanishes when $\alpha = \beta$ for any wire. Consider a ccw rotation angle β, which makes $\alpha \to \alpha + \beta$ and $\beta \to \alpha - \beta$

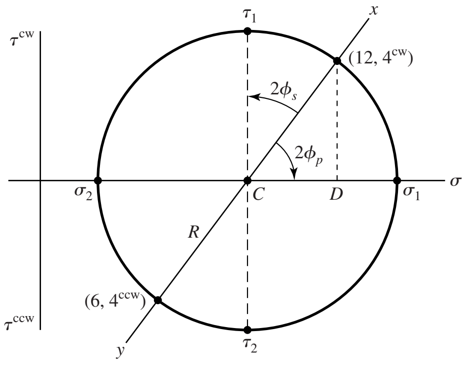
$$M_a = \frac{iWl}{i+1}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]$$

$$= \frac{2iWl}{i+1}\sin\alpha\sin\beta \doteq \frac{2iWl\beta}{i+1}\sin\alpha$$

There exists a correcting moment of opposite sense to arbitrary rotation β. An equation for an upward bend can be found by changing the sign of W. The moment will no longer be correcting. A curved, convex-upward bend of wire will produce stable equilibrium too, but the equation would change somewhat.

3-8

(a)



$$C = \frac{12+6}{2} = 9$$

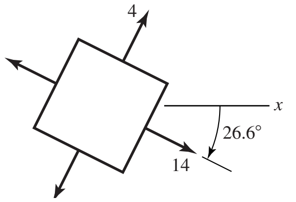
$$CD = \frac{12-6}{2} = 3$$

$$R = \sqrt{3^2 + 4^2} = 5$$

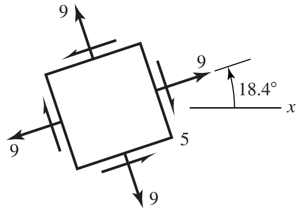
$$\sigma_1 = 5 + 9 = 14$$

$$\sigma_2 = 9 - 5 = 4$$

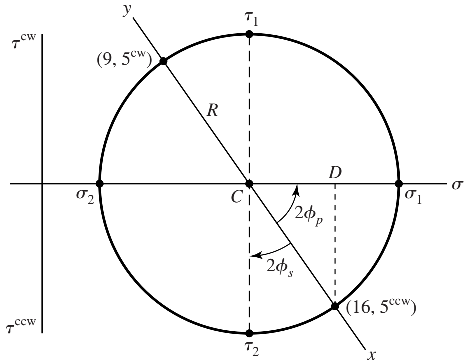
$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right) = 26.6^\circ \text{ cw}$$



$$\tau_1 = R = 5, \quad \phi_s = 45^\circ - 26.6^\circ = 18.4^\circ \text{ ccw}$$



(b)



$$C = \frac{9+16}{2} = 12.5$$

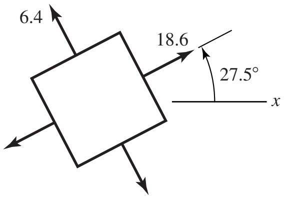
$$CD = \frac{16-9}{2} = 3.5$$

$$R = \sqrt{5^2 + 3.5^2} = 6.10$$

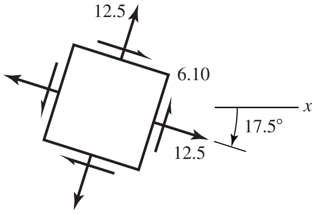
$$\sigma_1 = 6.1 + 12.5 = 18.6$$

$$\phi_p = \frac{1}{2}\tan^{-1}\frac{5}{3.5} = 27.5^\circ \text{ ccw}$$

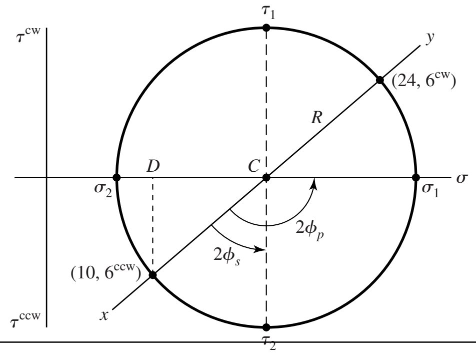
$$\sigma_2 = 12.5 - 6.1 = 6.4$$



$$\tau_1 = R = 6.10, \quad \phi_s = 45^\circ - 27.5^\circ = 17.5^\circ \text{ cw}$$



(c)



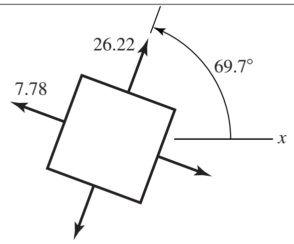
$$C = \frac{24+10}{2} = 17$$

$$CD = \frac{24-10}{2} = 7$$

$$R = \sqrt{7^2 + 6^2} = 9.22$$

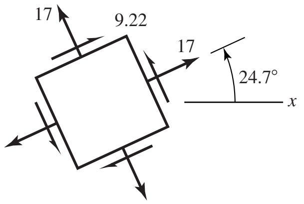
$$\sigma_1 = 17 + 9.22 = 26.22$$

$$\sigma_2 = 17 - 9.22 = 7.78$$

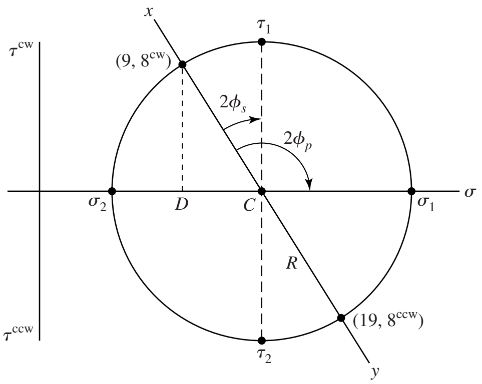


$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{7}{6}\right] = 69.7^\circ \text{ ccw}$$

$$\tau_1 = R = 9.22, \quad \phi_s = 69.7^\circ - 45^\circ = 24.7^\circ \text{ ccw}$$



(d)



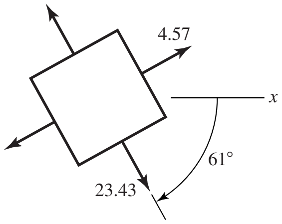
$$C = \frac{9+19}{2} = 14$$

$$CD = \frac{19-9}{2} = 5$$

$$R = \sqrt{5^2 + 8^2} = 9.434$$

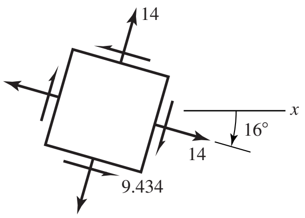
$$\sigma_1 = 14 + 9.43 = 23.43$$

$$\sigma_2 = 14 - 9.43 = 4.57$$



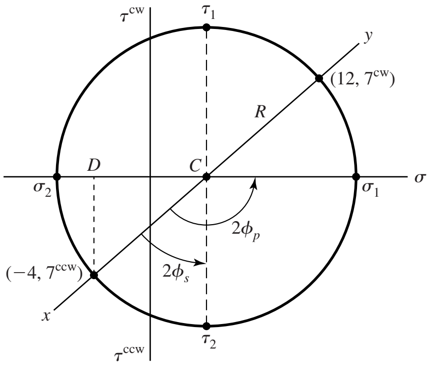
$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{5}{8}\right] = 61.0^\circ \text{ cw}$$

$$\tau_1 = R = 9.434, \quad \phi_s = 61^\circ - 45^\circ = 16^\circ \text{ cw}$$



3-9

(a)



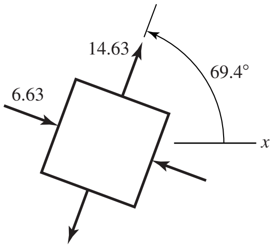
$$C = \frac{12-4}{2} = 4$$

$$CD = \frac{12+4}{2} = 8$$

$$R = \sqrt{8^2 + 7^2} = 10.63$$

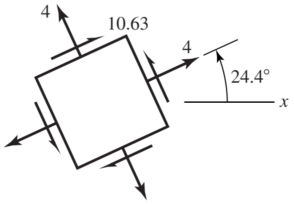
$$\sigma_1 = 4 + 10.63 = 14.63$$

$$\sigma_2 = 4 - 10.63 = -6.63$$

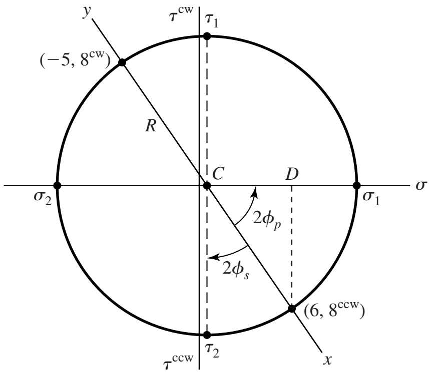


$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{8}{7}\right] = 69.4^\circ \text{ ccw}$$

$$\tau_1 = R = 10.63, \quad \phi_s = 69.4^\circ - 45^\circ = 24.4^\circ \text{ ccw}$$



(b)



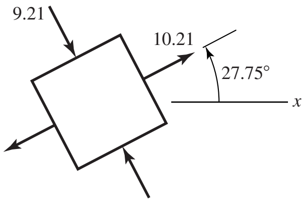
$$C = \frac{6-5}{2} = 0.5$$

$$CD = \frac{6+5}{2} = 5.5$$

$$R = \sqrt{5.5^2 + 8^2} = 9.71$$

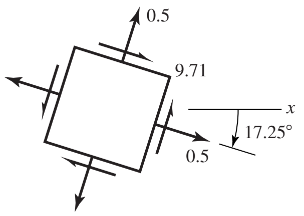
$$\sigma_1 = 0.5 + 9.71 = 10.21$$

$$\sigma_2 = 0.5 - 9.71 = -9.21$$

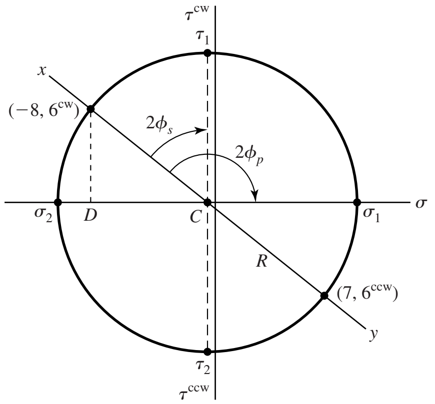


$$\phi_p = \frac{1}{2}\tan^{-1}\frac{8}{5.5} = 27.75^\circ \text{ ccw}$$

$$\tau_1 = R = 9.71, \quad \phi_s = 45^\circ - 27.75^\circ = 17.25^\circ \text{ cw}$$



(c)



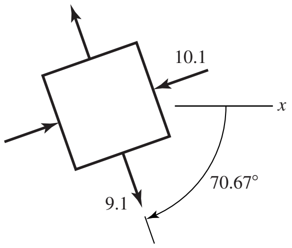
$$C = \frac{-8+7}{2} = -0.5$$

$$CD = \frac{8+7}{2} = 7.5$$

$$R = \sqrt{7.5^2 + 6^2} = 9.60$$

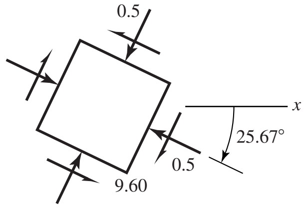
$$\sigma_1 = 9.60 - 0.5 = 9.10$$

$$\sigma_2 = -0.5 - 9.6 = -10.1$$

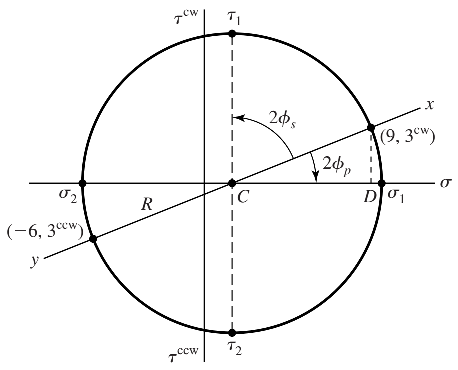


$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{7.5}{6}\right] = 70.67^\circ \text{ cw}$$

$$\tau_1 = R = 9.60, \quad \phi_s = 70.67^\circ - 45^\circ = 25.67^\circ \text{ cw}$$



(d)



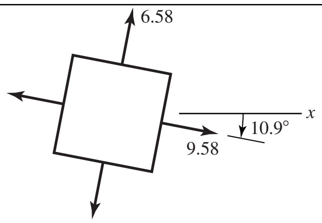
$$C = \frac{9-6}{2} = 1.5$$

$$CD = \frac{9+6}{2} = 7.5$$

$$R = \sqrt{7.5^2 + 3^2} = 8.078$$

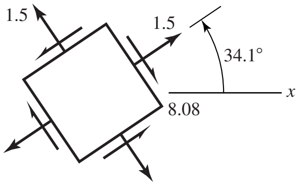
$$\sigma_1 = 1.5 + 8.078 = 9.58$$

$$\sigma_2 = 1.5 - 8.078 = -6.58$$



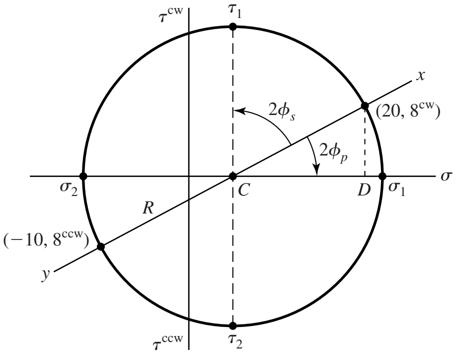
$$\phi_p = \frac{1}{2}\tan^{-1}\frac{3}{7.5} = 10.9^\circ \text{ cw}$$

$$\tau_1 = R = 8.078, \quad \phi_s = 45^\circ - 10.9^\circ = 34.1^\circ \text{ ccw}$$



3-10

(a)



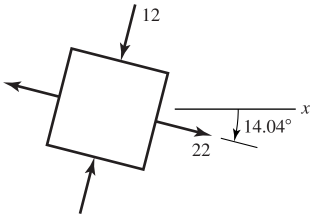
$$C = \frac{20-10}{2} = 5$$

$$CD = \frac{20+10}{2} = 15$$

$$R = \sqrt{15^2 + 8^2} = 17$$

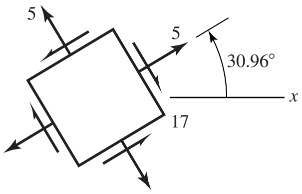
$$\sigma_1 = 5 + 17 = 22$$

$$\sigma_2 = 5 - 17 = -12$$

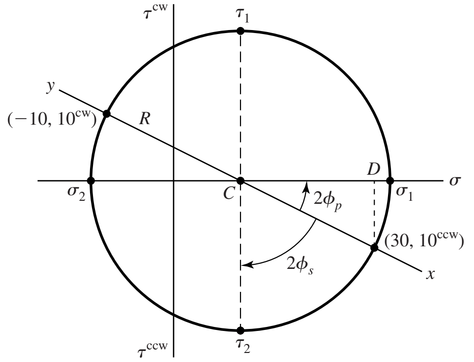


$$\phi_p = \frac{1}{2}\tan^{-1}\frac{8}{15} = 14.04^\circ \text{ cw}$$

$$\tau_1 = R = 17, \quad \phi_s = 45^\circ - 14.04^\circ = 30.96^\circ \text{ ccw}$$



(b)



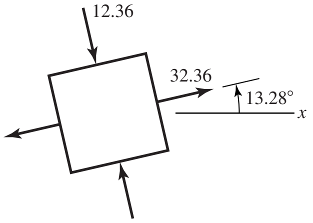
$$C = \frac{30 - 10}{2} = 10$$

$$CD = \frac{30 + 10}{2} = 20$$

$$R = \sqrt{20^2 + 10^2} = 22.36$$

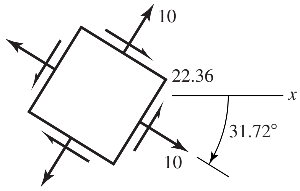
$$\sigma_1 = 10 + 22.36 = 32.36$$

$$\sigma_2 = 10 - 22.36 = -12.36$$

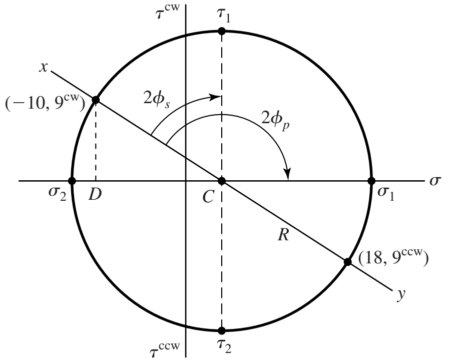


$$\phi_p = \frac{1}{2}\tan^{-1}\frac{10}{20} = 13.28^\circ \text{ ccw}$$

$$\tau_1 = R = 22.36, \quad \phi_s = 45^\circ - 13.28^\circ = 31.72^\circ \text{ cw}$$



(c)



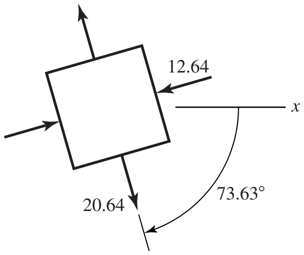
$$C = \frac{-10 + 18}{2} = 4$$

$$CD = \frac{10 + 18}{2} = 14$$

$$R = \sqrt{14^2 + 9^2} = 16.64$$

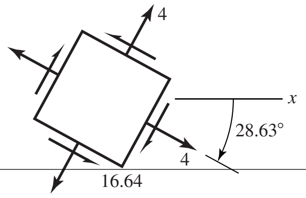
$$\sigma_1 = 4 + 16.64 = 20.64$$

$$\sigma_2 = 4 - 16.64 = -12.64$$

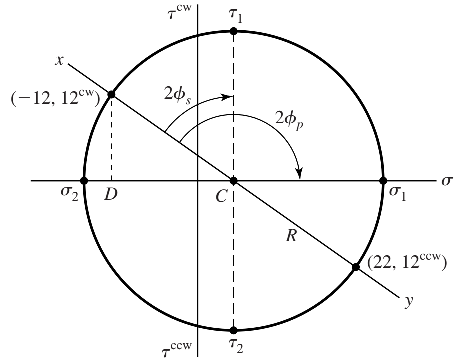


$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{14}{9}\right] = 73.63^\circ \text{ cw}$$

$$\tau_1 = R = 16.64, \quad \phi_s = 73.63^\circ - 45^\circ = 28.63^\circ \text{ cw}$$



(d)



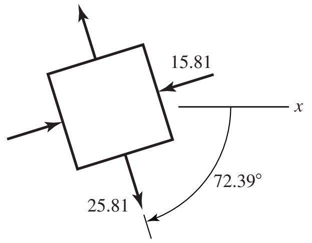
$$C = \frac{-12 + 22}{2} = 5$$

$$CD = \frac{12 + 22}{2} = 17$$

$$R = \sqrt{17^2 + 12^2} = 20.81$$

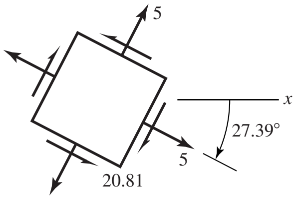
$$\sigma_1 = 5 + 20.81 = 25.81$$

$$\sigma_2 = 5 - 20.81 = -15.81$$



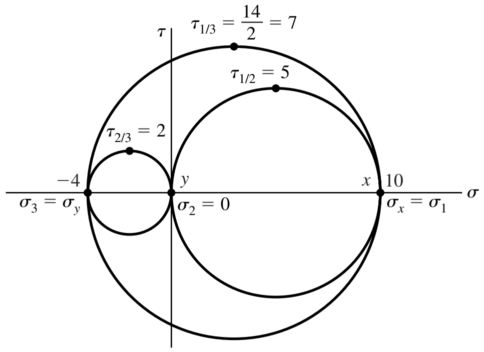
$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{17}{12}\right] = 72.39^\circ \text{ cw}$$

$$\tau_1 = R = 20.81, \quad \phi_s = 72.39^\circ - 45^\circ = 27.39^\circ \text{ cw}$$

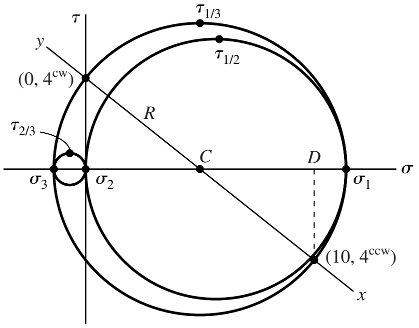


3-11

(a)



(b)



$$C = \frac{0 + 10}{2} = 5$$

$$CD = \frac{10 - 0}{2} = 5$$

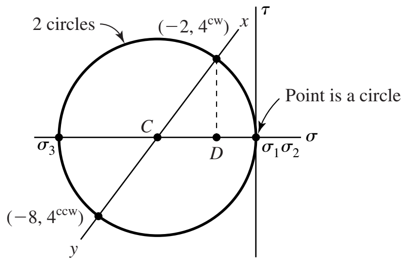
$$R = \sqrt{5^2 + 4^2} = 6.40$$

$$\sigma_1 = 5 + 6.40 = 11.40$$

$$\sigma_2 = 0, \quad \sigma_3 = 5 - 6.40 = -1.40$$

$$\tau_{1/3} = R = 6.40, \quad \tau_{1/2} = \frac{11.40}{2} = 5.70, \quad \tau_{2/3} = \frac{1.40}{2} = 0.70$$

(c)



$$C = \frac{-2-8}{2} = -5$$

$$CD = \frac{8-2}{2} = 3$$

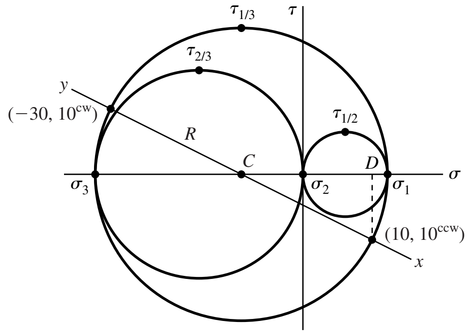
$$R = \sqrt{3^2 + 4^2} = 5$$

$$\sigma_1 = -5 + 5 = 0, \quad \sigma_2 = 0$$

$$\sigma_3 = -5 - 5 = -10$$

$$\tau_{1/3} = \frac{10}{2} = 5, \quad \tau_{1/2} = 0, \quad \tau_{2/3} = 5$$

(d)



$$C = \frac{10-30}{2} = -10$$

$$CD = \frac{10+30}{2} = 20$$

$$R = \sqrt{20^2 + 10^2} = 22.36$$

$$\sigma_1 = -10 + 22.36 = 12.36$$

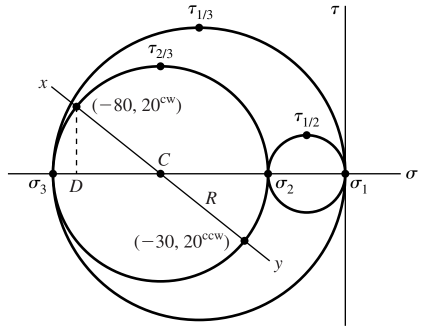
$$\sigma_2 = 0$$

$$\sigma_3 = -10 - 22.36 = -32.36$$

$$\tau_{1/3} = 22.36, \quad \tau_{1/2} = \frac{12.36}{2} = 6.18, \quad \tau_{2/3} = \frac{32.36}{2} = 16.18$$

3-12

(a)



$$C = \frac{-80-30}{2} = -55$$

$$CD = \frac{80-30}{2} = 25$$

$$R = \sqrt{25^2 + 20^2} = 32.02$$

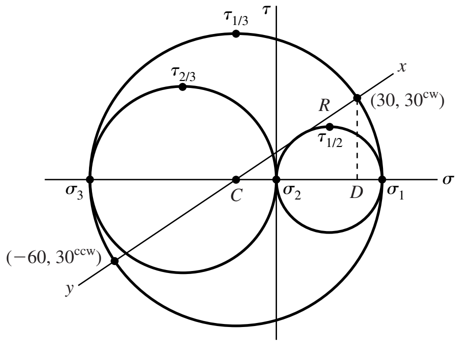
$$\sigma_1 = 0$$

$$\sigma_2 = -55 + 32.02 = -22.98 = -23.0$$

$$\sigma_3 = -55 - 32.0 = -87.0$$

$$\tau_{1/2} = \frac{23}{2} = 11.5, \quad \tau_{2/3} = 32.0, \quad \tau_{1/3} = \frac{87}{2} = 43.5$$

(b)



$$C = \frac{30 - 60}{2} = -15$$

$$CD = \frac{60 + 30}{2} = 45$$

$$R = \sqrt{45^2 + 30^2} = 54.1$$

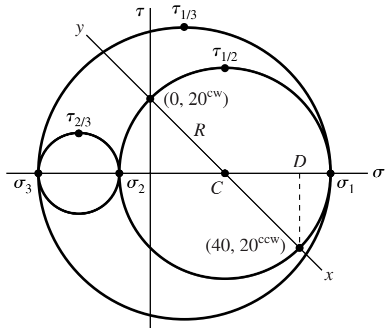
$$\sigma_1 = -15 + 54.1 = 39.1$$

$$\sigma_2 = 0$$

$$\sigma_3 = -15 - 54.1 = -69.1$$

$$\tau_{1/3} = \frac{39.1 + 69.1}{2} = 54.1, \quad \tau_{1/2} = \frac{39.1}{2} = 19.6, \quad \tau_{2/3} = \frac{69.1}{2} = 34.6$$

(c)



$$C = \frac{40 + 0}{2} = 20$$

$$CD = \frac{40 - 0}{2} = 20$$

$$R = \sqrt{20^2 + 20^2} = 28.3$$

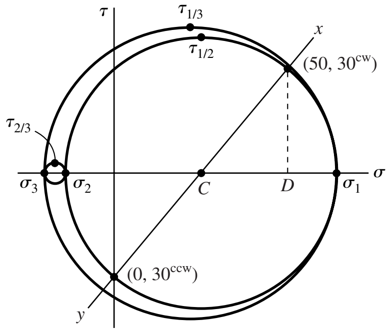
$$\sigma_1 = 20 + 28.3 = 48.3$$

$$\sigma_2 = 20 - 28.3 = -8.3$$

$$\sigma_3 = \sigma_z = -30$$

$$\tau_{1/3} = \frac{48.3 + 30}{2} = 39.1, \quad \tau_{1/2} = 28.3, \quad \tau_{2/3} = \frac{30 - 8.3}{2} = 10.9$$

(d)



$$C = \frac{50}{2} = 25$$

$$CD = \frac{50}{2} = 25$$

$$R = \sqrt{25^2 + 30^2} = 39.1$$

$$\sigma_1 = 25 + 39.1 = 64.1$$

$$\sigma_2 = 25 - 39.1 = -14.1$$

$$\sigma_3 = \sigma_z = -20$$

$$\tau_{1/3} = \frac{64.1 + 20}{2} = 42.1, \quad \tau_{1/2} = 39.1, \quad \tau_{2/3} = \frac{20 - 14.1}{2} = 2.95$$

3-13

$$\sigma = \frac{F}{A} = \frac{2000}{(\pi/4)(0.5^2)} = 10\,190 \text{ psi} = 10.19 \text{ kpsi} \quad \textit{Ans.}$$

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 10\,190\frac{72}{30(10^6)} = 0.024\,46 \text{ in} \quad \textit{Ans.}$$

$$\epsilon_1 = \frac{\delta}{L} = \frac{0.024\,46}{72} = 340(10^{-6}) = 340\mu \quad \textit{Ans.}$$

From Table A-5, $\nu = 0.292$

$$\epsilon_2 = -\nu\epsilon_1 = -0.292(340) = -99.3\mu \quad \textit{Ans.}$$

$$\Delta d = \epsilon_2 d = -99.3(10^{-6})(0.5) = -49.6(10^{-6}) \text{ in} \quad \textit{Ans.}$$

3-14 From Table A-5, $E = 71.7$ GPa

$$\delta = \sigma\frac{L}{E} = 135(10^6)\frac{3}{71.7(10^9)} = 5.65(10^{-3}) \text{ m} = 5.65 \text{ mm} \quad \textit{Ans.}$$

3-15 With $\sigma_z = 0$, solve the first two equations of Eq. (3-19) simultaneously. Place E on the left-hand side of both equations, and using Cramer's rule,

$$\sigma_x = \frac{\begin{vmatrix} E\epsilon_x & -\nu \\ E\epsilon_y & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\nu \\ -\nu & 1 \end{vmatrix}} = \frac{E\epsilon_x + \nu E\epsilon_y}{1-\nu^2} = \frac{E(\epsilon_x + \nu\epsilon_y)}{1-\nu^2}$$

Likewise,

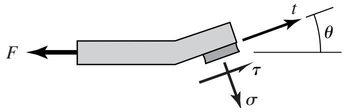
$$\sigma_y = \frac{E(\epsilon_y + \nu\epsilon_x)}{1-\nu^2}$$

From Table A-5, $E = 207$ GPa and $\nu = 0.292$. Thus,

$$\sigma_x = \frac{E(\epsilon_x + \nu\epsilon_y)}{1-\nu^2} = \frac{207(10^9)[0.0021 + 0.292(-0.000\,67)]}{1-0.292^2}(10^{-6}) = 431 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_y = \frac{207(10^9)[-0.000\,67 + 0.292(0.0021)]}{1-0.292^2}(10^{-6}) = -12.9 \text{ MPa} \quad \textit{Ans.}$$

3-16 The engineer has assumed the stress to be uniform. That is,



$$\sum F_t = -F\cos\theta + \tau A = 0 \quad \Rightarrow \quad \tau = \frac{F}{A}\cos\theta$$

When failure occurs in shear

$$S_{su} = \frac{F}{A}\cos\theta$$

The uniform stress assumption is common practice but is not exact. If interested in the details, see p. 570 of 6th edition.

3-17 From Eq. (3-15)

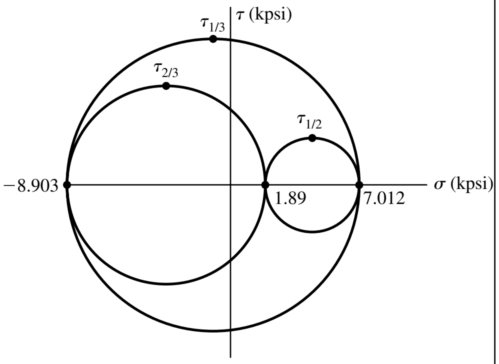
$$\sigma^3 - (-2+6-4)\sigma^2 + [-2(6) + (-2)(-4) + 6(-4) - 3^2 - 2^2 - (-5)^2]\sigma$$
$$- [-2(6)(-4) + 2(3)(2)(-5) - (-2)(2)^2 - 6(-5)^2 - (-4)(3)^2] = 0$$
$$\sigma^3 - 66\sigma + 118 = 0$$

Roots are: 7.012, 1.89, −8.903 kpsi *Ans.*

$$\tau_{1/2} = \frac{7.012 - 1.89}{2} = 2.56 \text{ kpsi}$$

$$\tau_{2/3} = \frac{8.903 + 1.89}{2} = 5.40 \text{ kpsi}$$

$$\tau_{\max} = \tau_{1/3} = \frac{8.903 + 7.012}{2} = 7.96 \text{ kpsi} \quad \textit{Ans.}$$



Note: For Probs. 3-17 to 3-19, one can also find the eigenvalues of the matrix

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z \end{bmatrix}$$

for the principal stresses

3-18 From Eq. (3-15)

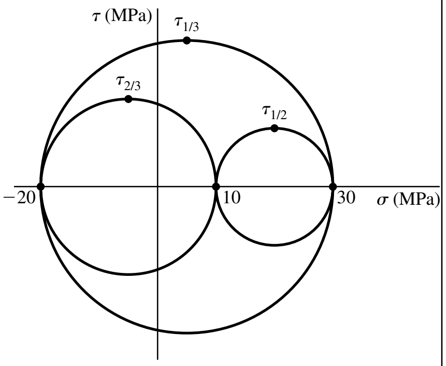
$$\sigma^3 - (10+0+10)\sigma^2 + \left[10(0) + 10(10) + 0(10) - 20^2 - \left(-10\sqrt{2}\right)^2 - 0^2\right]\sigma$$
$$- \left[10(0)(10) + 2(20)\left(-10\sqrt{2}\right)(0) - 10\left(-10\sqrt{2}\right)^2 - 0(0)^2 - 10(20)^2\right] = 0$$
$$\sigma^3 - 20\sigma^2 - 500\sigma + 6000 = 0$$

Roots are: 30, 10, −20 MPa *Ans.*

$$\tau_{1/2} = \frac{30 - 10}{2} = 10 \text{ MPa}$$

$$\tau_{2/3} = \frac{10 + 20}{2} = 15 \text{ MPa}$$

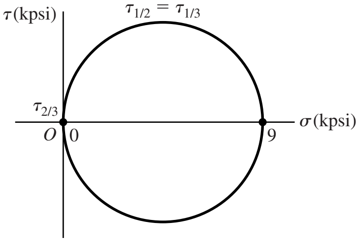
$$\tau_{\max} = \tau_{1/3} = \frac{30 + 20}{2} = 25 \text{ MPa} \quad \textit{Ans.}$$



3-19 From Eq. (3-15)

$$\sigma^3 - (1+4+4)\sigma^2 + [1(4) + 1(4) + 4(4) - 2^2 - (-4)^2 - (-2)^2]\sigma$$
$$-[1(4)(4) + 2(2)(-4)(-2) - 1(-4)^2 - 4(-2)^2 - 4(2)^2] = 0$$
$$\sigma^3 - 9\sigma^2 = 0$$

Roots are: 9, 0, 0 kpsi



$$\tau_{2/3} = 0, \quad \tau_{1/2} = \tau_{1/3} = \tau_{\max} = \frac{9}{2} = 4.5 \text{ kpsi} \quad \textit{Ans.}$$

3-20

(a) $R_1 = \dfrac{c}{l}F \quad M_{\max} = R_1 a = \dfrac{ac}{l}F$

$$\sigma = \frac{6M}{bh^2} = \frac{6}{bh^2}\frac{ac}{l}F \quad \Rightarrow \quad F = \frac{\sigma bh^2 l}{6ac} \quad \textit{Ans.}$$

(b) $\dfrac{F_m}{F} = \dfrac{(\sigma_m/\sigma)(b_m/b)\,(h_m/h)^2\,(l_m/l)}{(a_m/a)\,(c_m/c)} = \dfrac{1(s)(s)^2(s)}{(s)(s)} = s^2 \quad \textit{Ans.}$

For equal stress, the model load varies by the square of the scale factor.

3-21

$$R_1 = \frac{wl}{2}, \quad M_{\max}|_{x=l/2} = \frac{w}{2}\frac{l}{2}\left(l - \frac{l}{2}\right) = \frac{wl^2}{8}$$

$$\sigma = \frac{6M}{bh^2} = \frac{6}{bh^2}\frac{wl^2}{8} = \frac{3Wl}{4bh^2} \quad \Rightarrow \quad W = \frac{4}{3}\frac{\sigma bh^2}{l} \quad \textit{Ans.}$$

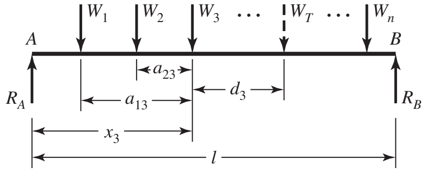
$$\frac{W_m}{W} = \frac{(\sigma_m/\sigma)(b_m/b)\,(h_m/h)^2}{l_m/l} = \frac{1(s)(s)^2}{s} = s^2 \quad \textit{Ans.}$$

$$\frac{w_m l_m}{wl} = s^2 \quad \Rightarrow \quad \frac{w_m}{w} = \frac{s^2}{s} = s \quad \textit{Ans.}$$

For equal stress, the model load w varies linearily with the scale factor.

3-22

(a) Can solve by iteration *or* derive equations for the general case.



Find maximum moment under wheel W_3

$W_T = \sum W$ at centroid of W's

$$R_A = \frac{l - x_3 - d_3}{l} W_T$$

Under wheel 3

$$M_3 = R_A x_3 - W_1 a_{13} - W_2 a_{23} = \frac{(l - x_3 - d_3)}{l} W_T x_3 - W_1 a_{13} - W_2 a_{23}$$

For maximum, $\dfrac{dM_3}{dx_3} = 0 = (l - d_3 - 2x_3)\dfrac{W_T}{l} \quad \Rightarrow \quad x_3 = \dfrac{l - d_3}{2}$

substitute into M, $\Rightarrow \quad M_3 = \dfrac{(l - d_3)^2}{4l} W_T - W_1 a_{13} - W_2 a_{23}$

This means the midpoint of d_3 intersects the midpoint of the beam

For wheel i $\quad x_i = \dfrac{l - d_i}{2}, \quad M_i = \dfrac{(l - d_i)^2}{4l} W_T - \displaystyle\sum_{j=1}^{i-1} W_j a_{ji}$

Note for wheel 1: $\Sigma W_j a_{ji} = 0$

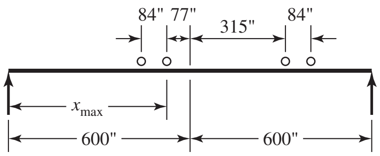
$$W_T = 104.4, \quad W_1 = W_2 = W_3 = W_4 = \frac{104.4}{4} = 26.1\,\text{kip}$$

Wheel 1: $\quad d_1 = \dfrac{476}{2} = 238\,\text{in}, \quad M_1 = \dfrac{(1200 - 238)^2}{4(1200)}(104.4) = 20\,128\,\text{kip}\cdot\text{in}$

Wheel 2: $\quad d_2 = 238 - 84 = 154\,\text{in}$

$$M_2 = \frac{(1200 - 154)^2}{4(1200)}(104.4) - 26.1(84) = 21\,605\,\text{kip}\cdot\text{in} = M_{\max}$$

Check if all of the wheels are on the rail



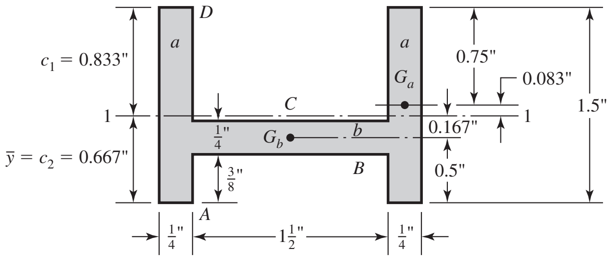
(b) $x_{\max} = 600 - 77 = 523\,\text{in}$

(c) See above sketch.

(d) inner axles

3-23

(a)



$$A_a = A_b = 0.25(1.5) = 0.375\,\text{in}^2$$

$$A = 3(0.375) = 1.125\,\text{in}^2$$

$$\bar{y} = \frac{2(0.375)(0.75) + 0.375(0.5)}{1.125} = 0.667\text{ in}$$

$$I_a = \frac{0.25(1.5)^3}{12} = 0.0703\,\text{in}^4$$

$$I_b = \frac{1.5(0.25)^3}{12} = 0.001\,95\,\text{in}^4$$

$$I_1 = 2[0.0703 + 0.375(0.083)^2] + [0.001\ 95 + 0.375(0.167)^2] = 0.158\,\text{in}^4 \quad \textit{Ans.}$$

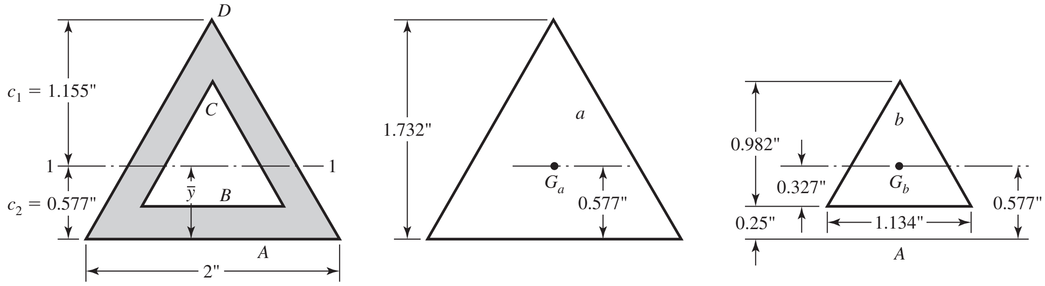
$$\sigma_A = \frac{10\,000(0.667)}{0.158} = 42(10)^3\text{ psi} \quad \textit{Ans.}$$

$$\sigma_B = \frac{10\,000(0.667 - 0.375)}{0.158} = 18.5(10)^3\text{ psi} \quad \textit{Ans.}$$

$$\sigma_C = \frac{10\,000(0.167 - 0.125)}{0.158} = 2.7(10)^3\text{ psi} \quad \textit{Ans.}$$

$$\sigma_D = -\frac{10\,000(0.833)}{0.158} = -52.7(10)^3\text{ psi} \quad \textit{Ans.}$$

(b)



Here we treat the hole as a negative area.

$$A_a = 1.732\text{ in}^2$$

$$A_b = 1.134\left(\frac{0.982}{2}\right) = 0.557\,\text{in}^2$$

$$A = 1.732 - 0.557 = 1.175\,\text{in}^2$$

$$\bar{y} = \frac{1.732(0.577) - 0.557(0.577)}{1.175} = 0.577 \text{ in} \quad \textit{Ans.}$$

$$I_a = \frac{bh^3}{36} = \frac{2(1.732)^3}{36} = 0.289 \text{ in}^4$$

$$I_b = \frac{1.134(0.982)^3}{36} = 0.0298\,\text{in}^4$$

$$I_1 = I_a - I_b = 0.289 - 0.0298 = 0.259\,\text{in}^4 \quad \textit{Ans.}$$

because the centroids are coincident.

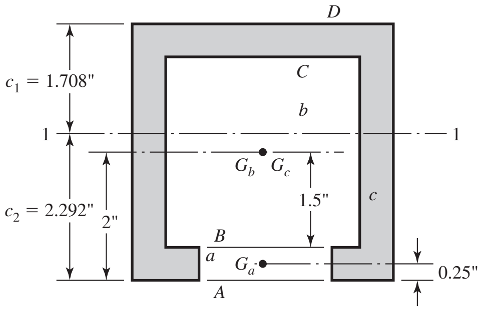
$$\sigma_A = \frac{10\,000(0.577)}{0.259} = 22.3(10)^3 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_B = \frac{10\,000(0.327)}{0.259} = 12.6(10)^3 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_C = -\frac{10\,000(0.982 - 0.327)}{0.259} = -25.3(10)^3 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_D = -\frac{10\,000(1.155)}{0.259} = -44.6(10)^3 \text{ psi} \quad \textit{Ans.}$$

(c) Use two negative areas.



$$A_a = 1\,\text{in}^2, \quad A_b = 9 \text{ in}^2, \quad A_c = 16\,\text{in}^2, \quad A = 16 - 9 - 1 = 6\,\text{in}^2;$$

$$\bar{y}_a = 0.25\,\text{in}, \quad \bar{y}_b = 2.0\,\text{in}, \quad \bar{y}_c = 2\,\text{in}$$

$$\bar{y} = \frac{16(2) - 9(2) - 1(0.25)}{6} = 2.292\,\text{in} \quad \textit{Ans.}$$

$$c_1 = 4 - 2.292 = 1.708\,\text{in}$$

$$I_a = \frac{2(0.5)^3}{12} = 0.020\,83\,\text{in}^4$$

$$I_b = \frac{3(3)^3}{12} = 6.75\,\text{in}^4$$

$$I_c = \frac{4(4)^3}{12} = 21.333\,\text{in}^4$$

$$I_1 = [21.333 + 16(0.292)^2] - [6.75 + 9(0.292)^2]$$
$$- [0.020\,83 + 1(2.292 - 0.25)^2]$$
$$= 10.99\text{ in}^4 \quad \textit{Ans.}$$

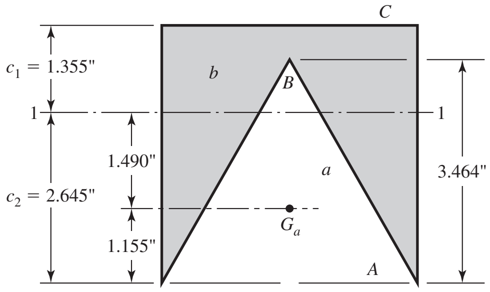
$$\sigma_A = \frac{10\,000(2.292)}{10.99} = 2086\text{ psi} \quad \textit{Ans.}$$

$$\sigma_B = \frac{10\,000(2.292 - 0.5)}{10.99} = 1631\text{ psi} \quad \textit{Ans.}$$

$$\sigma_C = -\frac{10\,000(1.708 - 0.5)}{10.99} = -1099\text{ psi} \quad \textit{Ans.}$$

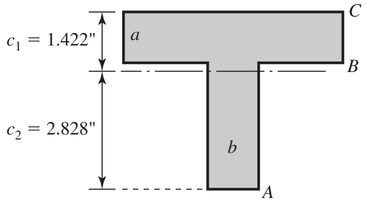
$$\sigma_D = -\frac{10\,000(1.708)}{10.99} = -1554\text{ psi} \quad \textit{Ans.}$$

(d) Use *a* as a negative area.



$$A_a = 6.928\text{ in}^2, \quad A_b = 16\text{ in}^2, \quad A = 9.072\text{ in}^2;$$
$$\bar{y}_a = 1.155\text{ in}, \quad \bar{y}_b = 2\text{ in}$$
$$\bar{y} = \frac{2(16) - 1.155(6.928)}{9.072} = 2.645\text{ in} \quad \textit{Ans.}$$
$$c_1 = 4 - 2.645 = 1.355\text{ in}$$
$$I_a = \frac{bh^3}{36} = \frac{4(3.464)^3}{36} = 4.618\text{ in}^4$$
$$I_b = \frac{4(4)^3}{12} = 21.33\text{ in}^4$$
$$I_1 = [21.33 + 16(0.645)^2] - [4.618 + 6.928(1.490)^2]$$
$$= 7.99\text{ in}^4 \quad \textit{Ans.}$$
$$\sigma_A = \frac{10\,000(2.645)}{7.99} = 3310\text{ psi} \quad \textit{Ans.}$$
$$\sigma_B = -\frac{10\,000(3.464 - 2.645)}{7.99} = -1025\text{ psi} \quad \textit{Ans.}$$
$$\sigma_C = -\frac{10\,000(1.355)}{7.99} = -1696\text{ psi} \quad \textit{Ans.}$$

(e)



$$A_a = 6(1.25) = 7.5\text{ in}^2$$

$$A_b = 3(1.5) = 4.5\text{ in}^2$$

$$A = A_c + A_b = 12\text{ in}^2$$

$$\bar{y} = \frac{3.625(7.5) + 1.5(4.5)}{12} = 2.828\text{ in} \quad \textit{Ans.}$$

$$I = \frac{1}{12}(6)(1.25)^3 + 7.5(3.625 - 2.828)^2 + \frac{1}{12}(1.5)(3)^3 + 4.5(2.828 - 1.5)^2$$

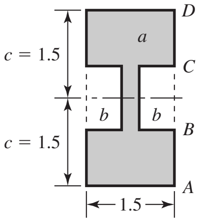
$$= 17.05\text{ in}^4 \quad \textit{Ans.}$$

$$\sigma_A = \frac{10\,000(2.828)}{17.05} = 1659\text{ psi} \quad \textit{Ans.}$$

$$\sigma_B = -\frac{10\,000(3 - 2.828)}{17.05} = -101\text{ psi} \quad \textit{Ans.}$$

$$\sigma_C = -\frac{10\,000(1.422)}{17.05} = -834\text{ psi} \quad \textit{Ans.}$$

(f)



Let a = total area

$$A = 1.5(3) - 1(1.25) = 3.25\text{ in}^2$$

$$I = I_a - 2I_b = \frac{1}{12}(1.5)(3)^3 - \frac{1}{12}(1.25)(1)^3$$

$$= 3.271\text{ in}^4 \quad \textit{Ans.}$$

$$\sigma_A = \frac{10\,000(1.5)}{3.271} = 4586\text{ psi}, \quad \sigma_D = -4586\text{ psi}$$

Ans.

$$\sigma_B = \frac{10\,000(0.5)}{3.271} = 1529\text{ psi}, \quad \sigma_C = -1529\text{ psi}$$

3-24

(a) The moment is maximum and constant between A and B

$$M = -50(20) = -1000\text{ lbf}\cdot\text{in}, \quad I = \frac{1}{12}(0.5)(2)^3 = 0.3333\text{ in}^4$$

$$\rho = \left|\frac{EI}{M}\right| = \frac{1.6(10^6)(0.3333)}{1000} = 533.3\text{ in}$$

$$(x, y) = (30, -533.3)\text{ in} \quad \textit{Ans.}$$

(b) The moment is maximum and constant between A and B

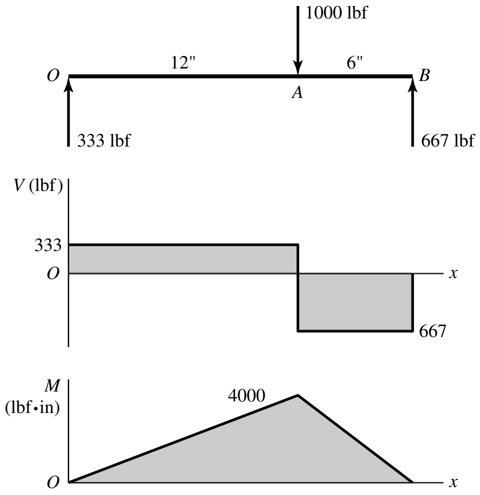
$$M = 50(5) = 250\text{ lbf}\cdot\text{in}, \quad I = 0.3333\text{ in}^4$$

$$\rho = \frac{1.6(10^6)(0.3333)}{250} = 2133\text{ in} \quad \textit{Ans.}$$

$$(x, y) = (20, 2133)\text{ in} \quad \textit{Ans.}$$

3-25

(a)



$$I = \frac{1}{12}(0.75)(1.5)^3 = 0.2109\,\text{in}^4$$

$$A = 0.75(1.5) = 1.125\,\text{in}$$

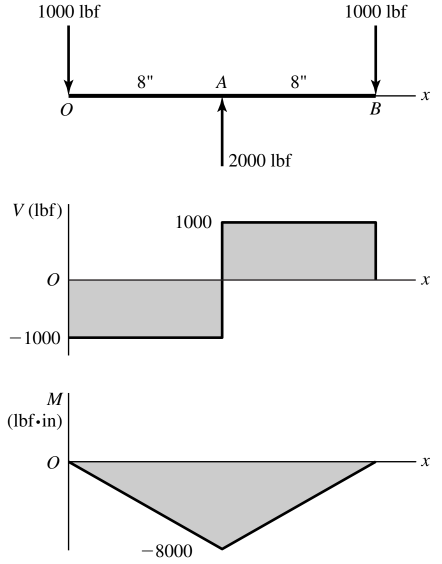
M_{max} is at A. At the bottom of the section,

$$\sigma_{\text{max}} = \frac{Mc}{I} = \frac{4000(0.75)}{0.2109} = 14\,225\,\text{psi} \quad \textit{Ans.}$$

Due to V, τ_{max} constant is between A and B at $y = 0$

$$\tau_{\text{max}} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{667}{1.125} = 889\,\text{psi} \quad \textit{Ans.}$$

(b)



$$I = \frac{1}{12}(1)(2)^3 = 0.6667\,\text{in}^4$$

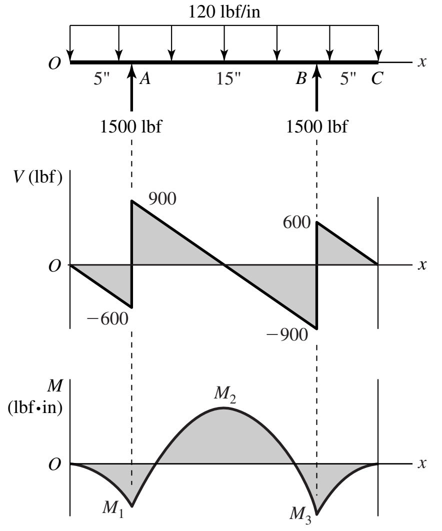
M_{max} is at A at the top of the beam

$$\sigma_{\text{max}} = \frac{8000(1)}{0.6667} = 12\,000\,\text{psi} \quad \textit{Ans.}$$

$$|V_{\text{max}}| = 1000\,\text{lbf} \quad \text{from } O \text{ to } B \text{ at } y = 0$$

$$\tau_{\text{max}} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{1000}{(2)(1)} = 750\,\text{psi} \quad \textit{Ans.}$$

(c)



$$I = \frac{1}{12}(0.75)(2)^3 = 0.5\,\text{in}^4$$

$$M_1 = -\frac{1}{2}600(5) = -1500\ \text{lbf}\cdot\text{in} = M_3$$

$$M_2 = -1500 + \frac{1}{2}(900)(7.5) = 1875\ \text{lbf}\cdot\text{in}$$

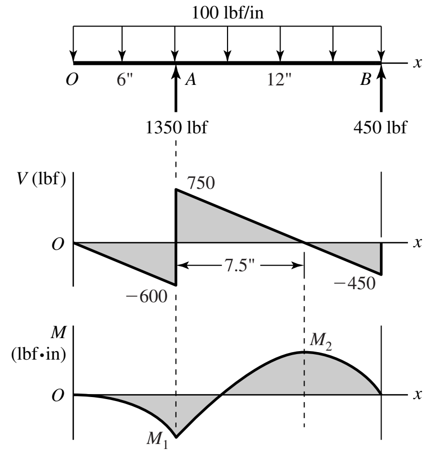
M_{max} is at span center. At the bottom of the beam,

$$\sigma_{\text{max}} = \frac{1875(1)}{0.5} = 3750\,\text{psi} \quad \textit{Ans.}$$

At A and B at $y = 0$

$$\tau_{\text{max}} = \frac{3}{2}\frac{900}{(0.75)(2)} = 900\,\text{psi} \quad \textit{Ans.}$$

(d)



$$I = \frac{1}{12}(1)(2)^3 = 0.6667\,\text{in}^4$$

$$M_1 = -\frac{600}{2}(6) = -1800\ \text{lbf}\cdot\text{in}$$

$$M_2 = -1800 + \frac{1}{2}750(7.5) = 1013\ \text{lbf}\cdot\text{in}$$

At A, top of beam

$$\sigma_{\max} = \frac{1800(1)}{0.6667} = 2700\,\text{psi} \quad \textit{Ans.}$$

At A, $y = 0$

$$\tau_{\max} = \frac{3}{2}\frac{750}{(2)(1)} = 563\,\text{psi} \quad \textit{Ans.}$$

3-26

$$M_{\max} = \frac{wl^2}{8} \quad \Rightarrow \quad \sigma_{\max} = \frac{wl^2c}{8I} \quad \Rightarrow \quad w = \frac{8\sigma I}{cl^2}$$

(a) $l = 12(12) = 144\,\text{in}$, $I = (1/12)(1.5)(9.5)^3 = 107.2\,\text{in}^4$

$$w = \frac{8(1200)(107.2)}{4.75(144^2)} = 10.4\ \text{lbf/in} \quad \textit{Ans.}$$

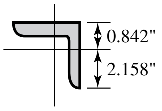
(b) $l = 48\,\text{in}$, $I = (\pi/64)(2^4 - 1.25^4) = 0.6656\,\text{in}^4$

$$w = \frac{8(12)(10^3)(0.6656)}{1(48)^2} = 27.7\ \text{lbf/in} \quad \textit{Ans.}$$

(c) $l = 48\,\text{in}$, $I \doteq (1/12)(2)(3^3) - (1/12)(1.625)(2.625^3) = 2.051\,\text{in}^4$

$$w = \frac{8(12)(10^3)(2.051)}{1.5(48)^2} = 57.0\ \text{lbf/in} \quad \textit{Ans.}$$

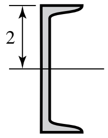
(d) $l = 72$ in; Table A-6, $I = 2(1.24) = 2.48\,\text{in}^4$



$$c_{\max} = 2.158"$$

$$w = \frac{8(12)(10^3)(2.48)}{2.158(72)^2} = 21.3\ \text{lbf/in} \quad \textit{Ans.}$$

(e) $l = 72$ in; Table A-7, $I = 3.85\,\text{in}^4$

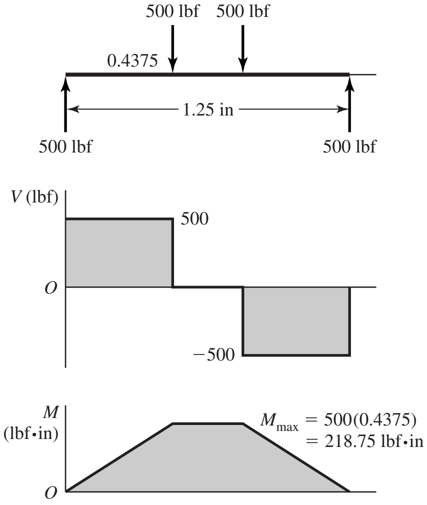


$$w = \frac{8(12)(10^3)(3.85)}{2(72^2)} = 35.6\ \text{lbf/in} \quad \textit{Ans.}$$

(f) $l = 72\,\text{in}$, $I = (1/12)(1)(4^3) = 5.333\,\text{in}^4$

$$w = \frac{8(12)(10^3)(5.333)}{(2)(72)^2} = 49.4\ \text{lbf/in} \quad \textit{Ans.}$$

3-27 **(a)** Model (c)



$$I = \frac{\pi}{64}(0.5^4) = 3.068(10^{-3})\ \text{in}^4$$

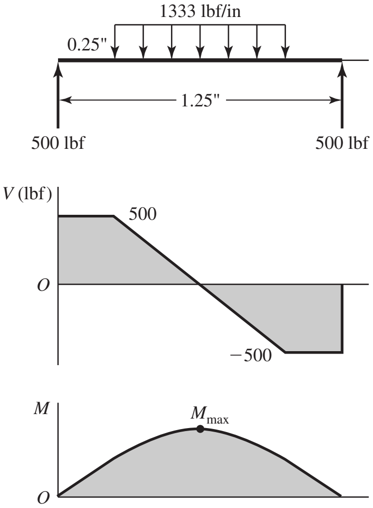
$$A = \frac{\pi}{4}(0.5^2) = 0.1963\ \text{in}^2$$

$$\sigma = \frac{Mc}{I} = \frac{218.75(0.25)}{3.068(10^{-3})}$$

$$= 17\,825\ \text{psi} = 17.8\ \text{kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{500}{0.1963} = 3400\ \text{psi} \quad \textit{Ans.}$$

(b) Model (d)



$$M_{\max} = 500(0.25) + \frac{1}{2}(500)(0.375)$$

$$= 218.75\ \text{lbf} \cdot \text{in}$$

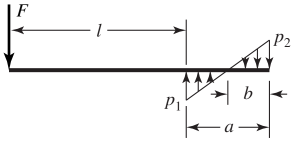
$$V_{\max} = 500\ \text{lbf}$$

Same M and V

$\therefore \sigma = 17.8$ kpsi *Ans.*

$\tau_{\max} = 3400$ psi *Ans.*

3-28



$$q = -F\langle x\rangle^{-1} + p_1\langle x-l\rangle^0 - \frac{p_1+p_2}{a}\langle x-l\rangle^1 + \text{ terms for } x > l+a$$

$$V = -F + p_1\langle x-l\rangle^1 - \frac{p_1+p_2}{2a}\langle x-l\rangle^2 + \text{ terms for } x > l+a$$

$$M = -Fx + \frac{p_1}{2}\langle x-l\rangle^2 - \frac{p_1+p_2}{6a}\langle x-l\rangle^3 + \text{ terms for } x > l+a$$

At $x = (l+a)^+$, $V = M = 0$, terms for $x > l+a = 0$

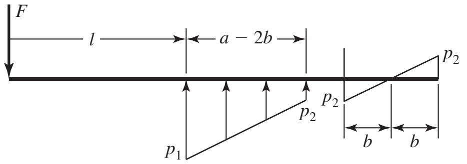
$$-F + p_1 a - \frac{p_1+p_2}{2a}a^2 = 0 \quad \Rightarrow \quad p_1 - p_2 = \frac{2F}{a} \tag{1}$$

$$-F(l+a)+\frac{p_1a^2}{2}-\frac{p_1+p_2}{6a}a^3=0 \quad \Rightarrow \quad 2p_1-p_2=\frac{6F(l+a)}{a^2} \tag{2}$$

From (1) and (2) $$p_1=\frac{2F}{a^2}(3l+2a), \quad p_2=\frac{2F}{a^2}(3l+a) \tag{3}$$

From similar triangles $$\frac{b}{p_2}=\frac{a}{p_1+p_2} \quad \Rightarrow \quad b=\frac{ap_2}{p_1+p_2} \tag{4}$$

$M_{\max}$ occurs where $V=0$



$$x_{\max}=l+a-2b$$

$$M_{\max}=-F(l+a-2b)+\frac{p_1}{2}(a-2b)^2-\frac{p_1+p_2}{6a}(a-2b)^3$$

$$=-Fl-F(a-2b)+\frac{p_1}{2}(a-2b)^2-\frac{p_1+p_2}{6a}(a-2b)^3$$

Normally $M_{\max}=-Fl$
The fractional increase in the magnitude is

$$\Delta=\frac{F(a-2b)-(p_1/2)(a-2b)^2-[(p_1+p_2)/6a](a-2b)^3}{Fl} \tag{5}$$

For example, consider $F=1500$ lbf, $a=1.2$ in, $l=1.5$ in

(3) $$p_1=\frac{2(1500)}{1.2^2}[3(1.5)+2(1.2)]=14\,375 \text{ lbf/in}$$

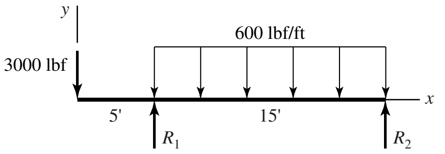
$$p_2=\frac{2(1500)}{1.2^2}[3(1.5)+1.2]=11\,875 \text{ lbf/in}$$

(4) $$b=1.2(11\,875)/(14\,375+11\,875)=0.5429 \text{ in}$$

Substituting into (5) yields

$$\Delta=0.036\,89 \quad \text{or} \quad 3.7\% \text{ higher than } -Fl$$

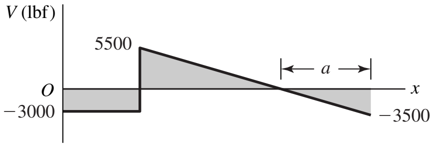
3-29

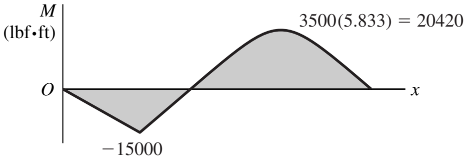


$$R_1 = \frac{600(15)}{2} + \frac{20}{15}3000 = 8500 \text{ lbf}$$

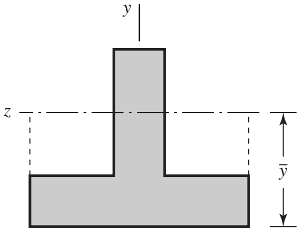
$$R_2 = \frac{600(15)}{2} - \frac{5}{15}3000 = 3500 \text{ lbf}$$

$$a = \frac{3500}{600} = 5.833 \text{ ft}$$





(a)



$$\bar{y} = \frac{1(12) + 5(12)}{24} = 3 \text{ in}$$

$$I_z = \frac{1}{3}[2(5^3) + 6(3^3) - 4(1^3)] = 136 \text{ in}^4$$

At $x = 5$ ft, $\quad y = -3$ in, $\quad \sigma_x = -\dfrac{-15000(12)(-3)}{136} = -3970$ psi

$y = 5$ in, $\quad \sigma_x = -\dfrac{-15000(12)5}{136} = 6620$ psi

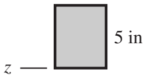
At $x = 14.17$ ft, $\quad y = -3$ in, $\quad \sigma_x = -\dfrac{20420(12)(-3)}{136} = 5405$ psi

$y = 5$ in, $\quad \sigma_x = -\dfrac{20420(12)5}{136} = -9010$ psi

Max tension = 6620 psi *Ans.*

Max compression = −9010 psi *Ans.*

(b) $V_{\max} = 5500$ lbf

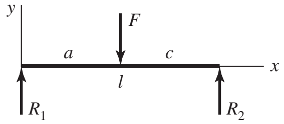


$$Q_{\text{n.a.}} = \bar{y}A = 2.5(5)(2) = 25 \text{ in}^3$$

$$\underset{\text{v}}{\tau_{\max}} = \frac{VQ}{Ib} = \frac{5500(25)}{136(2)} = 506 \text{ psi} \quad \textit{Ans.}$$

(c) $\tau_{\max} = \dfrac{|\sigma_{\max}|}{2} = \dfrac{9010}{2} = 4510$ psi *Ans.*

3-30



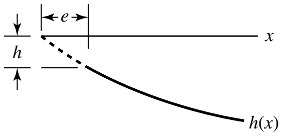
$$R_1 = \frac{c}{l}F$$

$$M = \frac{c}{l}Fx \quad 0 \le x \le a$$

$$\sigma = \frac{6M}{bh^2} = \frac{6(c/l)Fx}{bh^2} \quad \Rightarrow \quad h = \sqrt{\frac{6cFx}{bl\sigma_{\max}}} \quad 0 \le x \le a \quad \textit{Ans.}$$

3-31 From Prob. 3-30, $R_1 = \frac{c}{l}F = V, \quad 0 \le x \le a$

$$\tau_{\max} = \frac{3}{2}\frac{V}{bh} = \frac{3}{2}\frac{(c/l)F}{bh} \qquad \therefore\ h = \frac{3}{2}\frac{Fc}{lb\tau_{\max}} \quad \textit{Ans.}$$

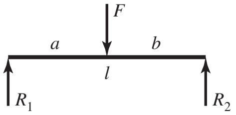


From Prob. 3-30 $= \sqrt{\frac{6Fcx}{lb\sigma_{\max}}}$ sub in $x = e$ and equate to h above

$$\frac{3}{2}\frac{Fc}{lb\tau_{\max}} = \sqrt{\frac{6Fce}{lb\sigma_{\max}}}$$

$$e = \frac{3}{8}\frac{Fc\sigma_{\max}}{lb\tau_{\max}^2} \quad \textit{Ans.}$$

3-32



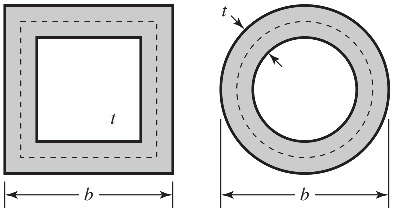
$$R_1 = \frac{b}{l}F$$

$$M = \frac{b}{l}Fx$$

$$\sigma_{\max} = \frac{32M}{\pi d^3} = \frac{32}{\pi d^3}\frac{b}{l}Fx$$

$$d = \left[\frac{32}{\pi}\frac{bFx}{l\sigma_{\max}}\right]^{1/3} \quad 0 \le x \le a \quad \textit{Ans.}$$

3-33



Square: $A_m = (b - t)^2$

$T_{\text{sq}} = 2A_m t\tau_{\text{all}} = 2(b - t)^2 t\tau_{\text{all}}$

Round: $A_m = \pi(b - t)^2/4$

$T_{\text{rd}} = 2\pi(b - t)^2 t\tau_{\text{all}}/4$

Ratio of torques

$$\frac{T_{\text{sq}}}{T_{\text{rd}}} = \frac{2(b-t)^2 t\tau_{\text{all}}}{\pi(b-t)^2 t\tau_{\text{all}}/2} = \frac{4}{\pi} = 1.27$$

Twist per unit length
square:

$$\theta_{\text{sq}} = \frac{2G\theta_1 t}{t\tau_{\text{all}}}\left(\frac{L}{A}\right)_m = C\left|\frac{L}{A}\right|_m = C\frac{4(b-t)}{(b-t)^2}$$

Round:

$$\theta_{\text{rd}} = C\left(\frac{L}{A}\right)_m = C\frac{\pi(b-t)}{\pi(b-t)^2/4} = C\frac{4(b-t)}{(b-t)^2}$$

Ratio equals 1, twists are the same.

Note the weight ratio is

$$\frac{W_{\text{sq}}}{W_{\text{rd}}} = \frac{\rho l(b-t)^2}{\rho l\pi(b-t)(t)} = \frac{b-t}{\pi t} \qquad \text{thin-walled assumes } b \geq 20t$$

$$= \frac{19}{\pi} = 6.04 \qquad \text{with } b = 20t$$

$$= 2.86 \qquad \text{with } b = 10t$$

3-34 $l = 40$ in, $\tau_{\text{all}} = 11\,500$ psi, $G = 11.5(10^6)$ psi, $t = 0.050$ in

$$r_m = r_i + t/2 = r_i + 0.025 \qquad \text{for } r_i > 0$$

$$= 0 \qquad \text{for } r_i = 0$$

$$A_m = (1-0.05)^2 - 4\left(r_m^2 - \frac{\pi}{4}r_m^2\right) = 0.95^2 - (4-\pi)r_m^2$$

$$L_m = 4(1 - 0.05 - 2r_m + 2\pi r_m/4) = 4[0.95 - (2 - \pi/2)r_m]$$

Eq. (3-45): $T = 2A_m t\tau = 2(0.05)(11\,500)A_m = 1150A_m$

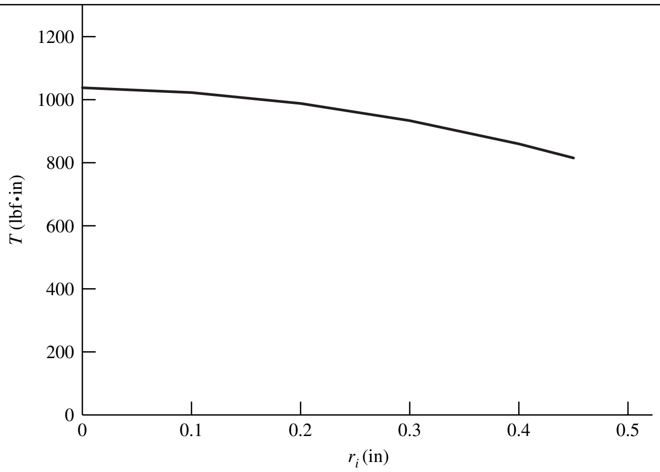
Eq. (3-46):

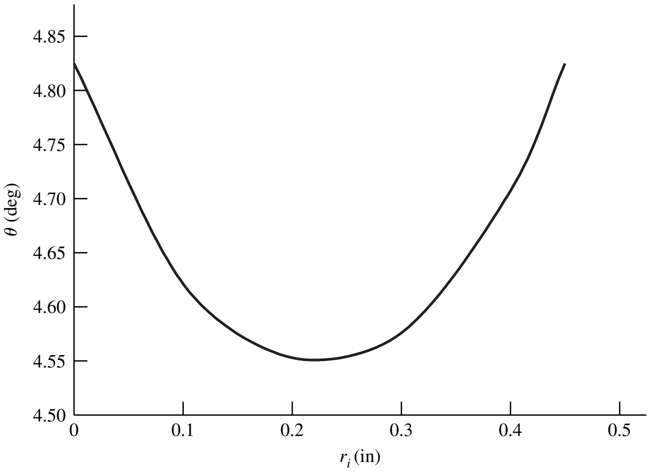
$$\theta(\text{deg}) = \theta_1 l\frac{180}{\pi} = \frac{TL_m l}{4GA_m^2 t}\frac{180}{\pi} = \frac{TL_m(40)}{4(11.5)(10^6)A_m^2(0.05)}\frac{180}{\pi}$$

$$= 9.9645(10^{-4})\frac{TL_m}{A_m^2}$$

Equations can then be put into a spreadsheet resulting in:

r_i	r_m	A_m	L_m	r_i	T(lbf · in)	r_i	θ(deg)
0	0	0.902 5	3.8	0	1037.9	0	4.825
0.10	0.125	0.889 087	3.585 398	0.10	1022.5	0.10	4.621
0.20	0.225	0.859 043	3.413 717	0.20	987.9	0.20	4.553
0.30	0.325	0.811 831	3.242 035	0.30	933.6	0.30	4.576
0.40	0.425	0.747 450	3.070 354	0.40	859.6	0.40	4.707
0.45	0.475	0.708 822	2.984 513	0.45	815.1	0.45	4.825





Torque carrying capacity reduces with r_i. However, this is based on an assumption of uniform stresses which is not the case for small r_i. Also note that weight also goes down with an increase in r_i.

3-35 From Eq. (3-47) where θ_1 is the same for each leg.

$$T_1 = \frac{1}{3}G\theta_1 L_1 c_1^3, \quad T_2 = \frac{1}{3}G\theta_1 L_2 c_2^3$$

$$T = T_1 + T_2 = \frac{1}{3}G\theta_1\left(L_1c_1^3 + L_2c_2^3\right) = \frac{1}{3}G\theta_1 \sum L_i c_i^3 \quad \textit{Ans.}$$

$$\tau_1 = G\theta_1 c_1, \quad \tau_2 = G\theta_1 c_2$$

$$\tau_{\max} = G\theta_1 c_{\max} \quad \textit{Ans.}$$

3-36

(a) $\tau_{\max} = G\theta_1 c_{\max}$

$$G\theta_1 = \frac{\tau_{\max}}{c_{\max}} = \frac{12\,000}{1/8} = 9.6(10^4)\ \text{psi/in}$$

$$T_{1/16} = \frac{1}{3}G\theta_1(Lc^3)_{1/16} = \frac{1}{3}(9.6)(10^4)(5/8)(1/16)^3 = 4.88\ \text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_{1/8} = \frac{1}{3}(9.6)(10^4)(5/8)(1/8)^3 = 39.06 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$\tau_{1/16} = 9.6(10^4)1/16 = 6000 \text{ psi}, \quad \tau_{1/8} = 9.6(10^4)1/8 = 12\,000 \text{ psi} \quad \textit{Ans.}$$

(b) $$\theta_1 = \frac{9.6(10^4)}{12(10^6)} = 87(10^{-3}) \text{ rad/in} = 0.458°\text{/in} \quad \textit{Ans.}$$

3-37 *Separate strips:* For each 1/16 in thick strip,

$$T = \frac{Lc^2\tau}{3} = \frac{(1)(1/16)^2(12\,000)}{3} = 15.625 \text{ lbf} \cdot \text{in}$$

$$\therefore T_{\max} = 2(15.625) = 31.25 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

For each strip,

$$\theta = \frac{3Tl}{Lc^3G} = \frac{3(15.625)(12)}{(1)(1/16)^3(12)(10^6)} = 0.192 \text{ rad} \quad \textit{Ans.}$$

$$k_t = T/\theta = 31.25/0.192 = 162.8 \text{ lbf} \cdot \text{in/rad} \quad \textit{Ans.}$$

Solid strip: From Eq. (3-47),

$$T_{\max} = \frac{Lc^2\tau}{3} = \frac{1(1/8)^2 12\,000}{3} = 62.5 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$\theta = \theta_1 l = \frac{\tau l}{Gc} = \frac{12\,000(12)}{12(10^6)(1/8)} = 0.0960 \text{ rad} \quad \textit{Ans.}$$

$$k_l = 62.5/0.0960 = 651 \text{ lbf} \cdot \text{in/rad} \quad \textit{Ans.}$$

3-38 $\tau_{\text{all}} = 60$ MPa, H = 35 kW

(a) $n = 2000$ rpm

Eq. (4-40) $$T = \frac{9.55H}{n} = \frac{9.55(35)10^3}{2000} = 167.1 \text{ N} \cdot \text{m}$$

$$\tau_{\max} = \frac{16T}{\pi d^3} \quad \Rightarrow \quad d = \left(\frac{16T}{\pi \tau_{\max}}\right)^{1/3} = \left[\frac{16(167.1)}{\pi(60)10^6}\right]^{1/3} = 24.2(10^{-3}) \text{ m} = 24.2 \text{ mm} \quad \textit{Ans.}$$

(b) $n = 200$ rpm $\quad \therefore T = 1671 \text{ N} \cdot \text{m}$

$$d = \left[\frac{16(1671)}{\pi(60)10^6}\right]^{1/3} = 52.2(10^{-3}) \text{ m} = 52.2 \text{ mm} \quad \textit{Ans.}$$

3-39 $\tau_{\text{all}} = 110 \text{ MPa}, \theta = 30°, \; d = 15 \text{ mm}, l = ?$

$$\tau = \frac{16T}{\pi d^3} \quad \Rightarrow \quad T = \frac{\pi}{16}\tau d^3$$

$$\theta = \frac{Tl}{JG}\left(\frac{180}{\pi}\right)$$

$$l = \frac{\pi}{180}\frac{JG\theta}{T} = \frac{\pi}{180}\left[\frac{\pi}{32}\frac{d^4 G\theta}{(\pi/16)\,\tau d^3}\right] = \frac{\pi}{360}\frac{dG\theta}{\tau}$$

$$= \frac{\pi}{360}\frac{(0.015)(79.3)(10^9)(30)}{110(10^6)} = 2.83\ \text{m} \quad \textit{Ans.}$$

3-40 $d = 3$ in, replaced by 3 in hollow with $t = 1/4$ in

(a) $$T_{\text{solid}} = \frac{\pi}{16}\tau(3^3) \quad T_{\text{hollow}} = \frac{\pi}{32}\tau\frac{(3^4 - 2.5^4)}{1.5}$$

$$\%\Delta T = \frac{(\pi/16)(3^3) - (\pi/32)\,[(3^4 - 2.5^4)/1.5]}{(\pi/16)(3^3)}(100) = 48.2\% \quad \textit{Ans.}$$

(b) $W_{\text{solid}} = kd^2 = k(3^2), \quad W_{\text{hollow}} = k(3^2 - 2.5^2)$

$$\%\Delta W = \frac{k(3^2) - k(3^2 - 2.5^2)}{k(3^2)}(100) = 69.4\% \quad \textit{Ans.}$$

3-41 $T = 5400\ \text{N}\cdot\text{m},\ \tau_{\text{all}} = 150\ \text{MPa}$

(a) $$\tau = \frac{Tc}{J} \quad\Rightarrow\quad 150(10^6) = \frac{5400(d/2)}{(\pi/32)[d^4 - (0.75d)^4]} = \frac{4.023(10^4)}{d^3}$$

$$d = \left(\frac{4.023(10^4)}{150(10^6)}\right)^{1/3} = 6.45(10^{-2})\,\text{m} = 64.5\ \text{mm}$$

From Table A-17, the next preferred size is $d = 80$ mm; $ID = 60$ mm *Ans.*

(b) $$J = \frac{\pi}{32}(0.08^4 - 0.06^4) = 2.749(10^{-6})\ \text{mm}^4$$

$$\tau_i = \frac{5400(0.030)}{2.749(10^{-6})} = 58.9(10^6)\ \text{Pa} = 58.9\ \text{MPa} \quad \textit{Ans.}$$

3-42

(a) $$T = \frac{63\,025H}{n} = \frac{63\,025(1)}{5} = 12\,605\ \text{lbf}\cdot\text{in}$$

$$\tau = \frac{16T}{\pi d_C^3} \quad\Rightarrow\quad d_C = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left[\frac{16(12\,605)}{\pi(14\,000)}\right]^{1/3} = 1.66\ \text{in} \quad \textit{Ans.}$$

From Table A-17, select 1 3/4 in

$$\tau_{\text{start}} = \frac{16(2)(12\,605)}{\pi(1.75^3)} = 23.96(10^3)\ \text{psi} = 23.96\ \text{kpsi}$$

(b) design activity

3-43 $\omega = 2\pi n/60 = 2\pi(8)/60 = 0.8378 \text{ rad/s}$

$$T = \frac{H}{\omega} = \frac{1000}{0.8378} = 1194 \text{ N}\cdot\text{m}$$

$$d_C = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left[\frac{16(1194)}{\pi(75)(10^6)}\right]^{1/3} = 4.328(10^{-2}) \text{ m} = 43.3 \text{ mm}$$

From Table A-17, select 45 mm *Ans.*

3-44 $s = \sqrt{A}, \quad d = \sqrt{4A/\pi}$

Square: Eq. (3-43) with $b = c$

$$\tau_{\max} = \frac{4.8T}{c^3}$$

$$(\tau_{\max})_{\text{sq}} = \frac{4.8T}{(A)^{3/2}}$$

Round:

$$(\tau_{\max})_{\text{rd}} = \frac{16}{\pi}\frac{T}{d^3} = \frac{16T}{\pi(4A/\pi)^{3/2}} = \frac{3.545T}{(A)^{3/2}}$$

$$\frac{(\tau_{\max})_{\text{sq}}}{(\tau_{\max})_{\text{rd}}} = \frac{4.8}{3.545} = 1.354$$

Square stress is 1.354 times the round stress *Ans.*

3-45 $s = \sqrt{A}, \quad d = \sqrt{4A/\pi}$

Square: Eq. (3-44) with $b = c$, $\beta = 0.141$

$$\theta_{\text{sq}} = \frac{Tl}{0.141c^4G} = \frac{Tl}{0.141(A)^{4/2}G}$$

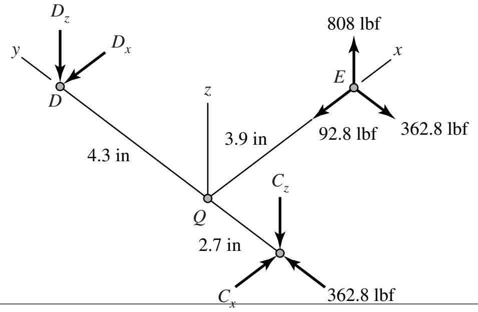
Round:

$$\theta_{\text{rd}} = \frac{Tl}{JG} = \frac{Tl}{(\pi/32)\,(4A/\pi)^{4/2}\,G} = \frac{6.2832Tl}{(A)^{4/2}G}$$

$$\frac{\theta_{\text{sq}}}{\theta_{\text{rd}}} = \frac{1/0.141}{6.2832} = 1.129$$

Square has greater θ by a factor of 1.13 *Ans.*

3-46



$$\left(\sum M_D\right)_z = 7C_x - 4.3(92.8) - 3.9(362.8) = 0$$

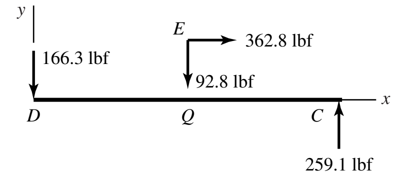
$$C_x = 259.1 \text{ lbf}$$

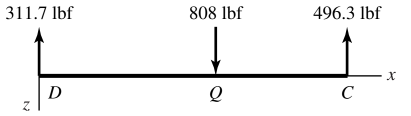
$$\left(\sum M_C\right)_z = -7D_x - 2.7(92.8) + 3.9(362.8) = 0$$

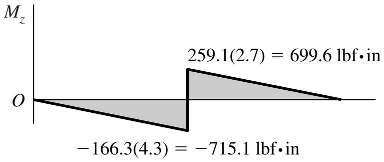
$$D_x = 166.3 \text{ lbf}$$

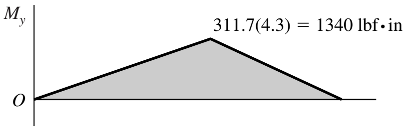
$$\left(\sum M_D\right)_x \quad \Rightarrow \quad C_z = \frac{4.3}{7} 808 = 496.3 \text{ lbf}$$

$$\left(\sum M_C\right)_x \quad \Rightarrow \quad D_z = \frac{2.7}{7} 808 = 311.7 \text{ lbf}$$









$$\underset{x=4.3^+_{\text{in}}}{\text{Torque}} : T = 808(3.9) = 3151 \text{ lbf} \cdot \text{in}$$

$$\underset{x=4.3^+_{\text{in}}}{\text{Bending } Q} : M = \sqrt{699.6^2 + 1340^2} = 1512 \text{ lbf} \cdot \text{in}$$

Torque:

$$\tau = \frac{16T}{\pi d^3} = \frac{16(3151)}{\pi(1.25^3)} = 8217 \text{ psi}$$

Bending:

$$\sigma_b = \pm\frac{32(1512)}{\pi(1.25^3)} = \pm 7885 \text{ psi}$$

Axial:

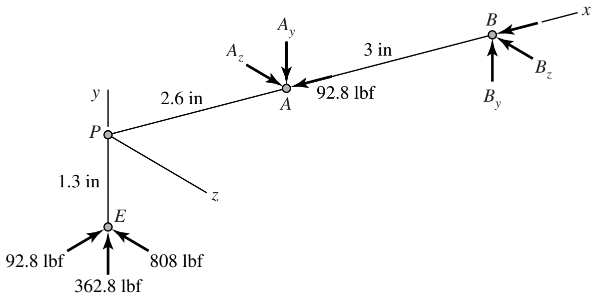
$$\sigma_a = -\frac{F}{A} = -\frac{362.8}{(\pi/4)(1.25^2)} = -296 \text{ psi}$$

$$|\sigma_{\max}| = 7885 + 296 = 8181 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{8181}{2}\right)^2 + 8217^2} = 9179 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_{\substack{\max\\ \text{tens.}}} = \frac{7885-296}{2} + \sqrt{\left(\frac{7885-296}{2}\right)^2 + 8217^2} = 12\,845 \text{ psi} \quad \textit{Ans.}$$

3-47



$$\left(\sum M_B\right)_z = -5.6(362.8) + 1.3(92.8) + 3A_y = 0$$

$$A_y = 637.0 \text{ lbf}$$

$$\left(\sum M_A\right)_z = -2.6(362.8) + 1.3(92.8) + 3B_y = 0$$

$$B_y = 274.2 \text{ lbf}$$

$$\left(\sum M_B\right)_y = 0 \quad \Rightarrow \quad A_z = \frac{5.6}{3}808 = 1508.3 \text{ lbf}$$

$$\left(\sum M_A\right)_y = 0 \quad \Rightarrow \quad B_z = \frac{2.6}{3}808 = 700.3 \text{ lbf}$$

Torsion: $T = 808(1.3) = 1050 \text{ lbf} \cdot \text{in}$

$$\tau = \frac{16(1050)}{\pi(1^3)} = 5348 \text{ psi}$$

Bending: $M_p = 92.8(1.3) = 120.6 \text{ lbf} \cdot \text{in}$

$$M_A = 3\sqrt{B_y^2 + B_z^2} = 3\sqrt{274.2^2 + 700.3^2}$$

$$= 2256 \text{ lbf} \cdot \text{in} = M_{\max}$$

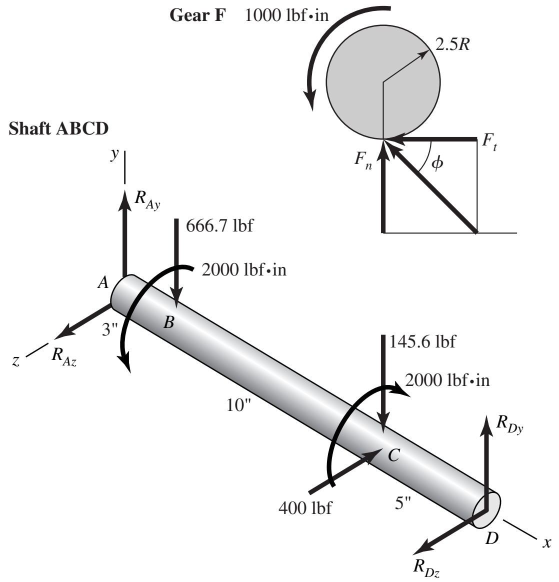
$$\sigma_b = \pm\frac{32(2256)}{\pi(1^3)} = \pm 22\,980 \text{ psi}$$

$$\underset{\text{inAP}}{\text{Axial:}}\ \sigma = -\frac{92.8}{(\pi/4)1^2} = -120 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{-22980-120}{2}\right)^2 + 5348^2} = 12\,730 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_{\substack{\max\\ \text{tens}}} = \frac{22980-120}{2} + \sqrt{\left(\frac{22980-120}{2}\right)^2 + 5348^2} = 24\,049 \text{ psi} \quad \textit{Ans.}$$

3-48



$$F_t = \frac{1000}{2.5} = 400\,\text{lbf}$$

$$F_n = 400 \tan 20 = 145.6 \text{ lbf}$$

$$\text{Torque at } C \quad T_C = 400(5) = 2000 \text{ lbf} \cdot \text{in}$$

$$P = \frac{2000}{3} = 666.7\,\text{lbf}$$

$$\sum (M_A)_z = 0 \quad \Rightarrow \quad 18R_{Dy} - 145.6(13) - 666.7(3) = 0 \quad \Rightarrow \quad R_{Dy} = 216.3 \text{ lbf}$$

$$\sum (M_A)_y = 0 \quad \Rightarrow \quad -18R_{Dz} + 400(13) = 0 \quad \Rightarrow \quad R_{Dz} = 288.9 \text{ lbf}$$

$$\sum F_y = 0 \quad \Rightarrow \quad R_{Ay} + 216.3 - 666.7 - 145.6 = 0 \quad \Rightarrow \quad R_{Ay} = 596.0 \text{ lbf}$$

$$\sum F_z = 0 \quad \Rightarrow \quad R_{Az} + 288.9 - 400 = 0 \quad \Rightarrow \quad R_{Az} = 111.1 \text{ lbf}$$

$$M_B = 3\sqrt{596^2 + 111.1^2} = 1819 \text{ lbf} \cdot \text{in}$$

$$M_C = 5\sqrt{216.3^2 + 288.9^2} = 1805 \text{ lbf} \cdot \text{in}$$

$\therefore$ Maximum stresses occur at B. *Ans.*

$$\sigma_B = \frac{32M_B}{\pi d^3} = \frac{32(1819)}{\pi(1.25^3)} = 9486 \text{ psi}$$

$$\tau_B = \frac{16T_B}{\pi d^3} = \frac{16(2000)}{\pi(1.25^3)} = 5215 \text{ psi}$$

$$\sigma_{\max} = \frac{\sigma_B}{2} + \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \frac{9486}{2} + \sqrt{\left(\frac{9486}{2}\right)^2 + 5215^2} = 11\,792 \text{ psi} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = 7049 \text{ psi} \quad \textit{Ans.}$$

3-49 $r = d/2$

(a) For top, $\theta = 90^\circ$,

$$\sigma_r = \frac{\sigma}{2}[1 - 1 + (1 - 1)(1 - 3)\cos 180] = 0 \quad \textit{Ans.}$$

$$\sigma_\theta = \frac{\sigma}{2}[1 + 1 - (1+3)\cos 180] = 3\sigma \quad \textit{Ans.}$$

$$\tau_{r\theta} = -\frac{\sigma}{2}(1-1)(1+3)\sin 180 = 0 \quad \textit{Ans.}$$

For side, $\theta = 0^\circ$,

$$\sigma_r = \frac{\sigma}{2}[1 - 1 + (1-1)(1-3)\cos 0] = 0 \quad \textit{Ans.}$$

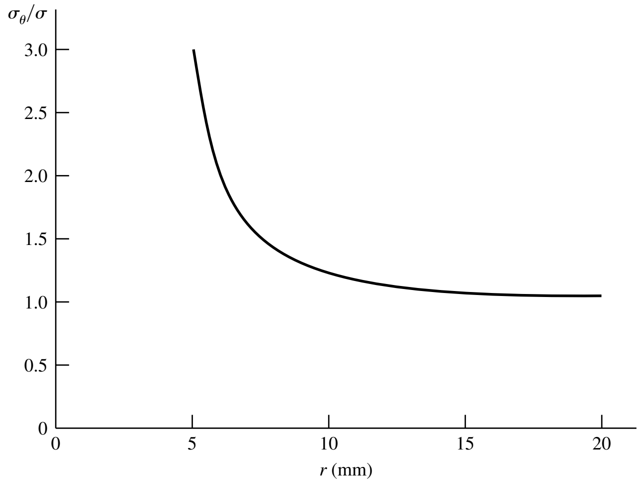
$$\sigma_\theta = \frac{\sigma}{2}[1 + 1 - (1+3)\cos 0] = -\sigma \quad \textit{Ans.}$$

$$\tau_{r\theta} = -\frac{\sigma}{2}(1-1)(1+3)\sin 0 = 0 \quad \textit{Ans.}$$

(b)

$$\sigma_\theta/\sigma = \frac{1}{2}\left[1 + \frac{100}{4r^2} - \left(1 + \frac{3}{16}\frac{10^4}{r^4}\right)\cos 180\right] = \frac{1}{2}\left(2 + \frac{25}{r^2} + \frac{3}{16}\frac{10^4}{r^4}\right)$$

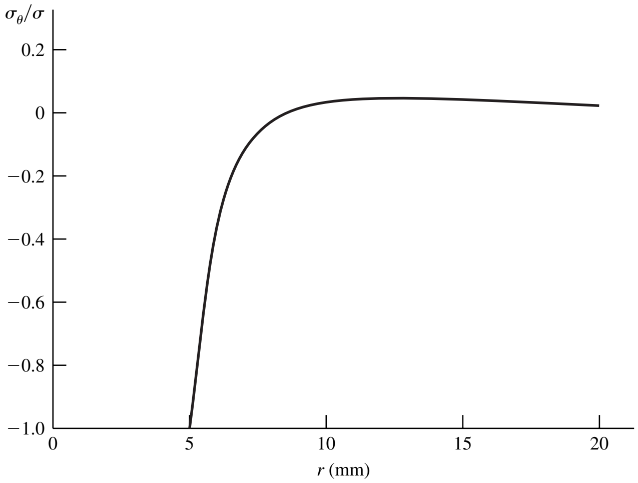
r	σ_θ/σ
5	3.000
6	2.071
7	1.646
8	1.424
9	1.297
10	1.219
11	1.167
12	1.132
13	1.107
14	1.088
15	1.074
16	1.063
17	1.054
18	1.048
19	1.042
20	1.037



(c)

$$\sigma_\theta/\sigma = \frac{1}{2}\left[1+\frac{100}{4r^2}-\left(1+\frac{3}{16}\frac{10^4}{r^4}\right)\cos 0\right] = \frac{1}{2}\left(\frac{25}{r^2}-\frac{3}{16}\frac{10^4}{r^4}\right)$$

r	σ_θ/σ
5	−1.000
6	−0.376
7	−0.135
8	−0.034
9	0.011
10	0.031
11	0.039
12	0.042
13	0.041
14	0.039
15	0.037
16	0.035
17	0.032
18	0.030
19	0.027
20	0.025



3-50

$$D/d = \frac{1.5}{1} = 1.5$$

$$r/d = \frac{1/8}{1} = 0.125$$

Fig. A-15-8: $K_{ts} \doteq 1.39$

Fig. A-15-9: $K_t \doteq 1.60$

$$\sigma_A = K_t\frac{Mc}{I} = \frac{32K_tM}{\pi d^3} = \frac{32(1.6)(200)(14)}{\pi(1^3)} = 45\,630 \text{ psi}$$

$$\tau_A = K_{ts}\frac{Tc}{J} = \frac{16K_{ts}T}{\pi d^3} = \frac{16(1.39)(200)(15)}{\pi(1^3)} = 21\,240 \text{ psi}$$

$$\sigma_{\max} = \frac{\sigma_A}{2}+\sqrt{\left(\frac{\sigma_A}{2}\right)^2+\tau_A^2} = \frac{45.63}{2}+\sqrt{\left(\frac{45.63}{2}\right)^2+21.24^2}$$

$$= 54.0 \text{ kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{45.63}{2}\right)^2+21.24^2} = 31.2 \text{ kpsi} \quad \textit{Ans.}$$

3-51 As shown in Fig. 3-32, the maximum stresses occur at the inside fiber where $r = r_i$. Therefore, from Eq. (3-50)

$$\sigma_{t,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right)$$

$$= p_i \left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) \quad Ans.$$

$$\sigma_{r,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right) = -p_i \quad Ans.$$

3-52 If $p_i = 0$, Eq. (3-49) becomes

$$\sigma_t = \frac{-p_o r_o^2 - r_i^2 r_o^2 p_o / r^2}{r_o^2 - r_i^2}$$

$$= -\frac{p_o r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

The maximum tangential stress occurs at $r = r_i$. So

$$\sigma_{t,\max} = -\frac{2 p_o r_o^2}{r_o^2 - r_i^2} \quad Ans.$$

For σ_r, we have

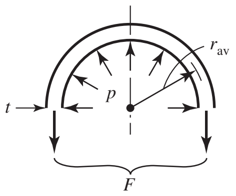
$$\sigma_r = \frac{-p_o r_o^2 + r_i^2 r_o^2 p_o / r^2}{r_o^2 - r_i^2}$$

$$= \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2} - 1\right)$$

So $\sigma_r = 0$ at $r = r_i$. Thus at $r = r_o$

$$\sigma_{r,\max} = \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2 - r_o^2}{r_o^2}\right) = -p_o \quad Ans.$$

3-53



$$F = pA = \pi r_{av}^2 p$$

$$\sigma_1 = \sigma_2 = \frac{F}{A_{wall}} = \frac{\pi r_{av}^2 p}{2\pi r_{av} t} = \frac{p r_{av}}{2t} \quad Ans.$$

3-54 $\sigma_t > \sigma_l > \sigma_r$

$\tau_{\max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$ where σ_l is intermediate in value. From Prob. 4-50

$$\tau_{\max} = \frac{1}{2}(\sigma_{t,\,\max} - \sigma_{r,\,\max})$$

$$\tau_{\max} = \frac{p_i}{2}\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} + 1\right)$$

Now solve for p_i using $r_o = 75$ mm, $r_i = 69$ mm, and $\tau_{\max} = 25$ MPa. This gives $p_i = 3.84$ MPa *Ans.*

3-55 Given $r_o = 5$ in, $r_i = 4.625$ in and referring to the solution of Prob. 3-54,

$$\tau_{\max} = \frac{350}{2}\left[\frac{(5)^2 + (4.625)^2}{(5)^2 - (4.625)^2} + 1\right]$$

$$= 2\,424 \text{ psi} \quad \textit{Ans.}$$

3-56 From Table A-20, $S_y = 57$ kpsi; also, $r_o = 0.875$ in and $r_i = 0.625$ in

From Prob. 3-52

$$\sigma_{t,\max} = -\frac{2p_o r_o^2}{r_o^2 - r_i^2}$$

Rearranging

$$p_o = \frac{\left(r_o^2 - r_i^2\right)(0.8S_y)}{2r_o^2}$$

Solving, gives $p_o = 11\,200$ psi *Ans.*

3-57 From Table A-20, $S_y = 390$ MPa; also $r_o = 25$ mm, $r_i = 20$ mm.

From Prob. 3-51

$$\sigma_{t,\max} = p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) \quad \text{therefore} \quad p_i = 0.8S_y\left(\frac{r_o^2 - r_i^2}{r_o^2 + r_i^2}\right)$$

solving gives $p_i = 68.5$ MPa *Ans.*

3-58 Since σ_t and σ_r are both positive and $\sigma_t > \sigma_r$

$$\tau_{\max} = (\sigma_t)_{\max}/2$$

where σ_t is max at r_i

Eq. (3-55) for $r = r_i = 0.375$ in

$$(\sigma_t)_{\max} = \frac{0.282}{386}\left[\frac{2\pi(7200)}{60}\right]^2\left(\frac{3+0.292}{8}\right)$$

$$\times\left[0.375^2 + 5^2 + \frac{(0.375^2)(5^2)}{0.375^2} - \frac{1+3(0.292)}{3+0.292}(0.375^2)\right] = 8556\text{ psi}$$

$$\tau_{\max} = \frac{8556}{2} = 4278\text{ psi} \quad \textit{Ans.}$$

Radial stress: $$\sigma_r = k\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)$$

Maxima: $$\frac{d\sigma_r}{dr} = k\left(2\frac{r_i^2 r_o^2}{r^3} - 2r\right) = 0 \quad \Rightarrow \quad r = \sqrt{r_i r_o} = \sqrt{0.375(5)} = 1.3693\text{ in}$$

$$(\sigma_r)_{\max} = \frac{0.282}{386}\left[\frac{2\pi(7200)}{60}\right]^2\left(\frac{3+0.292}{8}\right)\left[0.375^2 + 5^2 - \frac{0.375^2(5^2)}{1.3693^2} - 1.3693^2\right]$$

$$= 3656\text{ psi} \quad \textit{Ans.}$$

3-59

$$\omega = 2\pi(2069)/60 = 216.7\text{ rad/s},$$
$$\rho = 3320\text{ kg/m}^3,\ \nu = 0.24,\ r_i = 0.0125\text{ m},\ r_o = 0.15\text{ m};$$

use Eq. (3-55)

$$\sigma_t = 3320(216.7)^2\left(\frac{3+0.24}{8}\right)\left[(0.0125)^2 + (0.15)^2 + (0.15)^2 - \frac{1+3(0.24)}{3+0.24}(0.0125)^2\right](10)^{-6}$$

$$= 2.85\text{ MPa} \quad \textit{Ans.}$$

3-60

$$\rho = \frac{(6/16)}{386(1/16)(\pi/4)(6^2-1^2)} = 5.655(10^{-4})\text{ lbf}\cdot\text{s}^2/\text{in}^4$$

$\tau_{\max}$ is at bore and equals $\dfrac{\sigma_t}{2}$

Eq. (3-55)

$$(\sigma_t)_{\max} = 5.655(10^{-4})\left[\frac{2\pi(10\,000)}{60}\right]^2\left(\frac{3+0.20}{8}\right)\left[0.5^2 + 3^2 + 3^2 - \frac{1+3(0.20)}{3+0.20}(0.5)^2\right]$$

$$= 4496\text{ psi}$$

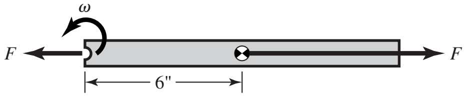
$$\tau_{\max} = \frac{4496}{2} = 2248\text{ psi} \quad \textit{Ans.}$$

3-61

$$\omega = 2\pi(3000)/60 = 314.2 \text{ rad/s}$$

$$m = \frac{0.282(1.25)(12)(0.125)}{386}$$

$$= 1.370(10^{-3}) \text{ lbf}\cdot\text{s}^2\text{/in}$$



$$F = m\omega^2 r = 1.370(10^{-3})(314.2^2)(6)$$

$$= 811.5 \text{ lbf}$$

$$A_{\text{nom}} = (1.25 - 0.5)(1/8) = 0.093\,75 \text{ in}^2$$

$$\sigma_{\text{nom}} = \frac{811.5}{0.093\,75} = 8656 \text{ psi} \quad \textit{Ans.}$$

Note: Stress concentration Fig. A-15-1 gives $K_t \doteq 2.25$ which increases $\sigma_{\max}$ and fatigue.

3-62 to 3-67

$$\nu = 0.292, \quad E = 30 \text{ Mpsi (207 GPa)}, \quad r_i = 0$$

$$R = 0.75 \text{ in (20 mm)}, \quad r_o = 1.5 \text{ in (40 mm)}$$

Eq. (3-57)

$$p_{\text{psi}} = \frac{30(10^6)\delta}{0.75^3}\left[\frac{(1.5^2 - 0.75^2)(0.75^2 - 0)}{2(1.5^2 - 0)}\right] = 1.5(10^7)\delta \qquad (1)$$

$$p_{\text{Pa}} = \frac{207(10^9)\delta}{0.020^3}\left[\frac{(0.04^2 - 0.02^2)(0.02^2 - 0)}{2(0.04^2 - 0)}\right] = 3.881(10^{12})\delta \qquad (2)$$

3-62

$$\delta_{\max} = \frac{1}{2}[40.042 - 40.000] = 0.021 \text{ mm} \quad \textit{Ans.}$$

$$\delta_{\min} = \frac{1}{2}[40.026 - 40.025] = 0.0005 \text{ mm} \quad \textit{Ans.}$$

From (2)

$$p_{\max} = 81.5 \text{ MPa}, \quad p_{\min} = 1.94 \text{ MPa} \quad \textit{Ans.}$$

3-63

$$\delta_{\max} = \frac{1}{2}(1.5016 - 1.5000) = 0.0008 \text{ in} \quad \textit{Ans.}$$

$$\delta_{\min} = \frac{1}{2}(1.5010 - 1.5010) = 0 \quad \textit{Ans.}$$

Eq. (1) $\quad p_{\max} = 12\,000 \text{ psi}, \quad p_{\min} = 0 \quad \textit{Ans.}$

3-64

$$\delta_{\max} = \frac{1}{2}(40.059 - 40.000) = 0.0295\,\text{mm} \quad \textit{Ans.}$$

$$\delta_{\min} = \frac{1}{2}(40.043 - 40.025) = 0.009\ \text{mm} \quad \textit{Ans.}$$

Eq. (2) $\quad p_{\max} = 114.5\,\text{MPa}, \quad p_{\min} = 34.9\ \text{MPa} \quad \textit{Ans.}$

3-65

$$\delta_{\max} = \frac{1}{2}(1.5023 - 1.5000) = 0.001\,15\,\text{in} \quad \textit{Ans.}$$

$$\delta_{\min} = \frac{1}{2}(1.5017 - 1.5010) = 0.000\,35\ \text{in} \quad \textit{Ans.}$$

Eq. (1) $\quad p_{\max} = 17\,250\,\text{psi} \quad p_{\min} = 5250\ \text{psi} \quad \textit{Ans.}$

3-66

$$\delta_{\max} = \frac{1}{2}(40.076 - 40.000) = 0.038\,\text{mm} \quad \textit{Ans.}$$

$$\delta_{\min} = \frac{1}{2}(40.060 - 40.025) = 0.0175\ \text{mm} \quad \textit{Ans.}$$

Eq. (2) $\quad p_{\max} = 147.5\,\text{MPa} \quad p_{\min} = 67.9\ \text{MPa} \quad \textit{Ans.}$

3-67

$$\delta_{\max} = \frac{1}{2}(1.5030 - 1.500) = 0.0015\,\text{in} \quad \textit{Ans.}$$

$$\delta_{\min} = \frac{1}{2}(1.5024 - 1.5010) = 0.0007\,\text{in} \quad \textit{Ans.}$$

Eq. (1) $\quad p_{\max} = 22\,500\,\text{psi} \quad p_{\min} = 10\,500\ \text{psi} \quad \textit{Ans.}$

3-68

$$\delta = \frac{1}{2}(1.002 - 1.000) = 0.001\,\text{in} \quad r_i = 0, \quad R = 0.5\,\text{in}, \quad r_o = 1\,\text{in}$$

$$\nu = 0.292, \quad E = 30\,\text{Mpsi}$$

Eq. (3-57)

$$p = \frac{30(10^6)(0.001)}{0.5^3}\left[\frac{(1^2 - 0.5^2)(0.5^2 - 0)}{2(1^2 - 0)}\right] = 2.25(10^4)\ \text{psi} \quad \textit{Ans.}$$

Eq. (3-50) for outer member at $r_i = 0.5$ in

$$(\sigma_t)_o = \frac{0.5^2(2.25)(10^4)}{1^2 - 0.5^2}\left(1 + \frac{1^2}{0.5^2}\right) = 37\,500\ \text{psi} \quad \textit{Ans.}$$

Inner member, from Prob. 3-52

$$(\sigma_t)_i = -\frac{p_o r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r_o^2}\right) = -\frac{2.25(10^4)(0.5^2)}{0.5^2 - 0}\left(1 + \frac{0}{0.5^2}\right) = -22\,500 \text{ psi} \quad \textit{Ans.}$$

3-69

$$\nu_i = 0.292, \quad E_i = 30(10^6) \text{ psi}, \quad \nu_o = 0.211, \quad E_o = 14.5(10^6) \text{ psi}$$

$$\delta = \frac{1}{2}(1.002 - 1.000) = 0.001 \text{ in}, \quad r_i = 0, \quad R = 0.5, \quad r_o = 1$$

Eq. (3-56)

$$0.001 = \left[\frac{0.5}{14.5(10^6)}\left(\frac{1^2 + 0.5^2}{1^2 - 0.5^2} + 0.211\right) + \frac{0.5}{30(10^6)}\left(\frac{0.5^2 + 0}{0.5^2 - 0} - 0.292\right)\right] p$$

$$p = 13\,064 \text{ psi} \quad \textit{Ans.}$$

Eq. (3-50) for outer member at $r_i = 0.5$ in

$$(\sigma_t)_o = \frac{0.5^2(13\,064)}{1^2 - 0.5^2}\left(1 + \frac{1^2}{0.5^2}\right) = 21\,770 \text{ psi} \quad \textit{Ans.}$$

Inner member, from Prob. 3-52

$$(\sigma_t)_i = -\frac{13\,064(0.5^2)}{0.5^2 - 0}\left(1 + \frac{0}{0.5^2}\right) = -13\,064 \text{ psi} \quad \textit{Ans.}$$

3-70

$$\delta_{\max} = \frac{1}{2}(1.003 - 1.000) = 0.0015 \text{ in} \quad r_i = 0, \quad R = 0.5 \text{ in}, \quad r_o = 1 \text{ in}$$

$$\delta_{\min} = \frac{1}{2}(1.002 - 1.001) = 0.0005 \text{ in}$$

Eq. (3-57)

$$p_{\max} = \frac{30(10^6)(0.0015)}{0.5^3}\left[\frac{(1^2 - 0.5^2)(0.5^2 - 0)}{2(1^2 - 0)}\right] = 33\,750 \text{ psi} \quad \textit{Ans.}$$

Eq. (3-50) for outer member at $r = 0.5$ in

$$(\sigma_t)_o = \frac{0.5^2(33\,750)}{1^2 - 0.5^2}\left(1 + \frac{1^2}{0.5^2}\right) = 56\,250 \text{ psi} \quad \textit{Ans.}$$

For inner member, from Prob. 3-52, with $r = 0.5$ in

$$(\sigma_t)_i = -33\,750 \text{ psi} \quad \textit{Ans.}$$

For $\delta_{\min}$ all answers are $0.0005/0.0015 = 1/3$ of above answers *Ans.*

3-71

$$\nu_i = 0.292, \quad E_i = 30 \text{ Mpsi}, \quad \nu_o = 0.334, \quad E_o = 10.4 \text{ Mpsi}$$

$$\delta_{\max} = \frac{1}{2}(2.005 - 2.000) = 0.0025 \text{ in}$$

$$\delta_{\min} = \frac{1}{2}(2.003 - 2.002) = 0.0005 \text{ in}$$

$$0.0025 = \left[\frac{1.0}{10.4(10^6)}\left(\frac{2^2+1^2}{2^2-1^2} + 0.334\right) + \frac{1.0}{30(10^6)}\left(\frac{1^2+0}{1^2-0} - 0.292\right)\right] p_{\max}$$

$$p_{\max} = 11\,576 \text{ psi} \quad \textit{Ans.}$$

Eq. (3-50) for outer member at $r = 1$ in

$$(\sigma_t)_o = \frac{1^2(11\,576)}{2^2 - 1^2}\left(1 + \frac{2^2}{1^2}\right) = 19\,293 \text{ psi} \quad \textit{Ans.}$$

Inner member from Prob. 3-52 with $r = 1$ in

$$(\sigma_t)_i = -11\,576 \text{ psi} \quad \textit{Ans.}$$

For $\delta_{\min}$ all above answers are $0.0005/0.0025 = 1/5$ *Ans.*

3-72

(a) Axial resistance

Normal force at fit interface

$$N = pA = p(2\pi Rl) = 2\pi pRl$$

Fully-developed friction force

$$F_{ax} = fN = 2\pi f pRl \quad \textit{Ans.}$$

(b) Torsional resistance at fully developed friction is

$$T = fRN = 2\pi f pR^2 l \quad \textit{Ans.}$$

3-73 $d = 1$ in, $r_i = 1.5$ in, $r_o = 2.5$ in.

From Table 3-4, for $R = 0.5$ in,

$$r_c = 1.5 + 0.5 = 2 \text{ in}$$

$$r_n = \frac{0.5^2}{2\left(2 - \sqrt{2^2 - 0.5^2}\right)} = 1.968\,245\,8 \text{ in}$$

$$e = r_c - r_n = 2.0 - 1.968\,245\,8 = 0.031\,754 \text{ in}$$

$$c_i = r_n - r_i = 1.9682 - 1.5 = 0.4682 \text{ in}$$

$$c_o = r_o - r_n = 2.5 - 1.9682 = 0.5318 \text{ in}$$

$$A = \pi d^2/4 = \pi(1)^2/4 = 0.7854 \text{ in}^2$$

$$M = Fr_c = 1000(2) = 2000 \text{ lbf} \cdot \text{in}$$

Using Eq. (3-65)

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{1000}{0.7854} + \frac{2000(0.4682)}{0.7854(0.031\,754)(1.5)} = 26\,300 \text{ psi} \quad Ans.$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{1000}{0.7854} - \frac{2000(0.5318)}{0.7854(0.031\,754)(2.5)} = -15\,800 \text{ psi} \quad Ans.$$

3-74 Section AA:

$D = 0.75$ in, $r_i = 0.75/2 = 0.375$ in, $r_o = 0.75/2 + 0.25 = 0.625$ in

From Table 3-4, for $R = 0.125$ in,

$$r_c = (0.75 + 0.25)/2 = 0.500 \text{ in}$$

$$r_n = \frac{0.125^2}{2\left(0.5 - \sqrt{0.5^2 - 0.125^2}\right)} = 0.492\,061\,5 \text{ in}$$

$$e = 0.5 - r_n = 0.007\,939 \text{ in}$$

$$c_o = r_o - r_n = 0.625 - 0.492\,06 = 0.132\,94 \text{ in}$$

$$c_i = r_n - r_i = 0.492\,06 - 0.375 = 0.117\,06 \text{ in}$$

$$A = \pi(0.25)^2/4 = 0.049\,087$$

$$M = Fr_c = 100(0.5) = 50 \text{ lbf}\cdot\text{in}$$

$$\sigma_i = \frac{100}{0.049\,09} + \frac{50(0.117\,06)}{0.049\,09(0.007\,939)(0.375)} = 42\,100 \text{ psi} \quad Ans.$$

$$\sigma_o = \frac{100}{0.049\,09} - \frac{50(0.132\,94)}{0.049\,09(0.007\,939)(0.625)} = -25\,250 \text{ psi} \quad Ans.$$

Section BB: Abscissa angle θ of line of radius centers is

$$\theta = \cos^{-1}\left(\frac{r_2 + d/2}{r_2 + d + D/2}\right)$$

$$= \cos^{-1}\left(\frac{0.375 + 0.25/2}{0.375 + 0.25 + 0.75/2}\right) = 60^\circ$$

$$M = F\frac{D+d}{2}\cos\theta = 100(0.5)\cos 60^\circ = 25 \text{ lbf}\cdot\text{in}$$

$$r_i = r_2 = 0.375 \text{ in}$$

$$r_o = r_2 + d = 0.375 + 0.25 = 0.625 \text{ in}$$

$$e = 0.007\,939 \text{ in} \quad \text{(as before)}$$

$$\sigma_i = \frac{F\cos\theta}{A} - \frac{Mc_i}{Aer_i}$$

$$= \frac{100\cos 60^\circ}{0.049\,09} - \frac{25(0.117\,06)}{0.049\,09(0.007\,939)0.375} = -19\,000 \text{ psi} \quad Ans.$$

$$\sigma_o = \frac{100\cos 60^\circ}{0.049\,09} + \frac{25(0.132\,94)}{0.049\,09(0.007\,939)0.625} = 14\,700 \text{ psi} \quad Ans.$$

On section BB, the shear stress due to the shear force is zero at the surface.

3-75 $r_i = 0.125$ in, $r_o = 0.125 + 0.1094 = 0.2344$ in

From Table 3-4 for $h = 0.1094$

$$
\begin{aligned}
r_c &= 0.125 + 0.1094/2 = 0.1797 \text{ in}\\
r_n &= 0.1094/\ln(0.2344/0.125) = 0.174\,006 \text{ in}\\
e &= r_c - r_n = 0.1797 - 0.174\,006 = 0.005\,694 \text{ in}\\
c_i &= r_n - r_i = 0.174\,006 - 0.125 = 0.049\,006 \text{ in}\\
c_o &= r_o - r_n = 0.2344 - 0.174\,006 = 0.060\,394 \text{ in}\\
A &= 0.75(0.1094) = 0.082\,050 \text{ in}^2\\
M &= F(4 + h/2) = 3(4 + 0.1094/2) = 12.16 \text{ lbf} \cdot \text{in}\\
\sigma_i &= -\frac{3}{0.082\,05} - \frac{12.16(0.0490)}{0.082\,05(0.005\,694)(0.125)} = -10\,240 \text{ psi} \quad \textit{Ans.}\\
\sigma_o &= -\frac{3}{0.082\,05} + \frac{12.16(0.0604)}{0.082\,05(0.005\,694)(0.2344)} = 6670 \text{ psi} \quad \textit{Ans.}
\end{aligned}
$$

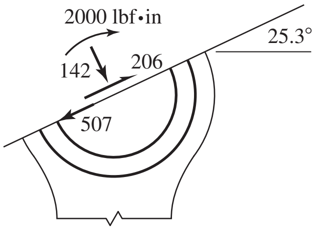
3-76 Find the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$.

$$
\begin{aligned}
F_x &= F_{1x} + F_{2x} = 250\cos 60^\circ + 333\cos 0^\circ\\
&= 458 \text{ lbf}\\
F_y &= F_{1y} + F_{2y} = 250\sin 60^\circ + 333\sin 0^\circ\\
&= 216.5 \text{ lbf}\\
F &= (458^2 + 216.5^2)^{1/2} = 506.6 \text{ lbf}
\end{aligned}
$$

This is the pin force on the lever which acts in a direction

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{216.5}{458} = 25.3^\circ$$

On the 25.3° surface from $\mathbf{F}_1$



$$
\begin{aligned}
F_t &= 250\cos(60^\circ - 25.3^\circ) = 206 \text{ lbf}\\
F_n &= 250\sin(60^\circ - 25.3^\circ) = 142 \text{ lbf}\\
r_c &= 1 + 3.5/2 = 2.75 \text{ in}\\
A &= 2[0.8125(0.375) + 1.25(0.375)]\\
&= 1.546\,875 \text{ in}^2
\end{aligned}
$$

The denominator of Eq. (3-63), given below, has four additive parts.

$$r_n = \frac{A}{\int (dA/r)}$$

For $\int dA/r$, add the results of the following equation for each of the four rectangles.

$$\int_{r_i}^{r_o} \frac{b dr}{r} = b \ln \frac{r_o}{r_i}, \qquad b = \text{width}$$

$$\int \frac{dA}{r} = 0.375 \ln \frac{1.8125}{1} + 1.25 \ln \frac{2.1875}{1.8125} + 1.25 \ln \frac{3.6875}{3.3125} + 0.375 \ln \frac{4.5}{3.6875}$$

$$= 0.666\,810\,6$$

$$r_n = \frac{1.546\,875}{0.666\,810\,6} = 2.3198 \text{ in}$$

$$e = r_c - r_n = 2.75 - 2.3198 = 0.4302 \text{ in}$$

$$c_i = r_n - r_i = 2.320 - 1 = 1.320 \text{ in}$$

$$c_o = r_o - r_n = 4.5 - 2.320 = 2.180 \text{ in}$$

Shear stress due to 206 lbf force is zero at inner and outer surfaces.

$$\sigma_i = -\frac{142}{1.547} + \frac{2000(1.32)}{1.547(0.4302)(1)} = 3875 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_o = -\frac{142}{1.547} - \frac{2000(2.18)}{1.547(0.4302)(4.5)} = -1548 \text{ psi} \quad \textit{Ans.}$$

3-77

$$A = (6 - 2 - 1)(0.75) = 2.25 \text{ in}^2$$

$$r_c = \frac{6+2}{2} = 4 \text{ in}$$

Similar to Prob. 3-76,

$$\int \frac{dA}{r} = 0.75 \ln \frac{3.5}{2} + 0.75 \ln \frac{6}{4.5} = 0.635\,473\,4 \text{ in}$$

$$r_n = \frac{A}{\int (dA/r)} = \frac{2.25}{0.635\,473\,4} = 3.5407 \text{ in}$$

$$e = 4 - 3.5407 = 0.4593 \text{ in}$$

$$\sigma_i = \frac{5000}{2.25} + \frac{20\,000(3.5407 - 2)}{2.25(0.4593)(2)} = 17\,130 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{5000}{2.25} - \frac{20\,000(6 - 3.5407)}{2.25(0.4593)(6)} = -5710 \text{ psi} \quad \textit{Ans.}$$

3-78

$$A = \int_{r_i}^{r_o} b\, dr = \int_2^6 \frac{2}{r}\, dr = 2 \ln \frac{6}{2}$$

$$= 2.197\,225 \text{ in}^2$$

$$r_c = \frac{1}{A}\int_{r_i}^{r_o} br\,dr = \frac{1}{2.197\,225}\int_2^6 \frac{2r}{r}\,dr$$

$$= \frac{2}{2.197\,225}(6-2) = 3.640\,957 \text{ in}$$

$$r_n = \frac{A}{\int_{r_i}^{r_o}(b/r)\,dr} = \frac{2.197\,225}{\int_2^6 (2/r^2)\,dr}$$

$$= \frac{2.197\,225}{2[1/2 - 1/6]} = 3.295\,837 \text{ in}$$

$$e = R - r_n = 3.640\,957 - 3.295\,837 = 0.345\,12$$

$$c_i = r_n - r_i = 3.2958 - 2 = 1.2958 \text{ in}$$

$$c_o = r_o - r_n = 6 - 3.2958 = 2.7042 \text{ in}$$

$$\sigma_i = \frac{20\,000}{2.197} + \frac{20\,000(3.641)(1.2958)}{2.197(0.345\,12)(2)} = 71\,330 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{20\,000}{2.197} - \frac{20\,000(3.641)(2.7042)}{2.197(0.345\,12)(6)} = -34\,180 \text{ psi} \quad \textit{Ans.}$$

3-79 $r_c = 12$ in, $M = 20(2+2) = 80$ kip · in

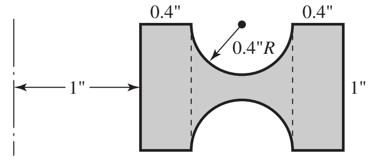
From statics book, $I = \dfrac{\pi}{4}a^3 b = \dfrac{\pi}{4}(2^3)1 = 2\pi \text{ in}^4$

Inside: $\sigma_i = \dfrac{F}{A} + \dfrac{My}{I}\dfrac{r_c}{r_i} = \dfrac{20}{2\pi} + \dfrac{80(2)}{2\pi}\dfrac{12}{10} = 33.7$ kpsi *Ans.*

Outside: $\sigma_o = \dfrac{F}{A} - \dfrac{My}{I}\dfrac{r_c}{r_o} = \dfrac{20}{2\pi} - \dfrac{80(2)}{2\pi}\dfrac{12}{14} = -18.6$ kpsi *Ans.*

Note: A much more accurate solution (see the 7th edition) yields $\sigma_i = 32.25$ kpsi and $\sigma_o = -19.40$ kpsi

3-80



For rectangle, $\displaystyle\int \frac{dA}{r} = b \ln r_o/r_i$

For circle, $\dfrac{A}{\int(dA/r)} = \dfrac{r^2}{2\left(r_c - \sqrt{r_c^2 - r^2}\right)}$, $A_o = \pi r^2$

$$\therefore \int \frac{dA}{r} = 2\pi\left(r_c - \sqrt{r_c^2 - r^2}\right)$$

$$\sum\int\frac{dA}{r} = 1\ln\frac{2.6}{1} - 2\pi\left(1.8-\sqrt{1.8^2-0.4^2}\right) = 0.672\,723\,4$$

$$A = 1(1.6) - \pi(0.4^2) = 1.097\,345\,2\ \text{in}^2$$

$$r_n = \frac{1.097\,345\,2}{0.672\,723\,4} = 1.6312\ \text{in}$$

$$e = 1.8 - r_n = 0.1688\ \text{in}$$

$$c_i = 1.6312 - 1 = 0.6312\ \text{in}$$

$$c_o = 2.6 - 1.6312 = 0.9688\ \text{in}$$

$$M = 3000(5.8) = 17\,400\ \text{lbf}\cdot\text{in}$$

$$\sigma_i = \frac{3}{1.0973} + \frac{17.4(0.6312)}{1.0973(0.1688)(1)} = 62.03\ \text{kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{3}{1.0973} - \frac{17.4(0.9688)}{1.0973(0.1688)(2.6)} = -32.27\ \text{kpsi} \quad \textit{Ans.}$$

3-81 From Eq. (3-68)

$$a = KF^{1/3} = F^{1/3}\left\{\frac{3}{8}\,\frac{2[(1-\nu^2)/E]}{2(1/d)}\right\}^{1/3}$$

Use $\nu = 0.292$, F in newtons, E in N/mm^2 and d in mm, then

$$K = \left\{\frac{3}{8}\frac{[(1-0.292^2)/207\,000]}{1/25}\right\}^{1/3} = 0.0346$$

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi(KF^{1/3})^2}$$

$$= \frac{3F^{1/3}}{2\pi K^2} = \frac{3F^{1/3}}{2\pi(0.0346)^2}$$

$$= 399F^{1/3}\ \text{MPa} = |\sigma_{\max}| \qquad \textit{Ans.}$$

$$\tau_{\max} = 0.3p_{\max}$$

$$= 120F^{1/3}\ \text{MPa} \qquad \textit{Ans.}$$

3-82 From Prob. 3-81,

$$K = \left\{\frac{3}{8}\frac{2[(1-0.292^2)/207\,000]}{1/25+0}\right\}^{1/3} = 0.0436$$

$$p_{\max} = \frac{3F^{1/3}}{2\pi K^2} = \frac{3F^{1/3}}{2\pi(0.0436)^2} = 251F^{1/3}$$

and so, $\sigma_z = -251F^{1/3}$ MPa *Ans.*

$$\tau_{\max} = 0.3(251)F^{1/3} = 75.3F^{1/3}\ \text{MPa} \quad \textit{Ans.}$$

$$z = 0.48a = 0.48(0.0436)18^{1/3} = 0.055\ \text{mm} \quad \textit{Ans.}$$

3-83 $\nu_1 = 0.334$, $E_1 = 10.4$ Mpsi, $l = 2$ in, $d_1 = 1$ in, $\nu_2 = 0.211$, $E_2 = 14.5$ Mpsi, $d_2 = -8$ in.

With $b = K_c F^{1/2}$, from Eq. (3-73),

$$K_c = \left(\frac{2}{\pi(2)}\,\frac{(1-0.334^2)/[10.4(10^6)] + (1-0.211^2)/[14.5(10^6)]}{1-0.125}\right)^{1/2}$$

$$= 0.000\,234\,6$$

Be sure to check σ_x for both ν_1 and ν_2. Shear stress is maximum in the aluminum roller. So,

$$\tau_{\max} = 0.3 p_{\max}$$

$$p_{\max} = \frac{4000}{0.3} = 13\,300 \text{ psi}$$

Since $p_{\max} = 2F/(\pi b l)$ we have

$$p_{\max} = \frac{2F}{\pi l K_c F^{1/2}} = \frac{2F^{1/2}}{\pi l K_c}$$

So,

$$F = \left(\frac{\pi l K_c p_{\max}}{2}\right)^2$$

$$= \left(\frac{\pi(2)(0.000\,234\,6)(13\,300)}{2}\right)^2$$

$$= 96.1 \text{ lbf} \quad \textit{Ans.}$$

3-84 Good class problem

3-85 From Table A-5, $\nu = 0.211$

$$\frac{\sigma_x}{p_{\max}} = (1+\nu) - \frac{1}{2} = (1+0.211) - \frac{1}{2} = 0.711$$

$$\frac{\sigma_y}{p_{\max}} = 0.711$$

$$\frac{\sigma_z}{p_{\max}} = 1$$

These are principal stresses

$$\frac{\tau_{\max}}{p_{\max}} = \frac{1}{2}(\sigma_1 - \sigma_3) = \frac{1}{2}(1 - 0.711) = 0.1445$$

3-86 From Table A-5: $\nu_1 = 0.211$, $\nu_2 = 0.292$, $E_1 = 14.5(10^6)$ psi, $E_2 = 30(10^6)$ psi, $d_1 = 6$ in, $d_2 = \infty$, $l = 2$ in

(a) Eq. (3-73):

$$b = \sqrt{\frac{2(800)}{\pi(2)} \frac{(1 - 0.211^2)/14.5(10^6) + (1 - 0.292^2)/[30(10^6)]}{1/6 + 1/\infty}}$$

$$= 0.012\,135 \text{ in}$$

$$p_{\max} = \frac{2(800)}{\pi(0.012\,135)(2)} = 20\,984 \text{ psi}$$

For $z = 0$ in,

$$\sigma_{x1} = -2\nu_1 p_{\max} = -2(0.211)20\,984 = -8855 \text{ psi in wheel}$$

$$\sigma_{x2} = -2(0.292)20\,984 = -12\,254 \text{ psi}$$

In plate

$$\sigma_y = -p_{\max} = -20\,984 \text{ psi}$$

$$\sigma_z = -20\,984 \text{ psi}$$

These are principal stresses.

(b) For $z = 0.010$ in,

$$\sigma_{x1} = -4177 \text{ psi} \quad \text{in wheel}$$

$$\sigma_{x2} = -5781 \text{ psi} \quad \text{in plate}$$

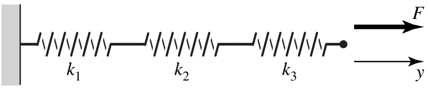
$$\sigma_y = -3604 \text{ psi}$$

$$\sigma_z = -16\,194 \text{ psi}$$

Chapter 4

4-1

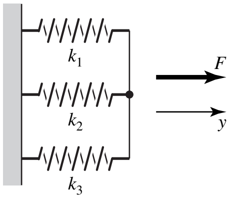
(a)



$$k = \frac{F}{y}; \quad y = \frac{F}{k_1} + \frac{F}{k_2} + \frac{F}{k_3}$$

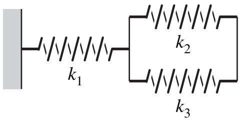
so
$$k = \frac{1}{(1/k_1) + (1/k_2) + (1/k_3)} \quad \textit{Ans.}$$

(b)



$$F = k_1 y + k_2 y + k_3 y$$
$$k = F/y = k_1 + k_2 + k_3 \quad \textit{Ans.}$$

(c)



$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2 + k_3} \qquad k = \left(\frac{1}{k_1} + \frac{1}{k_2 + k_3}\right)^{-1}$$

4-2 For a torsion bar, $k_T = T/\theta = Fl/\theta$, and so $\theta = Fl/k_T$. For a cantilever, $k_C = F/\delta$, $\delta = F/k_C$. For the assembly, $k = F/y$, $y = F/k = l\theta + \delta$

So
$$y = \frac{F}{k} = \frac{Fl^2}{k_T} + \frac{F}{k_C}$$

Or
$$k = \frac{1}{(l^2/k_T) + (1/k_C)} \quad \textit{Ans.}$$

4-3 For a torsion bar, $k = T/\theta = GJ/l$ where $J = \pi d^4/32$. So $k = \pi d^4 G/(32l) = Kd^4/l$. The springs, 1 and 2, are in parallel so

$$k = k_1 + k_2 = K\frac{d^4}{l_1} + K\frac{d^4}{l_2}$$
$$= Kd^4\left(\frac{1}{x} + \frac{1}{l - x}\right)$$

And
$$\theta = \frac{T}{k} = \frac{T}{Kd^4\left(\frac{1}{x} + \frac{1}{l - x}\right)}$$

Then
$$T = k\theta = \frac{Kd^4}{x}\theta + \frac{Kd^4\theta}{l - x}$$

Thus $$T_1 = \frac{Kd^4}{x}\theta; \qquad T_2 = \frac{Kd^4\theta}{l-x}$$

If $x = l/2$, then $T_1 = T_2$. If $x < l/2$, then $T_1 > T_2$

Using $\tau = 16T/\pi d^3$ and $\theta = 32Tl/(G\pi d^4)$ gives

$$T = \frac{\pi d^3 \tau}{16}$$

and so

$$\theta_{\text{all}} = \frac{32l}{G\pi d^4} \cdot \frac{\pi d^3 \tau}{16} = \frac{2l\tau_{\text{all}}}{Gd}$$

Thus, if $x < l/2$, the allowable twist is

$$\theta_{\text{all}} = \frac{2x\tau_{\text{all}}}{Gd} \quad \textit{Ans.}$$

Since $$k = Kd^4\left(\frac{1}{x} + \frac{1}{l-x}\right)$$

$$= \frac{\pi G d^4}{32}\left(\frac{1}{x} + \frac{1}{l-x}\right) \quad \textit{Ans.}$$

Then the maximum torque is found to be

$$T_{\max} = \frac{\pi d^3 x \tau_{\text{all}}}{16}\left(\frac{1}{x} + \frac{1}{l-x}\right) \quad \textit{Ans.}$$

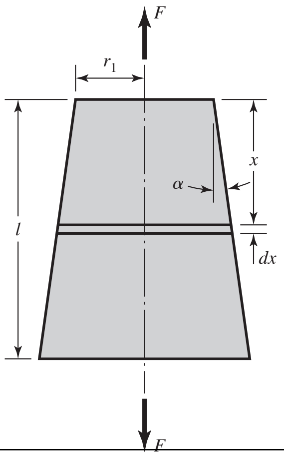
4-4 Both legs have the same twist angle. From Prob. 4-3, for equal shear, d is linear in x. Thus, $d_1 = 0.2d_2$ *Ans.*

$$k = \frac{\pi G}{32}\left[\frac{(0.2d_2)^4}{0.2l} + \frac{d_2^4}{0.8l}\right] = \frac{\pi G}{32l}\left(1.258d_2^4\right) \quad \textit{Ans.}$$

$$\theta_{\text{all}} = \frac{2(0.8l)\tau_{\text{all}}}{Gd_2} \quad \textit{Ans.}$$

$$T_{\max} = k\theta_{\text{all}} = 0.198d_2^3\tau_{\text{all}} \quad \textit{Ans.}$$

4-5



$$A = \pi r^2 = \pi(r_1 + x\tan\alpha)^2$$

$$d\delta = \frac{F\,dx}{AE} = \frac{F\,dx}{E\pi(r_1 + x\tan\alpha)^2}$$

$$\delta = \frac{F}{\pi E}\int_0^l \frac{dx}{(r_1 + x\tan\alpha)^2}$$

$$= \frac{F}{\pi E}\left(-\frac{1}{\tan\alpha(r_1 + x\tan\alpha)}\right)_0^l$$

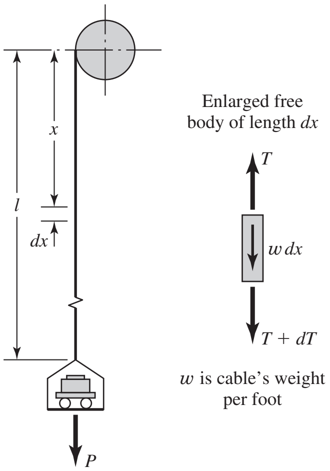
$$= \frac{F}{\pi E}\frac{1}{r_1(r_1 + l\tan\alpha)}$$

Then

$$k = \frac{F}{\delta} = \frac{\pi E r_1(r_1 + l\tan\alpha)}{l}$$

$$= \frac{EA_1}{l}\left(1 + \frac{2l}{d_1}\tan\alpha\right) \quad \textit{Ans.}$$

4-6



$$\sum F = (T + dT) + w\,dx - T = 0$$

$$\frac{dT}{dx} = -w$$

Solution is $T = -wx + c$

$$T|_{x=0} = P + wl = c$$

$$T = -wx + P + wl$$

$$T = P + w(l - x)$$

The infinitesmal stretch of the free body of original length dx is

$$d\delta = \frac{T\,dx}{AE}$$

$$= \frac{P + w(l - x)}{AE}\,dx$$

Integrating,

$$\delta = \int_0^l \frac{[P + w(l - x)]\,dx}{AE}$$

$$\delta = \frac{Pl}{AE} + \frac{wl^2}{2AE} \quad \textit{Ans.}$$

4-7

$$M = wlx - \frac{wl^2}{2} - \frac{wx^2}{2}$$

$$EI\frac{dy}{dx} = \frac{wlx^2}{2} - \frac{wl^2}{2}x - \frac{wx^3}{6} + C_1, \qquad \frac{dy}{dx} = 0 \text{ at } x = 0, \quad \therefore C_1 = 0$$

$$EIy = \frac{wlx^3}{6} - \frac{wl^2x^2}{4} - \frac{wx^4}{24} + C_2, \qquad y = 0 \text{ at } x = 0, \quad \therefore C_2 = 0$$

$$y = \frac{wx^2}{24EI}(4lx - 6l^2 - x^2) \quad \textit{Ans.}$$

4-8

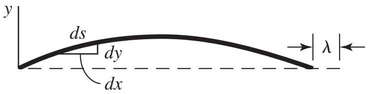
$$M = M_1 = M_B$$

$$EI\frac{dy}{dx} = M_B x + C_1, \qquad \frac{dy}{dx} = 0 \text{ at } x = 0, \quad \therefore C_1 = 0$$

$$EIy = \frac{M_B x^2}{2} + C_2, \qquad y = 0 \text{ at } x = 0, \quad \therefore C_2 = 0$$

$$y = \frac{M_B x^2}{2EI} \quad \textit{Ans.}$$

4-9



$$ds = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Expand right-hand term by Binomial theorem

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} = 1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2 + \cdots$$

Since dy/dx is small compared to 1, use only the first two terms,

$$\begin{aligned} d\lambda &= ds - dx \\ &= dx\left[1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2\right] - dx \\ &= \frac{1}{2}\left(\frac{dy}{dx}\right)^2 dx \\ \therefore \lambda &= \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx \quad \textit{Ans.} \end{aligned}$$

This contraction becomes important in a nonlinear, non-breaking extension spring.

4-10 $y = Cx^2(4lx - x^2 - 6l^2) \qquad \text{where } C = \dfrac{w}{24EI}$

$$\frac{dy}{dx} = Cx(12lx - 4x^2 - 12l^2) = 4Cx(3lx - x^2 - 3l^2)$$

$$\left(\frac{dy}{dx}\right)^2 = 16C^2(15l^2x^4 - 6lx^5 - 18x^3l^3 + x^6 + 9l^4x^2)$$

$$\lambda = \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx = 8C^2\int_0^l (15l^2x^4 - 6lx^5 - 18x^3l^3 + x^6 + 9l^4x^2)\,dx$$

$$= 8C^2\left(\frac{9}{14}l^7\right) = 8\left(\frac{w}{24EI}\right)^2\left(\frac{9}{14}l^7\right) = \frac{1}{112}\left(\frac{w}{EI}\right)^2 l^7 \qquad \textit{Ans.}$$

4-11 $y = Cx(2lx^2 - x^3 - l^3)$ where $C = \dfrac{w}{24EI}$

$$\frac{dy}{dx} = C(6lx^2 - 4x^3 - l^3)$$

$$\left(\frac{dy}{dx}\right)^2 = C^2(36l^2x^4 - 48lx^5 - 12l^4x^2 + 16x^6 + 8x^3l^3 + l^6)$$

$$\lambda = \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx = \frac{1}{2}C^2\int_0^l (36l^2x^4 - 48lx^5 - 12l^4x^2 + 16x^6 + 8x^3l^3 + l^6)\, dx$$

$$= C^2\left(\frac{17}{70}l^7\right) = \left(\frac{w}{24EI}\right)^2\left(\frac{17}{70}l^7\right) = \frac{17}{40\,320}\left(\frac{w}{EI}\right)^2 l^7 \qquad \textit{Ans.}$$

4-12

$$I = 2(5.56) = 11.12 \text{ in}^4$$

$$y_{\max} = y_1 + y_2 = -\frac{wl^4}{8EI} + \frac{Fa^2}{6EI}(a - 3l)$$

Here $w = 50/12 = 4.167$ lbf/in, and $a = 7(12) = 84$ in, and $l = 10(12) = 120$ in.

$$y_1 = -\frac{4.167(120)^4}{8(30)(10^6)(11.12)} = -0.324 \text{ in}$$

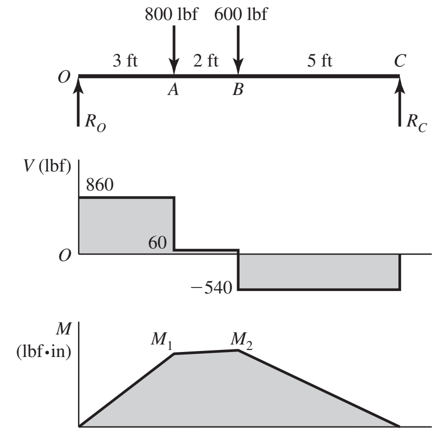
$$y_2 = -\frac{600(84)^2[3(120) - 84]}{6(30)(10^6)(11.12)} = -0.584 \text{ in}$$

So $$y_{\max} = -0.324 - 0.584 = -0.908 \text{ in} \quad \textit{Ans.}$$

$$\begin{aligned} M_0 &= -Fa - (wl^2/2) \\ &= -600(84) - [4.167(120)^2/2] \\ &= -80\,400 \text{ lbf}\cdot\text{in} \\ c &= 4 - 1.18 = 2.82 \text{ in} \\ \sigma_{\max} &= \frac{-My}{I} = -\frac{(-80\,400)(-2.82)}{11.12}(10^{-3}) \\ &= -20.4 \text{ kpsi} \quad \textit{Ans.} \end{aligned}$$

$\sigma_{\max}$ is at the bottom of the section.

4-13



$$R_O = \frac{7}{10}(800) + \frac{5}{10}(600) = 860\,\text{lbf}$$

$$R_C = \frac{3}{10}(800) + \frac{5}{10}(600) = 540\,\text{lbf}$$

$$M_1 = 860(3)(12) = 30.96(10^3)\ \text{lbf} \cdot \text{in}$$

$$M_2 = 30.96(10^3) + 60(2)(12)$$
$$= 32.40(10^3)\ \text{lbf} \cdot \text{in}$$

$$\sigma_{\max} = \frac{M_{\max}}{Z} \quad \Rightarrow \quad 6 = \frac{32.40}{Z} \qquad Z = 5.4\ \text{in}^3$$

$$y|_{x=5\text{ft}} = \frac{F_1 a[l - (l/2)]}{6EIl}\left[\left(\frac{l}{2}\right)^2 + a^2 - 2l\frac{l}{2}\right] - \frac{F_2 l^3}{48EI}$$

$$-\frac{1}{16} = \frac{800(36)(60)}{6(30)(10^6)I(120)}[60^2 + 36^2 - 120^2] - \frac{600(120^3)}{48(30)(10^6)I}$$

$$I = 23.69\ \text{in}^4 \quad \Rightarrow \quad I/2 = 11.84\ \text{in}^4$$

Select two 6 in-8.2 lbf/ft channels; from Table A-7, $I = 2(13.1) = 26.2\ \text{in}^4,\ Z = 2(4.38)\ \text{in}^3$

$$y_{\max} = \frac{23.69}{26.2}\left(-\frac{1}{16}\right) = -0.0565\,\text{in}$$

$$\sigma_{\max} = \frac{32.40}{2(4.38)} = 3.70\,\text{kpsi}$$

4-14

$$I = \frac{\pi}{64}(40^4) = 125.66(10^3)\ \text{mm}^4$$

Superpose beams A-9-6 and A-9-7,

$$y_A = \frac{1500(600)400}{6(207)10^9(125.66)10^3(1000)}(400^2 + 600^2 - 1000^2)(10^3)^2$$
$$+ \frac{2000(400)}{24(207)10^9(125.66)10^3}[2(1000)400^2 - 400^3 - 1000^3]10^3$$

$$y_A = -2.061\ \text{mm} \quad \textit{Ans.}$$

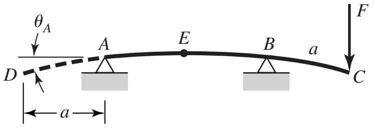
$$y|_{x=500} = \frac{1500(400)500}{24(207)10^9(125.66)10^3(1000)}[500^2 + 400^2 - 2(1000)500](10^3)^2$$
$$- \frac{5(2000)1000^4}{384(207)10^9(125.66)10^3}10^3 = -2.135\,\text{mm} \quad \textit{Ans.}$$

$$\%\ \text{difference} = \frac{2.135 - 2.061}{2.061}(100) = 3.59\% \quad \textit{Ans.}$$

4-15

$$I = \frac{1}{12}(9)(35^3) = 32.156(10^3)\ \text{mm}^4$$

From Table A-9-10



$$y_C = -\frac{Fa^2}{3EI}(l + a)$$

$$\frac{dy_{AB}}{dx} = \frac{Fa}{6EIl}(l^2 - 3x^2)$$

Thus,

$$\theta_A = \frac{Fal^2}{6EIl} = \frac{Fal}{6EI}$$

$$y_D = -\theta_A a = -\frac{Fa^2 l}{6EI}$$

With both loads,

$$y_D = -\frac{Fa^2 l}{6EI} - \frac{Fa^2}{3EI}(l + a)$$

$$= -\frac{Fa^2}{6EI}(3l + 2a) = -\frac{500(250^2)}{6(207)(10^9)(32.156)(10^3)}[3(500) + 2(250)](10^3)^2$$

$$= -1.565\ \text{mm} \quad \textit{Ans.}$$

$$y_E = \frac{2Fa(l/2)}{6EIl}\left[l^2 - \left(\frac{l}{2}\right)^2\right] = \frac{Fal^2}{8EI}$$

$$= \frac{500(250)(500^2)(10^3)^2}{8(207)(10^9)(32.156)(10^3)} = 0.587\ \text{mm} \quad \textit{Ans.}$$

4-16 $a = 36$ in, $l = 72$ in, $I = 13\ \text{in}^4$, $E = 30$ Mpsi

$$y = \frac{F_1 a^2}{6EI}(a - 3l) - \frac{F_2 l^3}{3EI}$$

$$= \frac{400(36)^2(36 - 216)}{6(30)(10^6)(13)} - \frac{400(72)^3}{3(30)(10^6)(13)}$$

$$= -0.1675\ \text{in} \quad \textit{Ans.}$$

4-17

$$I = 2(1.85) = 3.7\ \text{in}^4$$

Adding the weight of the channels, $2(5)/12 = 0.833$ lbf/in,

$$y_A = -\frac{wl^4}{8EI} - \frac{Fl^3}{3EI} = -\frac{10.833(48^4)}{8(30)(10^6)(3.7)} - \frac{220(48^3)}{3(30)(10^6)(3.7)}$$

$$= -0.1378\ \text{in} \quad \textit{Ans.}$$

4-18

$$I = \pi d^4/64 = \pi(2)^4/64 = 0.7854\ \text{in}^4$$

Tables A-9-5 and A-9-9

$$y = -\frac{F_2 l^3}{48EI} + \frac{F_1 a}{24EI}(4a^2 - 3l^2)$$

$$= -\frac{120(40)^3}{48(30)(10^6)(0.7854)} + \frac{85(10)(400 - 4800)}{24(30)(10^6)(0.7854)} = -0.0134\ \text{in} \quad \textit{Ans.}$$

4-19

(a) Useful relations

$$k = \frac{F}{y} = \frac{48EI}{l^3}$$

$$I = \frac{kl^3}{48E} = \frac{2400(48)^3}{48(30)10^6} = 0.1843\ \text{in}^4$$

From $I = bh^3/12$

$$h = \sqrt[3]{\frac{12(0.1843)}{b}}$$

Form a table. First, Table A-17 gives likely available fractional sizes for b:

$$8\tfrac{1}{2},\ 9,\ 9\tfrac{1}{2},\ 10\ \text{in}$$

For h:

$$\frac{1}{2}, \frac{9}{16}, \frac{5}{8}, \frac{11}{16}, \frac{3}{4}$$

For available b what is necessary h for required I?

b	$\sqrt[3]{\frac{12(0.1843)}{b}}$	
8.5	0.638	
9.0	0.626 ←	choose 9" × $\frac{5"}{8}$ *Ans.*
9.5	0.615	
10.0	0.605	

(b)

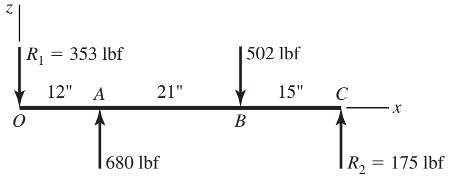
$$I = 9(0.625)^3/12 = 0.1831\ \text{in}^4$$

$$k = \frac{48EI}{l^3} = \frac{48(30)(10^6)(0.1831)}{48^3} = 2384\ \text{lbf/in}$$

$$F = \frac{4\sigma I}{cl} = \frac{4(90\,000)(0.1831)}{(0.625/2)(48)} = 4394\ \text{lbf}$$

$$y = \frac{F}{k} = \frac{4394}{2384} = 1.84\ \text{in} \quad \textit{Ans.}$$

4-20



$$\text{Torque} = (600 - 80)(9/2) = 2340\,\text{lbf} \cdot \text{in}$$

$$(T_2 - T_1)\frac{12}{2} = T_2(1 - 0.125)(6) = 2340$$

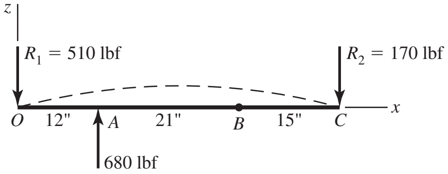
$$T_2 = \frac{2340}{6(0.875)} = 446\,\text{lbf}, \quad T_1 = 0.125(446) = 56\,\text{lbf}$$

$$\sum M_0 = 12(680) - 33(502) + 48R_2 = 0$$

$$R_2 = \frac{33(502) - 12(680)}{48} = 175\ \text{lbf}$$

$$R_1 = 680 - 502 + 175 = 353\ \text{lbf}$$

We will treat this as two separate problems and then sum the results.
First, consider the 680 lbf load as acting alone.



$$z_{OA} = -\frac{Fbx}{6EIl}(x^2 + b^2 - l^2); \quad \text{here } b = 36\text{"},$$

$$x = 12\text{"}, \quad l = 48\text{"}, \quad F = 680\ \text{lbf}$$

Also,

$$I = \frac{\pi d^4}{64} = \frac{\pi(1.5)^4}{64} = 0.2485\ \text{in}^4$$

$$z_A = -\frac{680(36)(12)(144 + 1296 - 2304)}{6(30)(10^6)(0.2485)(48)}$$

$$= +0.1182\ \text{in}$$

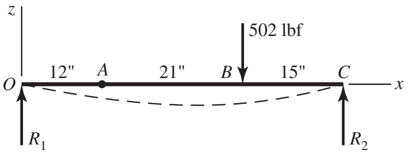
$$z_{AC} = -\frac{Fa(l - x)}{6EIl}(x^2 + a^2 - 2lx)$$

where $a = 12$" and $x = 21 + 12 = 33$"

$$z_B = -\frac{680(12)(15)(1089 + 144 - 3168)}{6(30)(10^6)(0.2485)(48)}$$

$$= +0.1103\ \text{in}$$

Next, consider the 502 lbf load as acting alone.



$$z_{OB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2), \quad \text{where } b = 15\text{"},$$

$$x = 12\text{"}, \quad l = 48\text{"}, \quad I = 0.2485 \text{ in}^4$$

Then, $$z_A = \frac{502(15)(12)(144 + 225 - 2304)}{6(30)(10^6)(0.2485)(48)} = -0.081\,44 \text{ in}$$

For z_B use $x = 33$"

$$z_B = \frac{502(15)(33)(1089 + 225 - 2304)}{6(30)(10^6)(0.2485)(48)}$$
$$= -0.1146 \text{ in}$$

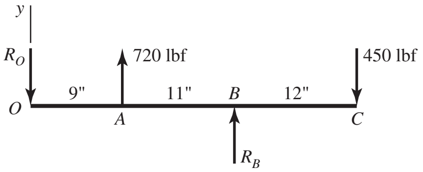
Therefore, by superposition

$$z_A = +0.1182 - 0.0814 = +0.0368 \text{ in} \quad \textit{Ans.}$$
$$z_B = +0.1103 - 0.1146 = -0.0043 \text{ in} \quad \textit{Ans.}$$

4-21

(a) Calculate torques and moment of inertia

$$T = (400 - 50)(16/2) = 2800 \text{ lbf} \cdot \text{in}$$
$$(8T_2 - T_2)(10/2) = 2800 \quad \Rightarrow \quad T_2 = 80 \text{ lbf}, \quad T_1 = 8(80) = 640 \text{ lbf}$$
$$I = \frac{\pi}{64}(1.25^4) = 0.1198 \text{ in}^4$$



Due to 720 lbf, flip beam A-9-6 such that $y_{AB} \rightarrow b = 9, x = 0, l = 20, F = -720$ lbf

$$\theta_B = \left.\frac{dy}{dx}\right|_{x=0} = -\frac{Fb}{6EIl}(3x^2 + b^2 - l^2)$$
$$= -\frac{-720(9)}{6(30)(10^6)(0.1198)(20)}(0 + 81 - 400) = -4.793(10^{-3}) \text{ rad}$$
$$y_C = -12\theta_B = -0.057\,52 \text{ in}$$

Due to 450 lbf, use beam A-9-10,

$$y_C = -\frac{Fa^2}{3EI}(l + a) = -\frac{450(144)(32)}{3(30)(10^6)(0.1198)} = -0.1923 \text{ in}$$

Adding the two deflections,

$$y_C = -0.057\,52 - 0.1923 = -0.2498 \text{ in} \quad \textit{Ans.}$$

(b) At O:
Due to 450 lbf:

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{Fa}{6EIl}(l^2 - 3x^2)\bigg|_{x=0} = \frac{Fal}{6EI}$$

$$\theta_O = -\frac{720(11)(0 + 11^2 - 400)}{6(30)(10^6)(0.1198)(20)} + \frac{450(12)(20)}{6(30)(10^6)(0.1198)} = 0.010\,13 \text{ rad} = 0.5805^\circ$$

At B:

$$\theta_B = -4.793(10^{-3}) + \frac{450(12)}{6(30)(10^6)(0.1198)(20)}[20^2 - 3(20^2)]$$

$$= -0.014\,81 \text{ rad} = 0.8485^\circ$$

$$I = 0.1198\left(\frac{0.8485^\circ}{0.06^\circ}\right) = 1.694 \text{ in}^4$$

$$d = \left(\frac{64I}{\pi}\right)^{1/4} = \left[\frac{64(1.694)}{\pi}\right]^{1/4} = 2.424 \text{ in}$$

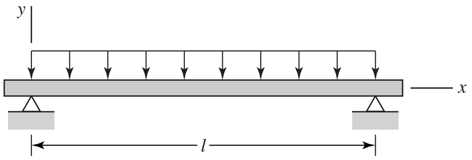
Use $d = 2.5$ in *Ans.*

$$I = \frac{\pi}{64}(2.5^4) = 1.917 \text{ in}^4$$

$$y_C = -0.2498\left(\frac{0.1198}{1.917}\right) = -0.015\,61 \text{ in} \quad \textit{Ans.}$$

4-22

(a) $l = 36(12) = 432$ in



$$y_{\max} = -\frac{5wl^4}{384EI} = -\frac{5(5000/12)(432)^4}{384(30)(10^6)(5450)}$$

$$= -1.16 \text{ in}$$

The frame is bowed up 1.16 in with respect to the bolsters. It is fabricated upside down and then inverted. *Ans.*

(b) The equation in xy-coordinates is for the center sill neutral surface

$$y = \frac{wx}{24EI}(2lx^2 - x^3 - l^3) \quad \textit{Ans.}$$

Differentiating this equation and solving for the slope at the left bolster gives

$$\frac{dy}{dx} = \frac{w}{24EI}(6lx^2 - 4x^3 - l^3)$$

Thus,
$$\left.\frac{dy}{dx}\right|_{x=0} = -\frac{wl^3}{24EI} = -\frac{(5000/12)(432)^3}{24(30)(10^6)(5450)}$$

$$= -0.008\,57$$

The slope at the right bolster is 0.008 57, so equation at left end is $y = -0.008\,57x$ and at the right end is $y = 0.008\,57(x - l)$. *Ans.*

4-23 From Table A-9-6,

$$y_L = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2)$$

$$y_L = \frac{Fb}{6EIl}(x^3 + b^2x - l^2x)$$

$$\frac{dy_L}{dx} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2)$$

$$\left.\frac{dy_L}{dx}\right|_{x=0} = \frac{Fb(b^2 - l^2)}{6EIl}$$

Let
$$\xi = \left|\frac{Fb(b^2 - l^2)}{6EIl}\right|$$

And set $I = \dfrac{\pi d_L^4}{64}$

And solve for d_L

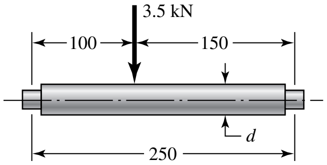
$$d_L = \left|\frac{32Fb(b^2 - l^2)}{3\pi El\xi}\right|^{1/4} \quad \textit{Ans.}$$

For the other end view, observe the figure of Table A-9-6 from the back of the page, noting that *a* and *b* interchange as do *x* and $-x$

$$d_R = \left|\frac{32Fa(l^2 - a^2)}{3\pi El\xi}\right|^{1/4} \quad \textit{Ans.}$$

For a uniform diameter shaft the necessary diameter is the larger of d_L and d_R.

4-24 Incorporating a design factor into the solution for d_L of Prob. 4-23,



$$d = \left[\frac{32n}{3\pi El\xi} Fb(l^2 - b^2)\right]^{1/4}$$

$$= \left|(\text{mm } 10^{-3}) \frac{\text{kN mm}^3}{\text{GPa mm}} \frac{10^3(10^{-9})}{10^9(10^{-3})}\right|^{1/4}$$

$$d = \sqrt[4]{\frac{32(1.28)(3.5)(150)|(250^2 - 150^2)|}{3\pi(207)(250)(0.001)} 10^{-12}}$$

$$= 36.4 \text{ mm} \quad \textit{Ans.}$$

4-25 The maximum occurs in the right section. Flip beam A-9-6 and use

$$y = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2) \quad \text{where } b = 100 \text{ mm}$$

$$\frac{dy}{dx} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2) = 0$$

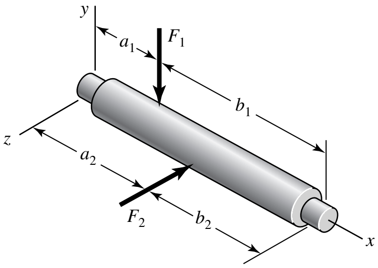
Solving for *x*,

$$x = \sqrt{\frac{l^2 - b^2}{3}} = \sqrt{\frac{250^2 - 100^2}{3}} = 132.29 \text{ mm} \quad \text{from right}$$

$$y = \frac{3.5(10^3)(0.1)(0.132\,29)}{6(207)(10^9)(\pi/64)(0.0364^4)(0.25)}[0.132\,29^2 + 0.1^2 - 0.25^2](10^3)$$

$$= -0.0606 \text{ mm} \quad \textit{Ans.}$$

4-26



The slope at $x = 0$ due to F_1 in the *xy* plane is

$$\theta_{xy} = \frac{F_1 b_1 (b_1^2 - l^2)}{6EIl}$$

and in the *xz* plane due to F_2 is

$$\theta_{xz} = \frac{F_2 b_2 (b_2^2 - l^2)}{6EIl}$$

For small angles, the slopes add as vectors. Thus

$$\theta_L = \left(\theta_{xy}^2 + \theta_{xz}^2\right)^{1/2}$$

$$= \left[\left(\frac{F_1 b_1 (b_1^2 - l^2)}{6EIl}\right)^2 + \left(\frac{F_2 b_2 (b_2^2 - l^2)}{6EIl}\right)^2\right]^{1/2}$$

Designating the slope constraint as ξ, we then have

$$\xi = |\theta_L| = \frac{1}{6EIl}\left\{\sum\left[F_i b_i\left(b_i^2 - l^2\right)\right]^2\right\}^{1/2}$$

Setting $I = \pi d^4/64$ and solving for d

$$d = \left|\frac{32}{3\pi E l \xi}\left\{\sum\left[F_i b_i\left(b_i^2 - l^2\right)\right]^2\right\}^{1/2}\right|^{1/4}$$

For the LH bearing, $E = 30$ Mpsi, $\xi = 0.001$, $b_1 = 12$, $b_2 = 6$, and $l = 16$. The result is $d_L = 1.31$ in. Using a similar flip beam procedure, we get $d_R = 1.36$ in for the RH bearing. So use $d = 1\ 3/8$ in *Ans.*

4-27 $I = \frac{\pi}{64}(1.375^4) = 0.17546\ \text{in}^4$. For the *xy* plane, use y_{BC} of Table A-9-6

$$y = \frac{100(4)(16-8)}{6(30)(10^6)(0.17546)(16)}[8^2 + 4^2 - 2(16)8] = -1.115(10^{-3})\ \text{in}$$

For the *xz* plane use y_{AB}

$$z = \frac{300(6)(8)}{6(30)(10^6)(0.17546)(16)}[8^2 + 6^2 - 16^2] = -4.445(10^{-3})\ \text{in}$$

$$\delta = (-1.115\mathbf{j} - 4.445\mathbf{k})(10^{-3})\ \text{in}$$

$$|\delta| = 4.583(10^{-3})\ \text{in} \quad \textit{Ans.}$$

4-28

$$d_L = \left|\frac{32n}{3\pi E l \xi}\left\{\sum\left[F_i b_i\left(b_i^2 - l^2\right)\right]^2\right\}^{1/2}\right|^{1/4}$$

$$= \left|\frac{32(1.5)}{3\pi(207)(10^9)(250)0.001}\left\{[3.5(150)(150^2 - 250^2)]^2 + [2.7(75)(75^2 - 250^2)]^2\right\}^{1/2}(10^3)^3\right|^{1/4}$$

$$= 39.2\ \text{mm}$$

$$d_R = \left|\frac{32(1.5)}{3\pi(207)10^9(250)0.001}\left\{[3.5(100)(100^2 - 250^2)]^2 + [2.7(175)(175^2 - 250^2)]^2\right\}^{1/2}(10^3)^3\right|^{1/4}$$

$$= 39.1\ \text{mm}$$

Choose $d \geq 39.2$ mm *Ans.*

4-29 From Table A-9-8 we have

$$y_L = \frac{M_B x}{6EIl}(x^2 + 3a^2 - 6al + 2l^2)$$

$$\frac{dy_L}{dx} = \frac{M_B}{6EIl}(3x^2 + 3a^2 - 6al + 2l^2)$$

At $x = 0$, the LH slope is

$$\theta_L = \frac{dy_L}{dx} = \frac{M_B}{6EIl}(3a^2 - 6al + 2l^2)$$

from which

$$\xi = |\theta_L| = \frac{M_B}{6EIl}(l^2 - 3b^2)$$

Setting $I = \pi d^4/64$ and solving for d

$$d = \left| \frac{32M_B(l^2 - 3b^2)}{3\pi El\xi} \right|^{1/4}$$

For a multiplicity of moments, the slopes add vectorially and

$$d_L = \left| \frac{32}{3\pi El\xi} \left\{ \sum \left[M_i \left(l^2 - 3b_i^2 \right) \right]^2 \right\}^{1/2} \right|^{1/4}$$

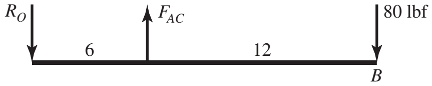
$$d_R = \left| \frac{32}{3\pi El\xi} \left\{ \sum \left[M_i \left(3a_i^2 - l^2 \right) \right]^2 \right\}^{1/2} \right|^{1/4}$$

The greatest slope is at the LH bearing. So

$$d = \left| \frac{32(1200)[9^2 - 3(4^2)]}{3\pi(30)(10^6)(9)(0.002)} \right|^{1/4} = 0.706 \text{ in}$$

So use $d = 3/4$ in *Ans.*

4-30



$$6F_{AC} = 18(80)$$
$$F_{AC} = 240 \text{ lbf}$$
$$R_O = 160 \text{ lbf}$$

$$I = \frac{1}{12}(0.25)(2^3) = 0.1667 \text{ in}^4$$

Initially, ignore the stretch of AC. From Table A-9-10

$$y_{B1} = -\frac{Fa^2}{3EI}(l + a) = -\frac{80(12^2)}{3(10)(10^6)(0.1667)}(6 + 12) = -0.041\,47 \text{ in}$$

Stretch of AC: $\delta = \left(\frac{FL}{AE} \right)_{AC} = \frac{240(12)}{(\pi/4)(1/2)^2(10)(10^6)} = 1.4668(10^{-3})$ in

Due to stretch of AC

$$y_{B2} = -3\delta = -4.400(10^{-3}) \text{ in}$$

By superposition, $y_B = -0.041\,47 - 0.0044 = -0.045\,87$ in *Ans.*

4-31

$$\theta = \frac{TL}{JG} = \frac{(0.1F)(1.5)}{(\pi/32)(0.012^4)(79.3)(10^9)} = 9.292(10^{-4})F$$

Due to twist

$$\delta_{B1} = 0.1(\theta) = 9.292(10^{-5})F$$

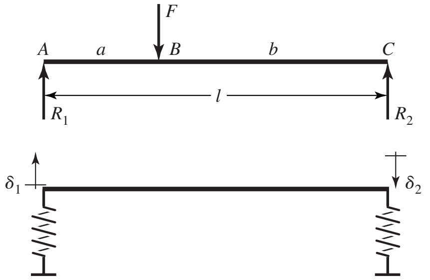
Due to bending

$$\delta_{B2} = \frac{FL^3}{3EI} = \frac{F(0.1^3)}{3(207)(10^9)(\pi/64)(0.012^4)} = 1.582(10^{-6})F$$

$$\delta_B = 1.582(10^{-6})F + 9.292(10^{-5})F = 9.450(10^{-5})F$$

$$k = \frac{1}{9.450(10^{-5})} = 10.58(10^3)\ \text{N/m} = 10.58\ \text{kN/m} \quad \textit{Ans.}$$

4-32



$$R_1 = \frac{Fb}{l} \quad R_2 = \frac{Fa}{l}$$

$$\delta_1 = \frac{R_1}{k_1} \quad \delta_2 = \frac{R_2}{k_2}$$

Spring deflection

$$y_S = -\delta_1 + \left(\frac{\delta_1 - \delta_2}{l}\right)x = -\frac{Fb}{k_1 l} + \left(\frac{Fb}{k_1 l^2} - \frac{Fa}{k_2 l^2}\right)x$$

$$y_{AB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2) + \frac{Fx}{l^2}\left(\frac{b}{k_1} - \frac{a}{k_2}\right) - \frac{Fb}{k_1 l} \quad \textit{Ans.}$$

$$y_{BC} = \frac{Fa(l-x)}{6EIl}(x^2 + a^2 - 2lx) + \frac{Fx}{l^2}\left(\frac{b}{k_1} - \frac{a}{k_2}\right) - \frac{Fb}{k_1 l} \quad \textit{Ans.}$$

4-33 See Prob. 4-32 for deflection due to springs. Replace Fb/l and Fa/l with $wl/2$

$$y_S = -\frac{wl}{2k_1} + \left(\frac{wl}{2k_1 l} - \frac{wl}{2k_2 l}\right)x = \frac{wx}{2}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) - \frac{wl}{2k_1}$$

$$y = \frac{wx}{24EI}(2lx^2 - x^3 - l^3) + \frac{wx}{2}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) - \frac{wl}{2k_1} \quad \textit{Ans.}$$

4-34 Let the load be at $x > l/2$. The maximum deflection will be in Section AB (Table A-9-10)

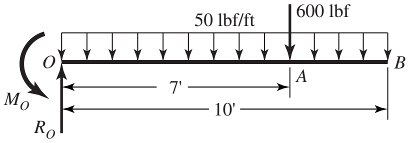
$$y_{AB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2)$$

$$\frac{dy_{AB}}{dx} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2) = 0 \quad \Rightarrow \quad 3x^2 + b^2 - l^2 = 0$$

$$x = \sqrt{\frac{l^2 - b^2}{3}}, \qquad x_{\max} = \sqrt{\frac{l^2}{3}} = 0.577l \quad Ans.$$

For $x < l/2 \quad x_{\min} = l - 0.577l = 0.423l \quad Ans.$

4-35



$$M_O = 50(10)(60) + 600(84)$$
$$= 80\,400 \text{ lbf} \cdot \text{in}$$
$$R_O = 50(10) + 600 = 1100 \text{ lbf}$$

$I = 11.12$ in^4 from Prob. 4-12

$$M = -80\,400 + 1100x - \frac{4.167x^2}{2} - 600\langle x - 84\rangle^1$$

$$EI\frac{dy}{dx} = -80\,400x + 550x^2 - 0.6944x^3 - 300\langle x - 84\rangle^2 + C_1$$

$\frac{dy}{dx} = 0$ at $x = 0 \quad \therefore C_1 = 0$

$$EIy = -402\,00x^2 + 183.33x^3 - 0.1736x^4 - 100\langle x - 84\rangle^3 + C_2$$

$y = 0$ at $x = 0 \quad \therefore C_2 = 0$

$$y_B = \frac{1}{30(10^6)(11.12)}[-40\,200(120^2) + 183.33(120^3) - 0.1736(120^4) - 100(120 - 84)^3]$$
$$= -0.9075 \text{ in} \quad Ans.$$

4-36 See Prob. 4-13 for reactions: $R_O = 860$ lbf, $R_C = 540$ lbf

$$M = 860x - 800\langle x - 36\rangle^1 - 600\langle x - 60\rangle^1$$

$$EI\frac{dy}{dx} = 430x^2 - 400\langle x - 36\rangle^2 - 300\langle x - 60\rangle^2 + C_1$$

$$EIy = 143.33x^3 - 133.33\langle x - 36\rangle^3 - 100\langle x - 60\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

$y = 0$ at $x = 120$ in $\Rightarrow C_1 = -1.2254(10^6)$ lbf $\cdot$ in^2

Substituting C_1 and C_2 and evaluating at $x = 60$,

$$EIy = 30(10^6)I\left(-\frac{1}{16}\right) = 143.33(60^3) - 133.33(60 - 36)^3 - 1.2254(10^6)(60)$$
$$I = 23.68 \text{ in}^4$$

Agrees with Prob. 4-13. The rest of the solution is the same.

4-37

$$I = \frac{\pi}{64}(40^4) = 125.66(10^3)\ \text{mm}^4$$

$$R_O = 2(500) + \frac{600}{1000}1500 = 1900\ \text{N}$$

$$M = 1900x - \frac{2000}{2}x^2 - 1500\langle x - 0.4\rangle^1 \ \text{ where } x \text{ is in meters}$$

$$EI\frac{dy}{dx} = 950x^2 - \frac{1000}{3}x^3 - 750\langle x - 0.4\rangle^2 + C_1$$

$$EIy = \frac{900}{3}x^3 - \frac{250}{3}x^4 - 250\langle x - 0.4\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

$y = 0$ at $x = 1\ \text{m} \Rightarrow C_1 = -179.33\ \text{N}\cdot\text{m}^2$

Substituting C_1 and C_2 and evaluating y at $x = 0.4$ m,

$$y_A = \frac{1}{207(10^9)125.66(10^{-9})}\left[\frac{950}{3}(0.4^3) - \frac{250}{3}(0.4^4) - 179.33(0.4)\right]10^3$$

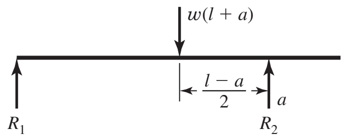
$$= -2.061\ \text{mm} \quad \textit{Ans.}$$

$$y|_{x=500} = \frac{1}{207(10^9)125.66(10^{-9})}\left[\frac{950}{3}(0.5^3) - \frac{250}{3}(0.5^4) - 250(0.5 - 0.4)^3 - 179.33(0.5)\right]10^3$$

$$= -2.135\ \text{mm} \quad \textit{Ans.}$$

$$\%\ \text{difference} = \frac{2.135 - 2.061}{2.061}(100) = 3.59\% \quad \textit{Ans.}$$

4-38



$$R_1 = \frac{w(l + a)[(l - a)/2)]}{l}$$

$$= \frac{w}{2l}(l^2 - a^2)$$

$$R_2 = w(l + a) - \frac{w}{2l}(l^2 - a^2) = \frac{w}{2l}(l + a)^2$$

$$M = \frac{w}{2l}(l^2 - a^2)x - \frac{wx^2}{2} + \frac{w}{2l}(l + a)^2\langle x - l\rangle^1$$

$$EI\frac{dy}{dx} = \frac{w}{4l}(l^2 - a^2)x^2 - \frac{w}{6}x^3 + \frac{w}{4l}(l + a)^2\langle x - l\rangle^2 + C_1$$

$$EIy = \frac{w}{12l}(l^2 - a^2)x^3 - \frac{w}{24}x^4 + \frac{w}{12l}(l + a)^2\langle x - l\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

$y = 0$ at $x = l$

$$0 = \frac{w}{12l}(l^2 - a^2)l^3 - \frac{w}{24}l^4 + C_1 l \quad \Rightarrow \quad C_1 = \frac{wl}{24}(2a^2 - l^2)$$

$$y = \frac{w}{24EIl}[2(l^2 - a^2)x^3 - lx^4 + 2(l+a)^2\langle x - l\rangle^3 + l^2(2a^2 - l^2)x] \quad Ans.$$

4-39 $R_A = R_B = 500$ N, and $I = \frac{1}{12}(9)35^3 = 32.156(10^3)$ mm^4

For first half of beam, $M = -500x + 500\langle x - 0.25\rangle^1$ where x is in meters

$$EI\frac{dy}{dx} = -250x^2 + 250\langle x - 0.25\rangle^2 + C_1$$

At $x = 0.5$ m, $dy/dx = 0 \Rightarrow 0 = -250(0.5^2) + 250(0.5 - 0.250)^2 + C_1 \Rightarrow C_1 = 46.875\ \text{N}\cdot\text{m}^2$

$$EIy = -\frac{250}{3}x^3 + \frac{250}{3}\langle x - 0.25\rangle^3 + 46.875x + C_2$$

$$y = 0 \text{ at } x = 0.25 \text{ m} \Rightarrow 0 = -\frac{250}{3}0.25^3 + 46.875(0.25) + C_2 \Rightarrow C_2 = -10.417\ \text{N}\cdot\text{m}^3$$

$$\therefore EIy = -\frac{250}{3}x^3 + \frac{250}{3}\langle x - 0.25\rangle^3 + 46.875x - 10.42$$

Evaluating y at A and the center,

$$y_A = \frac{1}{207(10^9)32.156(10^{-9})}\left[-\frac{250}{3}(0^3) + \frac{250}{3}(0)^3 + 46.875(0) - 10.417\right]10^3$$

$$= -1.565 \text{ mm} \quad Ans.$$

$$y|_{x=0.5\text{m}} = \frac{1}{207(10^9)32.156(10^{-9})}\left[-\frac{250}{3}(0.5^3) + \frac{250}{3}(0.5 - 0.25)^3 + 46.875(0.5) - 10.417\right]10^3$$

$$= -2.135 \text{ mm} \quad Ans.$$

4-40 From Prob. 4-30, $R_O = 160$ lbf ↓, $F_{AC} = 240$ lbf $\quad I = 0.1667$ in^4

$$M = -160x + 240\langle x - 6\rangle^1$$

$$EI\frac{dy}{dx} = -80x^2 + 120\langle x - 6\rangle^2 + C_1$$

$$EIy = -26.67x^3 + 40\langle x - 6\rangle^3 + C_1 x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

$$y_A = -\left(\frac{FL}{AE}\right)_{AC} = -\frac{240(12)}{(\pi/4)(1/2)^2(10)(10^6)} = -1.4668(10^{-3}) \text{ in}$$

at $x = 6$

$$10(10^6)(0.1667)(-1.4668)(10^{-3}) = -26.67(6^3) + C_1(6)$$

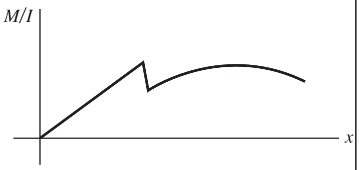
$$C_1 = 552.58\ \text{lbf}\cdot\text{in}^2$$

$$y_B = \frac{1}{10(10^6)(0.1667)}[-26.67(18^3) + 40(18-6)^3 + 552.58(18)]$$

$$= -0.045\,87 \text{ in} \quad Ans.$$

4-41

$$I_1 = \frac{\pi}{64}(1.5^4) = 0.2485 \text{ in}^4 \qquad I_2 = \frac{\pi}{64}(2^4) = 0.7854 \text{ in}^4$$



$$R_1 = \frac{200}{2}(12) = 1200 \text{ lbf}$$

For $0 \le x \le 16$ in, $M = 1200x - \frac{200}{2}\langle x-4\rangle^2$

$$\frac{M}{I} = \frac{1200x}{I_1} - 4800\left(\frac{1}{I_1} - \frac{1}{I_2}\right)\langle x-4\rangle^0 - 1200\left(\frac{1}{I_1} - \frac{1}{I_2}\right)\langle x-4\rangle^1 - \frac{100}{I_2}\langle x-4\rangle^2$$

$$= 4829x - 13\,204\langle x-4\rangle^0 - 3301.1\langle x-4\rangle^1 - 127.32\langle x-4\rangle^2$$

$$E\frac{dy}{dx} = 2414.5x^2 - 13\,204\langle x-4\rangle^1 - 1651\langle x-4\rangle^2 - 42.44\langle x-4\rangle^3 + C_1$$

Boundary Condition: $\frac{dy}{dx} = 0$ at $x = 10$ in

$$0 = 2414.5(10^2) - 13\,204(10-4)^1 - 1651(10-4)^2 - 42.44(10-4)^3 + C_1$$

$$C_1 = -9.362(10^4)$$

$$Ey = 804.83x^3 - 6602\langle x-4\rangle^2 - 550.3\langle x-4\rangle^3 - 10.61\langle x-4\rangle^4 - 9.362(10^4)x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

For $0 \le x \le 16$ in

$$y = \frac{1}{30(10^6)}[804.83x^3 - 6602\langle x-4\rangle^2 - 550.3\langle x-4\rangle^3 - 10.61\langle x-4\rangle^4 - 9.362(10^4)x] \quad Ans.$$

at $x = 10$ in

$$y|_{x=10} = \frac{1}{30(10^6)}[804.83(10^3) - 6602(10-4)^2 - 550.3(10-4)^3 - 10.61(10-4)^4 - 9.362(10^4)(10)]$$

$$= -0.016\,72 \text{ in} \quad Ans.$$

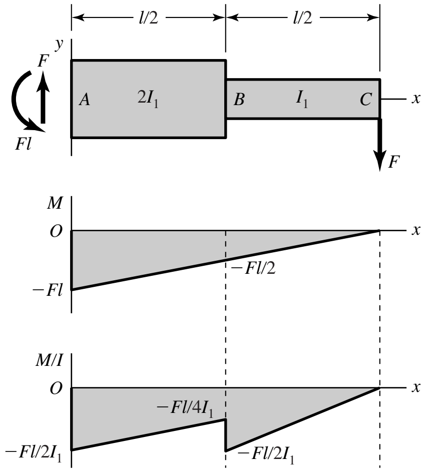
4-42 $q = F\langle x\rangle^{-1} - Fl\langle x\rangle^{-2} - F\langle x-l\rangle^{-1}$

Integrations produce

$$V = F\langle x\rangle^0 - Fl\langle x\rangle^{-1} - F\langle x-l\rangle^0$$

$$M = F\langle x\rangle^1 - Fl\langle x\rangle^0 - F\langle x-l\rangle^1 = Fx - Fl$$

Plots for M and M/I are shown below



M/I can be expressed by singularity functions as

$$\frac{M}{I} = \frac{F}{2I_1}x - \frac{Fl}{2I_1} - \frac{Fl}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^0 + \frac{F}{2I_1}\left\langle x - \frac{l}{2}\right\rangle^1$$

where the step down and increase in slope at $x = l/2$ are given by the last two terms.

Since $E\,d^2y/dx^2 = M/I$, two integrations yield

$$E\frac{dy}{dx} = \frac{F}{4I_1}x^2 - \frac{Fl}{2I_1}x - \frac{Fl}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^1 + \frac{F}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^2 + C_1$$

$$Ey = \frac{F}{12I_1}x^3 - \frac{Fl}{4I_1}x^2 - \frac{Fl}{8I_1}\left\langle x - \frac{l}{2}\right\rangle^2 + \frac{F}{12I_1}\left\langle x - \frac{l}{2}\right\rangle^3 + C_1x + C_2$$

At $x = 0$, $y = dy/dx = 0$. This gives $C_1 = C_2 = 0$, and

$$y = \frac{F}{24EI_1}\left(2x^3 - 6lx^2 - 3l\left\langle x - \frac{l}{2}\right\rangle^2 + 2\left\langle x - \frac{l}{2}\right\rangle^3\right)$$

At $x = l/2$ and l,

$$y|_{x=l/2} = \frac{F}{24EI_1}\left[2\left(\frac{l}{2}\right)^3 - 6l\left(\frac{l}{2}\right)^2 - 3l(0) + 2(0)\right] = -\frac{5Fl^3}{96EI_1} \qquad Ans.$$

$$y|_{x=l} = \frac{F}{24EI_1}\left[2(l)^3 - 6l(l)^2 - 3l\left(l - \frac{l}{2}\right)^2 + 2\left(l - \frac{l}{2}\right)^3\right] = -\frac{3Fl^3}{16EI_1} \qquad Ans.$$

The answers are identical to Ex. 4-11.

4-43 Define δ_{ij} as the deflection in the direction of the load at station i due to a unit load at station j. If U is the potential energy of strain for a body obeying Hooke's law, apply P_1 first. Then

$$U = \frac{1}{2}P_1(P_1\delta_{11})$$

When the second load is added, U becomes

$$U = \frac{1}{2}P_1(P_1\delta_{11}) + \frac{1}{2}P_2(P_2\,\delta_{22}) + P_1(P_2\,\delta_{12})$$

For loading in the reverse order

$$U' = \frac{1}{2}P_2(P_2\,\delta_{22}) + \frac{1}{2}P_1(P_1\delta_{11}) + P_2(P_1\,\delta_{21})$$

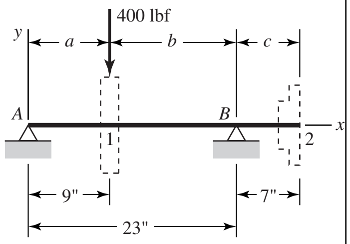
Since the order of loading is immaterial $U = U'$ and

$$P_1P_2\delta_{12} = P_2P_1\delta_{21} \qquad \text{when } P_1 = P_2,\ \delta_{12} = \delta_{21}$$

which states that the deflection at station 1 due to a unit load at station 2 is the same as the deflection at station 2 due to a unit load at 1. δ is sometimes called an *influence coefficient*.

4-44

(a) From Table A-9-10



$$y_{AB} = \frac{Fcx(l^2 - x^2)}{6EIl}$$

$$\delta_{12} = \frac{y}{F}\bigg|_{x=a} = \frac{ca(l^2 - a^2)}{6EIl}$$

$$y_2 = F\delta_{21} = F\delta_{12} = \frac{Fca(l^2 - a^2)}{6EIl}$$

Substituting $I = \dfrac{\pi d^4}{64}$

$$y_2 = \frac{400(7)(9)(23^2 - 9^2)(64)}{6(30)(10^6)(\pi)(2)^4(23)} = 0.00347 \text{ in} \quad \textit{Ans.}$$

(b) The slope of the shaft at *left* bearing at $x = 0$ is

$$\theta = \frac{Fb(b^2 - l^2)}{6EIl}$$

Viewing the illustration in Section 6 of Table A-9 from the back of the page provides the correct view of this problem. Noting that a is to be interchanged with b and $-x$ with x leads to

$$\theta = \frac{Fa(l^2 - a^2)}{6EIl} = \frac{Fa(l^2 - a^2)(64)}{6E\pi d^4 l}$$

$$\theta = \frac{400(9)(23^2 - 9^2)(64)}{6(30)(10^6)(\pi)(2)^4(23)} = 0.000\,496 \text{ in/in}$$

So $y_2 = 7\theta = 7(0.000\,496) = 0.00347$ in *Ans.*

4-45 Place a dummy load Q at the center. Then,

$$M = \frac{wx}{2}(l-x) + \frac{Qx}{2}$$

$$U = 2\int_0^{l/2} \frac{M^2 dx}{2EI}, \quad y_{\max} = \left.\frac{\partial U}{\partial Q}\right|_{Q=0}$$

$$y_{\max} = 2\left[\int_0^{l/2} \frac{2M}{2EI}\left(\frac{\partial M}{\partial Q}\right) dx\right]_{Q=0}$$

$$y_{\max} = \frac{2}{EI}\left\{\int_0^{l/2}\left[\frac{wx}{2}(l-x) + \frac{Qx}{2}\right]\frac{x}{2}\,dx\right\}_{Q=0}$$

Set $Q = 0$ and integrate

$$y_{\max} = \frac{w}{2EI}\left(\frac{lx^3}{3} - \frac{x^4}{4}\right)_0^{l/2}$$

$$y_{\max} = \frac{5wl^4}{384EI} \quad \textit{Ans.}$$

4-46

$$I = 2(1.85) = 3.7 \text{ in}^4$$

Adding weight of channels of 0.833 lbf · in,

$$M = -Fx - \frac{10.833}{2}x^2 = -Fx - 5.417x^2 \qquad \frac{\partial M}{\partial F} = -x$$

$$\delta_B = \frac{1}{EI}\int_0^{48} M\frac{\partial M}{\partial F}\,dx = \frac{1}{EI}\int_0^{48}(Fx + 5.417x^2)(x)\,dx$$

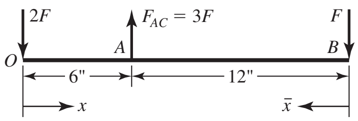
$$= \frac{(220/3)(48^3) + (5.417/4)(48^4)}{30(10^6)(3.7)} = 0.1378 \text{ in} \qquad \text{in direction of 220 lbf}$$

$$\therefore y_B = -0.1378 \text{ in} \quad \textit{Ans.}$$

4-47

$$I_{OB} = \frac{1}{12}(0.25)(2^3) = 0.1667 \text{ in}^4, \quad A_{AC} = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.196\,35 \text{ in}^2$$

$$F_{AC} = 3F, \quad \frac{\partial F_{AC}}{\partial F} = 3$$



right left

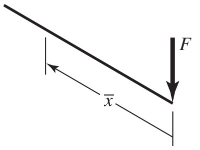
$$M = -F\bar{x} \quad M = -2Fx$$

$$\frac{\partial M}{\partial F} = -\bar{x} \quad \frac{\partial M}{\partial F} = -2x$$

$$U = \frac{1}{2EI}\int_0^l M^2 dx + \frac{F_{AC}^2 L_{AC}}{2A_{AC}E}$$

$$\begin{aligned}
\delta_B = \frac{\partial U}{\partial F} &= \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial F}\,dx + \frac{F_{AC}(\partial F_{AC}/\partial F)L_{AC}}{A_{AC}E} \\
&= \frac{1}{EI}\left[\int_0^{12} -F\bar{x}(-\bar{x})\,d\bar{x} + \int_0^6 (-2Fx)(-2x)\,dx\right] + \frac{3F(3)(12)}{A_{AC}E} \\
&= \frac{1}{EI}\left[\frac{F}{3}(12^3) + 4F\left(\frac{6^3}{3}\right)\right] + \frac{108F}{A_{AC}E} \\
&= \frac{864F}{EI} + \frac{108F}{A_{AC}E} \\
&= \frac{864(80)}{10(10^6)(0.1667)} + \frac{108(80)}{0.196\,35(10)(10^6)} = 0.045\,86 \text{ in} \quad \textit{Ans.}
\end{aligned}$$

4-48



Torsion $\qquad T = 0.1F \qquad \dfrac{\partial T}{\partial F} = 0.1$

Bending $\qquad M = -F\bar{x} \qquad \dfrac{\partial M}{\partial F} = -\bar{x}$

$$U = \frac{1}{2EI}\int M^2\,dx + \frac{T^2L}{2JG}$$

$$\begin{aligned}
\delta_B = \frac{\partial U}{\partial F} &= \frac{1}{EI}\int M\frac{\partial M}{\partial F}\,dx + \frac{T(\partial T/\partial F)L}{JG} \\
&= \frac{1}{EI}\int_0^{0.1} -F\bar{x}(-\bar{x})\,d\bar{x} + \frac{0.1F(0.1)(1.5)}{JG} \\
&= \frac{F}{3EI}(0.1^3) + \frac{0.015F}{JG}
\end{aligned}$$

Where

$$I = \frac{\pi}{64}(0.012)^4 = 1.0179(10^{-9})\text{ m}^4$$

$$J = 2I = 2.0358(10^{-9})\text{ m}^4$$

$$\delta_B = F\left[\frac{0.001}{3(207)(10^9)(1.0179)(10^{-9})} + \frac{0.015}{2.0358(10^{-9})(79.3)(10^9)}\right] = 9.45(10^{-5})F$$

$$k = \frac{1}{9.45(10^{-5})} = 10.58(10^3)\text{ N/m} = 10.58\text{ kN/m} \quad \textit{Ans.}$$

4-49 From Prob. 4-41, $I_1 = 0.2485\text{ in}^4$, $I_2 = 0.7854\text{ in}^4$

For a dummy load ↑ Q at the center

$$0 \le x \le 10\text{ in} \qquad M = 1200x - \frac{Q}{2}x - \frac{200}{2}\langle x-4\rangle^2, \qquad \frac{\partial M}{\partial Q} = \frac{-x}{2}$$

$$y|_{x=10} = \frac{\partial U}{\partial Q}\bigg|_{Q=0}$$

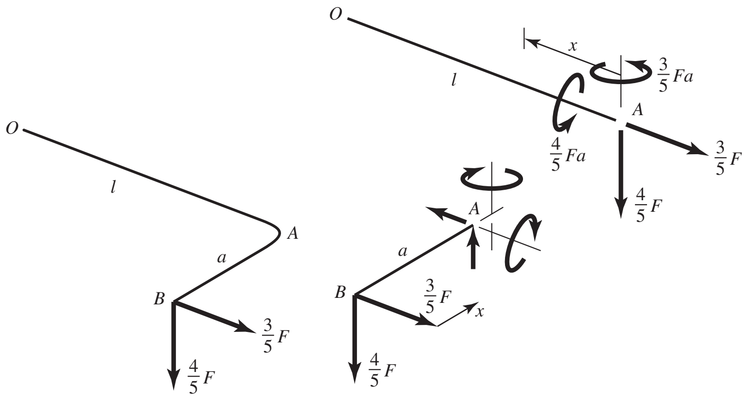
$$= \frac{2}{E}\left\{\frac{1}{I_1}\int_0^4 (1200x)\left(-\frac{x}{2}\right)dx + \frac{1}{I_2}\int_4^{10}[1200x - 100(x-4)^2]\left(-\frac{x}{2}\right)dx\right\}$$

$$= \frac{2}{E}\left[-\frac{200(4^3)}{I_1} - \frac{1.566(10^5)}{I_2}\right]$$

$$= -\frac{2}{30(10^6)}\left(\frac{1.28(10^4)}{0.2485} + \frac{1.566(10^5)}{0.7854}\right)$$

$$= -0.016\,73\text{ in} \quad \textit{Ans.}$$

4-50



AB

$$M = Fx \qquad \frac{\partial M}{\partial F} = x$$

OA

$$N = \frac{3}{5}F \qquad \frac{\partial N}{\partial F} = \frac{3}{5}$$

$$T = \frac{4}{5}Fa \qquad \frac{\partial T}{\partial F} = \frac{4}{5}a$$

$$M_1 = \frac{4}{5}F\bar{x} \qquad \frac{\partial M_1}{\partial F} = \frac{4}{5}\bar{x}$$

$$M_2 = \frac{3}{5}Fa \qquad \frac{\partial M_2}{\partial F} = \frac{3}{5}a$$

$$\delta_B = \frac{\partial u}{\partial F} = \frac{1}{EI}\int_0^a Fx(x)\,dx + \frac{(3/5)F(3/5)l}{AE} + \frac{(4/5)Fa(4a/5)l}{JG}$$

$$+ \frac{1}{EI}\int_0^l \frac{4}{5}F\bar{x}\left(\frac{4}{5}\bar{x}\right)d\bar{x} + \frac{1}{EI}\int_0^l \frac{3}{5}Fa\left(\frac{3}{5}a\right)d\bar{x}$$

$$= \frac{Fa^3}{3EI} + \frac{9}{25}\left(\frac{Fl}{AE}\right) + \frac{16}{25}\left(\frac{Fa^2 l}{JG}\right) + \frac{16}{75}\left(\frac{Fl^3}{EI}\right) + \frac{9}{25}\left(\frac{Fa^2 l}{EI}\right)$$

$$I = \frac{\pi}{64}d^4, \quad J = 2I, \quad A = \frac{\pi}{4}d^2$$

$$\delta_B = \frac{64Fa^3}{3E\pi d^4} + \frac{9}{25}\left(\frac{4Fl}{\pi d^2 E}\right) + \frac{16}{25}\left(\frac{32Fa^2 l}{\pi d^4 G}\right) + \frac{16}{75}\left(\frac{64Fl^3}{E\pi d^4}\right) + \frac{9}{25}\left(\frac{64Fa^2 l}{E\pi d^4}\right)$$

$$= \frac{4F}{75\pi Ed^4}\left(400a^3 + 27ld^2 + 384a^2 l\frac{E}{G} + 256l^3 + 432a^2 l\right) \quad \textit{Ans.}$$

4-51 The force applied to the copper and steel wire assembly is $F_c + F_s = 250$ lbf

Since $\delta_c = \delta_s$

$$\frac{F_c L}{3(\pi/4)(0.0801)^2(17.2)(10^6)} = \frac{F_s L}{(\pi/4)(0.0625)^2(30)(10^6)}$$

$$F_c = 2.825F_s$$

$$\therefore 3.825F_s = 250 \quad \Rightarrow \quad F_s = 65.36 \text{ lbf}, \quad F_c = 2.825F_s = 184.64 \text{ lbf}$$

$$\sigma_c = \frac{184.64}{3(\pi/4)(0.0801)^2} = 12\,200 \text{ psi} = 12.2 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_s = \frac{65.36}{(\pi/4)(0.0625^2)} = 21\,300 \text{ psi} = 21.3 \text{ kpsi} \quad \textit{Ans.}$$

4-52

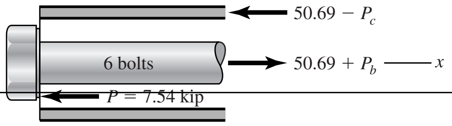
(a) Bolt stress $\sigma_b = 0.9(85) = 76.5$ kpsi *Ans.*

Bolt force $F_b = 6(76.5)\left(\frac{\pi}{4}\right)(0.375^2) = 50.69$ kips

Cylinder stress $\sigma_c = -\frac{F_b}{A_c} = -\frac{50.69}{(\pi/4)(4.5^2 - 4^2)} = -15.19$ kpsi *Ans.*

(b) Force from pressure

$$P = \frac{\pi D^2}{4}p = \frac{\pi(4^2)}{4}(600) = 7540 \text{ lbf} = 7.54 \text{ kip}$$



$$\sum F_x = 0$$

$$P_b + P_c = 7.54 \quad (1)$$

Since $\delta_c = \delta_b$, $$\frac{P_c L}{(\pi/4)(4.5^2 - 4^2)E} = \frac{P_b L}{6(\pi/4)(0.375^2)E}$$

$$P_c = 5.037 P_b \quad (2)$$

Substituting into Eq. (1)

$6.037 P_b = 7.54 \quad \Rightarrow \quad P_b = 1.249$ kip; and from Eq. (2), $P_c = 6.291$ kip

Using the results of (a) above, the total bolt and cylinder stresses are

$$\sigma_b = 76.5 + \frac{1.249}{6(\pi/4)(0.375^2)} = 78.4 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_c = -15.19 + \frac{6.291}{(\pi/4)(4.5^2 - 4^2)} = -13.3 \text{ kpsi} \quad \textit{Ans.}$$

4-53

$$T = T_c + T_s \quad \text{and} \quad \theta_c = \theta_s$$

Also, $$\frac{T_c L}{(GJ)_c} = \frac{T_s L}{(GJ)_s}$$

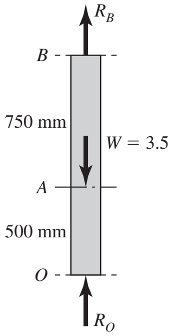
$$T_c = \frac{(GJ)_c}{(GJ)_s} T_s$$

Substituting into equation for T,

$$T = \left[1 + \frac{(GJ)_c}{(GJ)_s}\right] T_s$$

$$\%T_s = \frac{T_s}{T} = \frac{(GJ)_s}{(GJ)_s + (GJ)_c} \quad \textit{Ans.}$$

4-54



$$R_O + R_B = W \quad (1)$$

$$\delta_{OA} = \delta_{AB} \quad (2)$$

$$\frac{500 R_O}{AE} = \frac{750 R_B}{AE}, \quad R_O = \frac{3}{2} R_B$$

$$\frac{3}{2} R_B + R_B = 3.5$$

$$R_B = \frac{7}{5} = 1.4 \text{ kN} \quad \textit{Ans.}$$

$$R_O = 3.5 - 1.4 = 2.1 \text{ kN} \quad \textit{Ans.}$$

$$\sigma_O = -\frac{2100}{12(50)} = -3.50 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_B = \frac{1400}{12(50)} = 2.33 \text{ MPa} \quad \textit{Ans.}$$

4-55 Since $\theta_{OA} = \theta_{AB}$

$$\frac{T_{OA}(4)}{GJ} = \frac{T_{AB}(6)}{GJ}, \quad T_{OA} = \frac{3}{2}T_{AB}$$

Also $T_{OA} + T_{AB} = 50$

$$T_{AB}\left(\frac{3}{2}+1\right) = 50, \quad T_{AB} = \frac{50}{2.5} = 20\,\text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_{OA} = \frac{3}{2}T_{AB} = \frac{3}{2}(20) = 30\,\text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

4-56 Since $\theta_{OA} = \theta_{AB}$,

$$\frac{T_{OA}(4)}{G(\frac{\pi}{32}1.5^4)} = \frac{T_{AB}(6)}{G(\frac{\pi}{32}1.75^4)}, \quad T_{OA} = 0.80966T_{AB}$$

$$T_{OA} + T_{AB} = 50 \ \Rightarrow\ 0.80966\,T_{AB} + T_{AB} = 50 \ \Rightarrow\ T_{AB} = 27.63\ \text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_{OA} = 0.80966T_{AB} = 0.80966(27.63) = 22.37\ \text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

4-57

$$F_1 = F_2 \quad \Rightarrow \quad \frac{T_1}{r_1} = \frac{T_2}{r_2} \quad \Rightarrow \quad \frac{T_1}{1.25} = \frac{T_2}{3}$$

$$T_2 = \frac{3}{1.25}T_1$$

$$\therefore \theta_1 + \frac{3}{1.25}\theta_2 = \frac{4\pi}{180}\ \text{rad}$$

$$\frac{T_1(48)}{(\pi/32)(7/8)^4(11.5)(10^6)} + \frac{3}{1.25}\left[\frac{(3/1.25)T_1(48)}{(\pi/32)(1.25)^4(11.5)(10^6)}\right] = \frac{4\pi}{180}$$

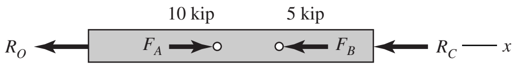
$$T_1 = 403.9\ \text{lbf}\cdot\text{in}$$

$$T_2 = \frac{3}{1.25}T_1 = 969.4\ \text{lbf}\cdot\text{in}$$

$$\tau_1 = \frac{16T_1}{\pi d^3} = \frac{16(403.9)}{\pi(7/8)^3} = 3071\ \text{psi} \quad \textit{Ans.}$$

$$\tau_2 = \frac{16(969.4)}{\pi(1.25)^3} = 2528\ \text{psi} \quad \textit{Ans.}$$

4-58



(1) Arbitrarily, choose R_C as redundant reaction

(2) $$\sum F_x = 0, \quad 10(10^3) - 5(10^3) - R_O - R_C = 0$$

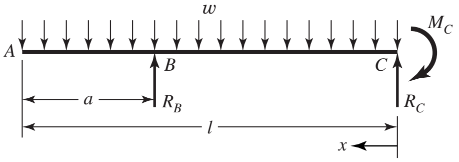
$$R_O + R_C = 5(10^3) \text{ lbf}$$

(3) $$\delta_C = \frac{[10(10^3) - 5(10^3) - R_C]20}{AE} - \frac{[5(10^3) + R_C]}{AE}(10) - \frac{R_C(15)}{AE} = 0$$

$$-45R_C + 5(10^4) = 0 \quad \Rightarrow \quad R_C = 1111 \text{ lbf} \quad \textit{Ans.}$$

$$R_O = 5000 - 1111 = 3889 \text{ lbf} \quad \textit{Ans.}$$

4-59



(1) Choose R_B as redundant reaction

(2) $$R_B + R_C = wl \quad (a) \qquad R_B(l - a) - \frac{wl^2}{2} + M_C = 0 \quad (b)$$

(3) $$y_B = \frac{R_B(l-a)^3}{3EI} + \frac{w(l-a)^2}{24EI}[4l(l-a) - (l-a)^2 - 6l^2] = 0$$

$$R_B = \frac{w}{8(l-a)}[6l^2 - 4l(l-a) + (l-a)^2]$$

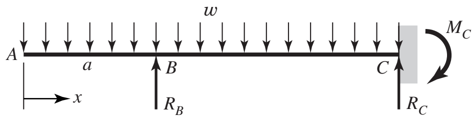
$$= \frac{w}{8(l-a)}(3l^2 + 2al + a^2) \quad \textit{Ans.}$$

Substituting,

Eq. (*a*) $$R_C = wl - R_B = \frac{w}{8(l-a)}(5l^2 - 10al - a^2) \quad \textit{Ans.}$$

Eq. (*b*) $$M_C = \frac{wl^2}{2} - R_B(l-a) = \frac{w}{8}(l^2 - 2al - a^2) \quad \textit{Ans.}$$

4-60



$$M = -\frac{wx^2}{2} + R_B\langle x - a\rangle^1, \qquad \frac{\partial M}{\partial R_B} = \langle x - a\rangle^1$$

$$\frac{\partial U}{\partial R_B} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial R_B}\,dx$$

$$= \frac{1}{EI}\int_0^a \frac{-wx^2}{2}(0)\,dx + \frac{1}{EI}\int_a^l \left[\frac{-wx^2}{2} + R_B(x-a)\right](x-a)\,dx = 0$$

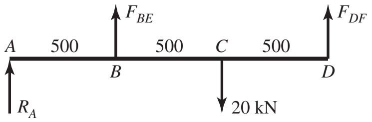
$$-\frac{w}{2}\left[\frac{1}{4}(l^4 - a^4) - \frac{a}{3}(l^3 - a^3)\right] + \frac{R_B}{3}\left[(l-a)^3 - (a-a)^3\right] = 0$$

$$R_B = \frac{w}{(l-a)^3}[3(L^4 - a^4) - 4a(l^3 - a^3)] = \frac{w}{8(l-a)}(3l^2 + 2al + a^2) \quad Ans.$$

$$R_C = wl - R_B = \frac{w}{8(l-a)}(5l^2 - 10al - a^2) \quad Ans.$$

$$M_C = \frac{wl^2}{2} - R_B(l-a) = \frac{w}{8}(l^2 - 2al - a^2) \quad Ans.$$

4-61



$$A = \frac{\pi}{4}(0.012^2) = 1.131(10^{-4})\ \mathrm{m}^2$$

(1)
$$R_A + F_{BE} + F_{DF} = 20\ \mathrm{kN} \qquad (a)$$
$$\sum M_A = 3F_{DF} - 2(20) + F_{BE} = 0$$
$$F_{BE} + 3F_{DF} = 40\ \mathrm{kN} \qquad (b)$$

(2)
$$M = R_A x + F_{BE}\langle x - 0.5\rangle^1 - 20(10^3)\langle x - 1\rangle^1$$
$$EI\frac{dy}{dx} = R_A\frac{x^2}{2} + \frac{F_{BE}}{2}\langle x - 0.5\rangle^2 - 10(10^3)\langle x - 1\rangle^2 + C_1$$
$$EIy = R_A\frac{x^3}{6} + \frac{F_{BE}}{6}\langle x - 0.5\rangle^3 - \frac{10}{3}(10^3)\langle x - 1\rangle^3 + C_1 x + C_2$$

(3) $y = 0$ at $x = 0 \quad \therefore C_2 = 0$

$$y_B = -\left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(1)}{1.131(10^{-4})209(10^9)} = -4.2305(10^{-8})F_{BE}$$

Substituting and evaluating at $x = 0.5$ m

$$EIy_B = 209(10^9)(8)(10^{-7})(-4.2305)(10^{-8})F_{BE} = R_A\frac{0.5^3}{6} + C_1(0.5)$$
$$2.0833(10^{-2})R_A + 7.0734(10^{-3})F_{BE} + 0.5C_1 = 0 \qquad (c)$$
$$y_D = -\left(\frac{Fl}{AE}\right)_{DF} = -\frac{F_{DF}(1)}{1.131(10^{-4})(209)(10^9)} = -4.2305(10^{-8})F_{DF}$$

Substituting and evaluating at $x = 1.5$ m

$$EIy_D = -7.0734(10^{-3})F_{DF} = R_A\frac{1.5^3}{6} + \frac{F_{BE}}{6}(1.5 - 0.5)^3 - \frac{10}{3}(10^3)(1.5 - 1)^3 + 1.5C_1$$
$$0.5625R_A + 0.16667F_{BE} + 7.0734(10^{-3})F_{DF} + 1.5C_1 = 416.67 \qquad (d)$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 2.0833(10^{-2}) & 7.0734(10^{-3}) & 0 & 0.5 \\ 0.5625 & 0.16667 & 7.0734(10^{-3}) & 1.5 \end{bmatrix} \begin{Bmatrix} R_A \\ F_{BE} \\ F_{DF} \\ C_1 \end{Bmatrix} = \begin{Bmatrix} 20000 \\ 40000 \\ 0 \\ 416.67 \end{Bmatrix}$$

Solve simultaneously or use software

$$R_A = -3885 \text{ N}, \quad F_{BE} = 15\,830 \text{ N}, \quad F_{DF} = 8058 \text{ N}, \quad C_1 = -62.045 \text{ N}\cdot\text{m}^2$$

$$\sigma_{BE} = \frac{15\,830}{(\pi/4)(12^2)} = 140 \text{ MPa} \quad \textit{Ans.}, \qquad \sigma_{DF} = \frac{8058}{(\pi/4)(12^2)} = 71.2 \text{ MPa} \quad \textit{Ans.}$$

$$EI = 209(10^9)(8)(10^{-7}) = 167.2(10^3) \text{ N}\cdot\text{m}^2$$

$$y = \frac{1}{167.2(10^3)}\left[-\frac{3885}{6}x^3 + \frac{15\,830}{6}\langle x-0.5\rangle^3 - \frac{10}{3}(10^3)\langle x-1\rangle^3 - 62.045x\right]$$

B: $x = 0.5$ m, $\quad y_B = -6.70(10^{-4})$ m $= -0.670$ mm *Ans.*

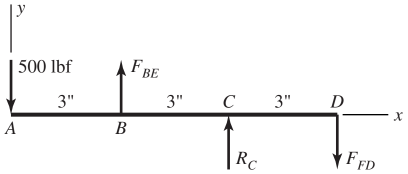
C: $x = 1$ m, $\quad y_C = \dfrac{1}{167.2(10^3)}\left[-\dfrac{3885}{6}(1^3) + \dfrac{15\,830}{6}(1-0.5)^3 - 62.045(1)\right]$

$$= -2.27(10^{-3}) \text{ m} = -2.27 \text{ mm} \quad \textit{Ans.}$$

D: $x = 1.5$, $\quad y_D = \dfrac{1}{167.2(10^3)}\left[-\dfrac{3885}{6}(1.5^3) + \dfrac{15\,830}{6}(1.5-0.5)^3 - \dfrac{10}{3}(10^3)(1.5-1)^3 - 62.045(1.5)\right]$

$$= -3.39(10^{-4}) \text{ m} = -0.339 \text{ mm} \quad \textit{Ans.}$$

4-62



$$EI = 30(10^6)(0.050) = 1.5(10^6) \text{ lbf}\cdot\text{in}^2$$

(1) $$R_C + F_{BE} - F_{FD} = 500 \qquad (a)$$

$$3R_C + 6F_{BE} = 9(500) = 4500 \qquad (b)$$

(2) $$M = -500x + F_{BE}\langle x-3\rangle^1 + R_C\langle x-6\rangle^1$$

$$EI\frac{dy}{dx} = -250x^2 + \frac{F_{BE}}{2}\langle x-3\rangle^2 + \frac{R_C}{2}\langle x-6\rangle^2 + C_1$$

$$EIy = -\frac{250}{3}x^3 + \frac{F_{BE}}{6}\langle x-3\rangle^3 + \frac{R_C}{6}\langle x-6\rangle^3 + C_1x + C_2$$

$$y_B = \left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(2)}{(\pi/4)(5/16)^2(30)(10^6)} = -8.692(10^{-7})F_{BE}$$

Substituting and evaluating at $x = 3$ in

$$EIy_B = 1.5(10^6)[-8.692(10^{-7})F_{BE}] = -\frac{250}{3}(3^3) + 3C_1 + C_2$$

$$1.3038F_{BE} + 3C_1 + C_2 = 2250 \qquad (c)$$

Since $y = 0$ at $x = 6$ in

$$EIy|_{=0} = -\frac{250}{3}(6^3) + \frac{F_{BE}}{6}(6-3)^3 + 6C_1 + C_2$$

$$4.5F_{BE} + 6C_1 + C_2 = 1.8(10^4) \qquad (d)$$

$$y_D = \left(\frac{Fl}{AE}\right)_{DF} = \frac{F_{DF}(2.5)}{(\pi/4)(5/16)^2(30)(10^6)} = 1.0865(10^{-6})F_{DF}$$

Substituting and evaluating at $x = 9$ in

$$EIy_D = 1.5(10^6)[1.0865(10^{-6})F_{DF}] = -\frac{250}{3}(9^3) + \frac{F_{BE}}{6}(9-3)^3 + \frac{R_C}{6}(9-6)^3 + 9C_1 + C_2$$

$$4.5R_C + 36F_{BE} - 1.6297F_{DF} + 9C_1 + C_2 = 6.075(10^4) \qquad (e)$$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 3 & 6 & 0 & 0 & 0 \\ 0 & 1.3038 & 0 & 3 & 1 \\ 0 & 4.5 & 0 & 6 & 1 \\ 4.5 & 36 & -1.6297 & 9 & 1 \end{bmatrix} \begin{Bmatrix} R_C \\ F_{BE} \\ F_{DF} \\ C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 500 \\ 4500 \\ 2250 \\ 1.8(10^4) \\ 6.075(10^4) \end{Bmatrix}$$

$$R_C = -590.4 \text{ lbf}, \quad F_{BE} = 1045.2 \text{ lbf}, \quad F_{DF} = -45.2 \text{ lbf}$$

$$C_1 = 4136.4 \text{ lbf} \cdot \text{in}^2, \quad C_2 = -11\,522 \text{ lbf} \cdot \text{in}^3$$

$$\sigma_{BE} = \frac{1045.2}{(\pi/4)(5/16)^2} = 13\,627 \text{ psi} = 13.6 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_{DF} = -\frac{45.2}{(\pi/4)(5/16)^2} = -589 \text{ psi} \quad \textit{Ans.}$$

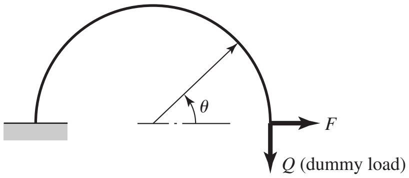
$$y_A = \frac{1}{1.5(10^6)}(-11\,522) = -0.007\,68 \text{ in} \quad \textit{Ans.}$$

$$y_B = \frac{1}{1.5(10^6)}\left[-\frac{250}{3}(3^3) + 4136.4(3) - 11\,522\right] = -0.000\,909 \text{ in} \quad \textit{Ans.}$$

$$y_D = \frac{1}{1.5(10^6)}\left[-\frac{250}{3}(9^3) + \frac{1045.2}{6}(9-3)^3 + \frac{-590.4}{6}(9-6)^3 + 4136.4(9) - 11\,522\right]$$

$$= -4.93(10^{-5}) \text{ in} \quad \textit{Ans.}$$

4-63



$$M = -PR\sin\theta + QR(1-\cos\theta) \qquad \frac{\partial M}{\partial Q} = R(1-\cos\theta)$$

$$\delta_Q = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \frac{1}{EI}\int_0^{\pi}(-PR\sin\theta)R(1-\cos\theta)R\,d\theta = -2\frac{PR^3}{EI}$$

Deflection is upward and equals $2(PR^3/EI)$ *Ans.*

4-64 Equation (4-28) becomes

$$U = 2\int_0^{\pi}\frac{M^2R\,d\theta}{2EI} \qquad R/h > 10$$

where $M = FR(1-\cos\theta)$ and $\dfrac{\partial M}{\partial F} = R(1-\cos\theta)$

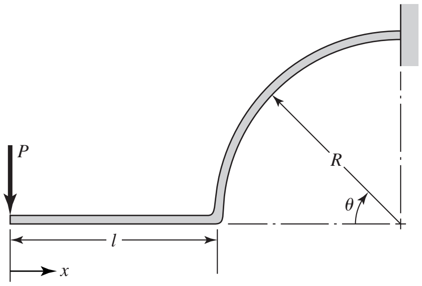
$$\begin{aligned}\delta &= \frac{\partial U}{\partial F} = \frac{2}{EI}\int_0^{\pi} M\frac{\partial M}{\partial F}R\,d\theta\\ &= \frac{2}{EI}\int_0^{\pi} FR^3(1-\cos\theta)^2\,d\theta\\ &= \frac{3\pi FR^3}{EI}\end{aligned}$$

Since $I = bh^3/12 = 4(6)^3/12 = 72\text{ mm}^4$ and $R = 81/2 = 40.5$ mm, we have

$$\delta = \frac{3\pi(40.5)^3F}{131(72)} = 66.4F\text{ mm} \quad \textit{Ans.}$$

where F is in kN.

4-65



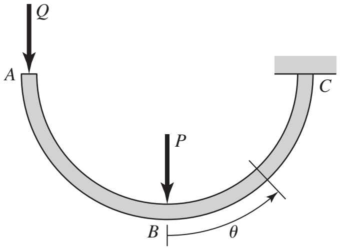
$$M = -Px, \quad \frac{\partial M}{\partial P} = -x \quad 0 \le x \le l$$

$$M = Pl + PR(1-\cos\theta), \quad \frac{\partial M}{\partial P} = l + R(1-\cos\theta) \quad 0 \le \theta \le l$$

$$\delta_P = \frac{1}{EI}\left\{\int_0^{l} -Px(-x)\,dx + \int_0^{\pi/2} P[l + R(1-\cos\theta)]^2R\,d\theta\right\}$$

$$= \frac{P}{12EI}\{4l^3 + 3R[2\pi l^2 + 4(\pi-2)lR + (3\pi-8)R^2]\} \quad \textit{Ans.}$$

4-66 *A*: Dummy load *Q* is applied at *A*. Bending in *AB* due only to *Q* which is zero.



$$M = PR\sin\theta + QR(1+\sin\theta), \quad \frac{\partial M}{\partial Q} = R(1+\sin\theta), \quad 0 \le \theta \le \frac{\pi}{2}$$

$$(\delta_A)_V = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \frac{1}{EI}\int_0^{\pi/2}(PR\sin\theta)[R(1+\sin\theta)]R\,d\theta$$

$$= \frac{PR^3}{EI}\left(-\cos\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\Bigg|_0^{\pi/2} = \frac{PR^3}{EI}\left(1+\frac{\pi}{4}\right)$$

$$= \frac{\pi+4}{4}\,\frac{PR^3}{EI} \quad \textit{Ans.}$$

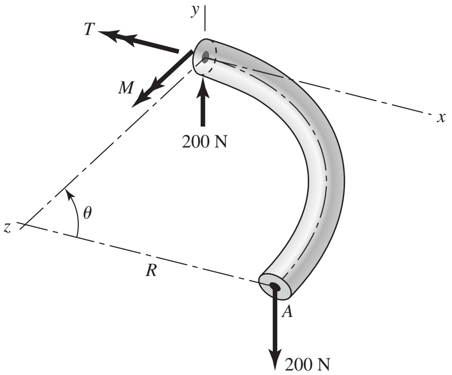
B:

$$M = PR\sin\theta, \quad \frac{\partial M}{\partial P} = R\sin\theta$$

$$(\delta_B)_V = \frac{\partial U}{\partial P} = \frac{1}{EI}\int_0^{\pi/2}(PR\sin\theta)(R\sin\theta)R\,d\theta$$

$$= \frac{\pi}{4}\,\frac{PR^3}{EI} \quad \textit{Ans.}$$

4-67



$$M = PR\sin\theta, \qquad \frac{\partial M}{\partial P} = R\sin\theta \quad 0 < \theta < \frac{\pi}{2}$$

$$T = PR(1-\cos\theta), \quad \frac{\partial T}{\partial P} = R(1-\cos\theta)$$

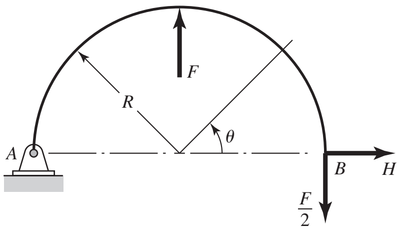
$$(\delta_A)_y = -\frac{\partial U}{\partial P} = -\left\{\frac{1}{EI}\int_0^{\pi/2} P(R\sin\theta)^2 R\,d\theta + \frac{1}{GJ}\int_0^{\pi/2} P[R(1-\cos\theta)]^2 R\,d\theta\right\}$$

Integrating and substituting $J = 2I$ and $G = E/[2(1+\nu)]$

$$(\delta_A)_y = -\frac{PR^3}{EI}\left[\frac{\pi}{4} + (1+\nu)\left(\frac{3\pi}{4} - 2\right)\right] = -[4\pi - 8 + (3\pi - 8)\nu]\frac{PR^3}{4EI}$$

$$= -[4\pi - 8 + (3\pi - 8)(0.29)]\frac{(200)(100)^3}{4(200)(10^3)(\pi/64)(5)^4} = -40.6\text{ mm}$$

4-68 Consider the horizontal reaction, to be applied at B, subject to the constraint $(\delta_B)_H = 0$.



(a) $(\delta_B)_H = \dfrac{\partial U}{\partial H} = 0$

Due to symmetry, consider half of the structure. F does not deflect horizontally.

$$M = \frac{FR}{2}(1-\cos\theta) - HR\sin\theta, \qquad \frac{\partial M}{\partial H} = -R\sin\theta, \quad 0 < \theta < \frac{\pi}{2}$$

$$\frac{\partial U}{\partial H} = \frac{1}{EI}\int_0^{\pi/2}\left[\frac{FR}{2}(1-\cos\theta) - HR\sin\theta\right](-R\sin\theta)R\,d\theta = 0$$

$$-\frac{F}{2} + \frac{F}{4} + H\frac{\pi}{4} = 0 \quad\Rightarrow\quad H = \frac{F}{\pi} \quad \textit{Ans.}$$

Reaction at A is the same where H goes to the left

(b) For $0 < \theta < \dfrac{\pi}{2}$, $\quad M = \dfrac{FR}{2}(1-\cos\theta) - \dfrac{FR}{\pi}\sin\theta$

$$M = \frac{FR}{2\pi}[\pi(1-\cos\theta) - 2\sin\theta] \quad \textit{Ans.}$$

Due to symmetry, the solution for the left side is identical.

(c)

$$\frac{\partial M}{\partial F} = \frac{R}{2\pi}[\pi(1-\cos\theta) - 2\sin\theta]$$

$$\delta_F = \frac{\partial U}{\partial F} = \frac{2}{EI}\int_0^{\pi/2}\frac{FR^2}{4\pi^2}[\pi(1-\cos\theta) - 2\sin\theta]^2 R\,d\theta$$

$$= \frac{FR^3}{2\pi^2 EI}\int_0^{\pi/2}(\pi^2 + \pi^2\cos^2\theta + 4\sin^2\theta - 2\pi^2\cos\theta - 4\pi\sin\theta + 4\pi\sin\theta\cos\theta)\,d\theta$$

$$= \frac{FR^3}{2\pi^2 EI}\left[\pi^2\left(\frac{\pi}{2}\right) + \pi^2\left(\frac{\pi}{4}\right) + 4\left(\frac{\pi}{4}\right) - 2\pi^2 - 4\pi + 2\pi\right]$$

$$= \frac{(3\pi^2 - 8\pi - 4)}{8\pi}\frac{FR^3}{EI} \quad \textit{Ans.}$$

4-69 Must use Eq. (4-33)

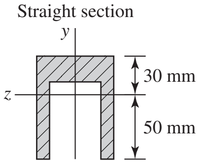
$$A = 80(60) - 40(60) = 2400\ \text{mm}^2$$

$$R = \frac{(25+40)(80)(60) - (25+20+30)(40)(60)}{2400} = 55\ \text{mm}$$

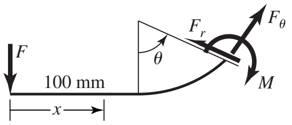
Section is equivalent to the “T” section of Table 3-4

$$r_n = \frac{60(20) + 20(60)}{60\ln[(25+20)/25] + 20\ln[(80+25)/(25+20)]} = 45.9654\ \text{mm}$$

$$e = R - r_n = 9.035\ \text{mm}$$



$$I_z = \frac{1}{12}(60)(20^3) + 60(20)(30-10)^2 + 2\left[\frac{1}{12}(10)(60^3) + 10(60)(50-30)^2\right] = 1.36(10^6)\ \text{mm}^4$$



For $0 \le x \le 100$ mm

$$M = -Fx, \quad \frac{\partial M}{\partial F} = -x; \quad V = F, \quad \frac{\partial V}{\partial F} = 1$$

For $\theta \le \pi/2$

$$F_r = F\cos\theta, \quad \frac{\partial F_r}{\partial F} = \cos\theta; \quad F_\theta = F\sin\theta, \quad \frac{\partial F_\theta}{\partial F} = \sin\theta$$

$$M = F(100 + 55\sin\theta), \quad \frac{\partial M}{\partial F} = (100 + 55\sin\theta)$$

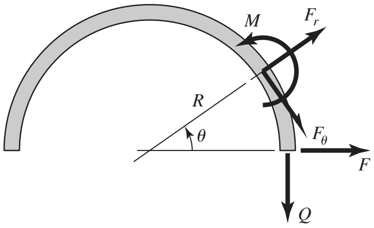
Use Eq. (5-34), integrate from 0 to $\pi/2$, double the results and add straight part

$$\delta = \frac{2}{E}\left\{\frac{1}{I}\int_0^{100} Fx^2dx + \int_0^{100}\frac{(1)F(1)\,dx}{2400(G/E)} + \int_0^{\pi/2} F\frac{(100+55\sin\theta)^2}{2400(9.035)}d\theta + \int_0^{\pi/2}\frac{F\sin^2\theta(55)}{2400}d\theta - \int_0^{\pi/2}\frac{F(100+55\sin\theta)}{2400}\sin\theta d\theta - \int_0^{\pi/2}\frac{F\sin\theta(100+55\sin\theta)}{2400}d\theta + \int_0^{\pi/2}\frac{(1)F\cos^2\theta(55)}{2400(G/E)}d\theta\right\}$$

Substitute
$I = 1.36(10^3)\ \text{mm}^2$, $F = 30(10^3)$ N, $E = 207(10^3)\ \text{N/mm}^2$, $G = 79(10^3)\ \text{N/mm}^2$

$$\delta = \frac{2}{207(10^3)}30(10^3)\left\{\frac{100^3}{3(1.36)(10^6)} + \frac{207}{79}\left(\frac{100}{2400}\right) + \frac{2.908(10^4)}{2400(9.035)} + \frac{55}{2400}\left(\frac{\pi}{4}\right) - \frac{2}{2400}(143.197) + \frac{207}{79}\left(\frac{55}{2400}\right)\left(\frac{\pi}{4}\right)\right\} = 0.476\ \text{mm} \quad \textit{Ans.}$$

4-70



$$M = FR\sin\theta - QR(1-\cos\theta), \quad \frac{\partial M}{\partial Q} = -R(1-\cos\theta)$$

$$F_\theta = Q\cos\theta + F\sin\theta, \quad \frac{\partial F_\theta}{\partial Q} = \cos\theta$$

$$\begin{aligned}\frac{\partial}{\partial Q}(MF_\theta) &= [FR\sin\theta - QR(1-\cos\theta)]\cos\theta \\ &\quad + [-R(1-\cos\theta)][Q\cos\theta + F\sin\theta]\end{aligned}$$

$$F_r = F\cos\theta - Q\sin\theta, \quad \frac{\partial F_r}{\partial Q} = -\sin\theta$$

From Eq. (4-33)

$$\begin{aligned}\delta = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} &= \frac{1}{AeE}\int_0^\pi (FR\sin\theta)[-R(1-\cos\theta)]\,d\theta + \frac{R}{AE}\int_0^\pi F\sin\theta\cos\theta d\theta \\ &\quad - \frac{1}{AE}\int_0^\pi [FR\sin\theta\cos\theta - FR\sin\theta(1-\cos\theta)]\,d\theta \\ &\quad + \frac{CR}{AG}\int_0^\pi -F\cos\theta\sin\theta d\theta \\ &= -\frac{2FR^2}{AeE} + 0 + \frac{2FR}{AE} + 0 = -\left(\frac{R}{e}-1\right)\frac{2FR}{AE} \quad \textit{Ans.}\end{aligned}$$

4-71 The cross section at *A* does not rotate, thus for a single quadrant we have

$$\frac{\partial U}{\partial M_A} = 0$$

The bending moment at an angle θ to the x axis is

$$M = M_A - \frac{F}{2}(R-x) = M_A - \frac{FR}{2}(1-\cos\theta) \qquad (1)$$

because $x = R\cos\theta$. Next,

$$U = \int \frac{M^2}{2EI}ds = \int_0^{\pi/2} \frac{M^2}{2EI}R\,d\theta$$

since $ds = R\,d\theta$. Then

$$\frac{\partial U}{\partial M_A} = \frac{R}{EI}\int_0^{\pi/2} M\frac{\partial M}{\partial M_A}\,d\theta = 0$$

But $\partial M/\partial M_A = 1$. Therefore

$$\int_0^{\pi/2} M\,d\theta = \int_0^{\pi/2}\left[M_A - \frac{FR}{2}(1-\cos\theta)\right]d\theta = 0$$

Since this term is zero, we have

$$M_A = \frac{FR}{2}\left(1-\frac{2}{\pi}\right)$$

Substituting into Eq. (1)

$$M = \frac{FR}{2}\left(\cos\theta - \frac{2}{\pi}\right)$$

The maximum occurs at B where $\theta = \pi/2$. It is

$$M_B = -\frac{FR}{\pi} \quad \textit{Ans.}$$

4-72 For one quadrant

$$M = \frac{FR}{2}\left(\cos\theta-\frac{2}{\pi}\right); \quad \frac{\partial M}{\partial F} = \frac{R}{2}\left(\cos\theta-\frac{2}{\pi}\right)$$

$$\delta = \frac{\partial U}{\partial F} = 4\int_0^{\pi/2}\frac{M}{EI}\frac{\partial M}{\partial F}R\,d\theta$$

$$= \frac{FR^3}{EI}\int_0^{\pi/2}\left(\cos\theta-\frac{2}{\pi}\right)^2 d\theta$$

$$= \frac{FR^3}{EI}\left(\frac{\pi}{4}-\frac{2}{\pi}\right) \quad \textit{Ans.}$$

4-73

$$P_{\text{cr}} = \frac{C\pi^2 EI}{l^2}$$

$$I = \frac{\pi}{64}(D^4-d^4) = \frac{\pi D^4}{64}(1-K^4)$$

$$P_{\text{cr}} = \frac{C\pi^2 E}{l^2}\left[\frac{\pi D^4}{64}(1-K^4)\right]$$

$$D = \left[\frac{64P_{\text{cr}}l^2}{\pi^3 CE(1-K^4)}\right]^{1/4} \quad \textit{Ans.}$$

4-74

$$A = \frac{\pi}{4}D^2(1-K^2), \quad I = \frac{\pi}{64}D^4(1-K^4) = \frac{\pi}{64}D^4(1-K^2)(1+K^2),$$

$$k^2 = \frac{I}{A} = \frac{D^2}{16}(1+K^2)$$

From Eq. (4-43)

$$\frac{P_{\text{cr}}}{(\pi/4)D^2(1-K^2)} = S_y - \frac{S_y^2 l^2}{4\pi^2 k^2 CE} = S_y - \frac{S_y^2 l^2}{4\pi^2(D^2/16)(1+K^2)CE}$$

$$4P_{\text{cr}} = \pi D^2(1-K^2)S_y - \frac{4S_y^2 l^2 \pi D^2(1-K^2)}{\pi^2 D^2(1+K^2)CE}$$

$$\pi D^2(1-K^2)S_y = 4P_{\text{cr}} + \frac{4S_y^2 l^2(1-K^2)}{\pi(1+K^2)CE}$$

$$D = \left[\frac{4P_{\text{cr}}}{\pi S_y(1-K^2)} + \frac{4S_y^2 l^2(1-K^2)}{\pi(1+K^2)CE\pi(1-K^2)S_y}\right]^{1/2}$$

$$= 2\left[\frac{P_{\text{cr}}}{\pi S_y(1-K^2)} + \frac{S_y l^2}{\pi^2 CE(1+K^2)}\right]^{1/2} \quad \textit{Ans.}$$

4-75 **(a)**

$$\overset{+}{\curvearrowleft}\sum M_A = 0, \qquad 2.5(180) - \frac{3}{\sqrt{3^2+1.75^2}}F_{BO}(1.75) = 0 \quad \Rightarrow \quad F_{BO} = 297.7 \text{ lbf}$$

Using $n_d = 5$, design for $F_{\text{cr}} = n_d F_{BO} = 5(297.7) = 1488$ lbf, $l = \sqrt{3^2+1.75^2} =$ 3.473 ft, $S_y = 24$ kpsi

In plane: $k = 0.2887h = 0.2887''$, $C = 1.0$

Try 1" × 1/2" section

$$\frac{l}{k} = \frac{3.473(12)}{0.2887} = 144.4$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2(1)(30)(10^6)}{24(10^3)}\right)^{1/2} = 157.1$$

Since $(l/k)_1 > (l/k)$ use Johnson formula

$$P_{\text{cr}} = (1)\left(\frac{1}{2}\right)\left[24(10^3) - \left(\frac{24(10^3)}{2\pi}144.4\right)^2\left(\frac{1}{1(30)(10^6)}\right)\right] = 6930 \text{ lbf}$$

Try 1" × 1/4": $P_{\text{cr}} = 3465$ lbf

Out of plane: $k = 0.2887(0.5) = 0.1444$ in, $C = 1.2$

$$\frac{l}{k} = \frac{3.473(12)}{0.1444} = 289$$

Since $(l/k)_1 < (l/k)$ use Euler equation

$$P_{\text{cr}} = 1(0.5)\frac{1.2(\pi^2)(30)(10^6)}{289^2} = 2127 \text{ lbf}$$

1/4" increases l/k by 2, $\left(\frac{l}{k}\right)^2$ by 4, and A by 1/2

Try 1" × 3/8": $k = 0.2887(0.375) = 0.1083$ in

$$\frac{l}{k} = 385, \quad P_{\text{cr}} = 1(0.375)\frac{1.2(\pi^2)(30)(10^6)}{385^2} = 899 \text{ lbf} \quad \text{(too low)}$$

Use 1" × 1/2" *Ans.*

(b) $\sigma_b = -\frac{P}{\pi d l} = -\frac{298}{\pi(0.5)(0.5)} = -379$ psi *No,* bearing stress is not significant.

4-76 This is a design problem with no one distinct solution.

4-77

$$F = 800\left(\frac{\pi}{4}\right)(3^2) = 5655 \text{ lbf}, \quad S_y = 37.5 \text{ kpsi}$$

$$P_{\text{cr}} = n_d F = 3(5655) = 17\,000 \text{ lbf}$$

(a) Assume Euler with $C = 1$

$$I = \frac{\pi}{64}d^4 = \frac{P_{\text{cr}} l^2}{C\pi^2 E} \quad \Rightarrow \quad d = \left[\frac{64 P_{\text{cr}} l^2}{\pi^3 C E}\right]^{1/4} = \left[\frac{64(17)(10^3)(60^2)}{\pi^3(1)(30)(10^6)}\right]^{1/4} = 1.433 \text{ in}$$

Use $d = 1.5$ in; $k = d/4 = 0.375$

$$\frac{l}{k} = \frac{60}{0.375} = 160$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2(1)(30)(10^6)}{37.5(10^3)}\right)^{1/2} = 126 \quad \therefore \text{ use Euler}$$

$$P_{\text{cr}} = \frac{\pi^2(30)(10^6)(\pi/64)(1.5^4)}{60^2} = 20\,440 \text{ lbf}$$

$d = 1.5$ in is satisfactory. *Ans.*

(b) $$d = \left[\frac{64(17)(10^3)(18^2)}{\pi^3(1)(30)(10^6)}\right]^{1/4} = 0.785 \text{ in}, \quad \text{so use } 0.875 \text{ in}$$

$$k = \frac{0.875}{4} = 0.2188 \text{ in}$$

$$l/k = \frac{18}{0.2188} = 82.3 \quad \text{try Johnson}$$

$$P_{\text{cr}} = \frac{\pi}{4}(0.875^2)\left[37.5(10^3) - \left(\frac{37.5(10^3)}{2\pi}82.3\right)^2 \frac{1}{1(30)(10^6)}\right] = 17\,714 \text{ lbf}$$

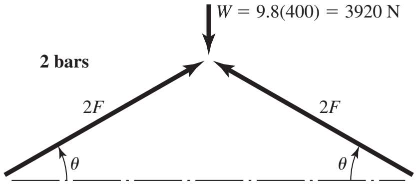
Use $d = 0.875$ in *Ans.*

(c)

$$n_{(a)} = \frac{20\,440}{5655} = 3.61 \quad \textit{Ans.}$$

$$n_{(b)} = \frac{17\,714}{5655} = 3.13 \quad \textit{Ans.}$$

4-78



$$4F\sin\theta = 3920$$

$$F = \frac{3920}{4\sin\theta}$$

In range of operation, F is maximum when $\theta = 15^\circ$

$$F_{\max} = \frac{3920}{4\sin 15} = 3786 \text{ N per bar}$$

$$P_{\text{cr}} = n_d F_{\max} = 2.5(3786) = 9465 \text{ N}$$

$l = 300$ mm, $h = 25$ mm

Try $b = 5$ mm: out of plane $k = (5/\sqrt{12}) = 1.443$ mm

$$\frac{l}{k} = \frac{300}{1.443} = 207.8$$

$$\left(\frac{l}{k}\right)_1 = \left[\frac{(2\pi^2)(1.4)(207)(10^9)}{380(10^6)}\right]^{1/2} = 123 \qquad \therefore \text{use Euler}$$

$$P_{\text{cr}} = (25)(5)\frac{(1.4\pi^2)(207)(10^3)}{(207.8)^2} = 8280 \text{ N}$$

Try: 5.5 mm: $k = 5.5/\sqrt{12} = 1.588$ mm

$$\frac{l}{k} = \frac{300}{1.588} = 189$$

$$P_{\text{cr}} = 25(5.5)\frac{(1.4\pi^2)(207)(10^3)}{189^2} = 11\,010 \text{ N}$$

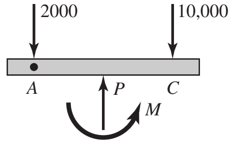
Use 25 × 5.5 mm bars *Ans.* The factor of safety is thus

$$n = \frac{11\,010}{3786} = 2.91 \quad Ans.$$

4-79

$$\sum F = 0 = 2000 + 10\,000 - P \quad \Rightarrow \quad P = 12\,000 \text{ lbf} \quad Ans.$$

$$\sum M_A = 12\,000\left(\frac{5.68}{2}\right) - 10\,000(5.68) + M = 0$$



$$M = 22\,720 \text{ lbf} \cdot \text{in}$$

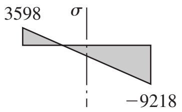
$$e = \frac{M}{P} = \frac{22}{12}\left(\frac{720}{000}\right) = 1.893 \text{ in} \quad Ans.$$

From Table A-8, $A = 4.271 \text{ in}^2,\ I = 7.090 \text{ in}^4$

$$k^2 = \frac{I}{A} = \frac{7.090}{4.271} = 1.66 \text{ in}^2$$

$$\sigma_c = -\frac{12\,000}{4.271}\left[1 + \frac{1.893(2)}{1.66}\right] = -9218 \text{ psi} \quad Ans.$$

$$\sigma_t = -\frac{12\,000}{4.271}\left[1 - \frac{1.893(2)}{1.66}\right] = 3598 \text{ psi}$$

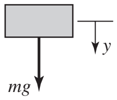


4-80 This is a design problem so the solutions will differ.

4-81 For free fall with $y \le h$

$$\sum F_y - m\ddot{y} = 0$$

$$mg - m\ddot{y} = 0, \quad \text{so } \ddot{y} = g$$

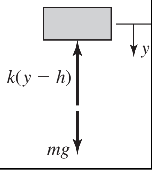


Using $y = a + bt + ct^2$, we have at $t = 0$, $y = 0$, and $\dot{y} = 0$, and so $a = 0$, $b = 0$, and $c = g/2$. Thus

$$y = \frac{1}{2}gt^2 \quad \text{and} \quad \dot{y} = gt \quad \text{for } y \le h$$

At impact, $y = h$, $t = (2h/g)^{1/2}$, and $v_0 = (2gh)^{1/2}$

After contact, the differential equatioin (D.E.) is



$$mg - k(y - h) - m\ddot{y} = 0 \qquad \text{for } y > h$$

Now let $x = y - h$; then $\dot{x} = \dot{y}$ and $\ddot{x} = \ddot{y}$. So the D.E. is $\ddot{x} + (k/m)x = g$ with solution $\omega = (k/m)^{1/2}$ and

$$x = A\cos\omega t' + B\sin\omega t' + \frac{mg}{k}$$

At contact, $t' = 0$, $x = 0$, and $\dot{x} = v_0$. Evaluating A and B then yields

$$x = -\frac{mg}{k}\cos\omega t' + \frac{v_0}{\omega}\sin\omega t' + \frac{mg}{k}$$

or

$$y = -\frac{W}{k}\cos\omega t' + \frac{v_0}{\omega}\sin\omega t' + \frac{W}{k} + h$$

and

$$\dot{y} = \frac{W\omega}{k}\sin\omega t' + v_0\cos\omega t'$$

To find $y_{\max}$ set $\dot{y} = 0$. Solving gives

$$\tan\omega t' = -\frac{v_0 k}{W\omega}$$

or

$$(\omega t')* = \tan^{-1}\left(-\frac{v_0 k}{W\omega}\right)$$

The first value of $(\omega t')*$ is a minimum and negative. So add π radians to it to find the maximum.

Numerical example: $h = 1$ in, $W = 30$ lbf, $k = 100$ lbf/in. Then

$$\omega = (k/m)^{1/2} = [100(386)/30]^{1/2} = 35.87 \text{ rad/s}$$

$$W/k = 30/100 = 0.3$$

$$v_0 = (2gh)^{1/2} = [2(386)(1)]^{1/2} = 27.78 \text{ in/s}$$

Then

$$y = -0.3\cos 35.87t' + \frac{27.78}{35.87}\sin 35.87t' + 0.3 + 1$$

For $y_{\max}$

$$\tan\omega t' = -\frac{v_0 k}{W\omega} = -\frac{27.78(100)}{30(35.87)} = -2.58$$

$$(\omega t')* = -1.20 \text{ rad (minimum)}$$

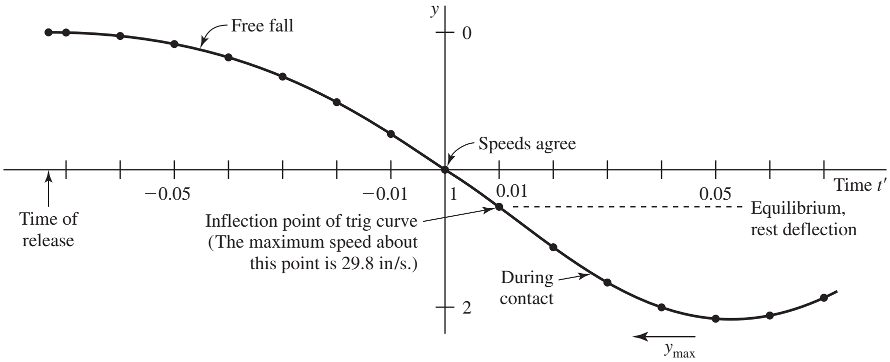
$$(\omega t')* = -1.20 + \pi = 1.940 \text{ (maximum)}$$

Then $t'* = 1.940/35.87 = 0.0541$ s. This means that the spring bottoms out at $t'*$ seconds. Then $(\omega t')* = 35.87(0.0541) = 1.94$ rad

So $$y_{\max} = -0.3\cos 1.94 + \frac{27.78}{35.87}\sin 1.94 + 0.3 + 1 = 2.130 \text{ in} \quad \textit{Ans.}$$

The maximum spring force is $F_{\max} = k(y_{\max} - h) = 100(2.130 - 1) = 113$ lbf *Ans.*

The action is illustrated by the graph below. *Applications:* Impact, such as a dropped package or a pogo stick with a passive rider. The idea has also been used for a one-legged robotic walking machine.

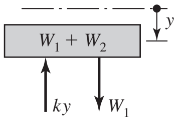


4-82 Choose $t' = 0$ at the instant of impact. At this instant, $v_1 = (2gh)^{1/2}$. Using momentum, $m_1 v_1 = m_2 v_2$. Thus

$$\frac{W_1}{g}(2gh)^{1/2} = \frac{W_1 + W_2}{g} v_2$$

$$v_2 = \frac{W_1(2gh)^{1/2}}{W_1 + W_2}$$

Therefore at $t' = 0$, $y = 0$, and $\dot{y} = v_2$



Let $W = W_1 + W_2$

Because the spring force at $y = 0$ includes a reaction to W_2, the D.E. is

$$\frac{W}{g}\ddot{y} = -ky + W_1$$

With $\omega = (kg/W)^{1/2}$ the solution is

$$y = A\cos\omega t' + B\sin\omega t' + W_1/k$$

$$\dot{y} = -A\omega\sin\omega t' + B\omega\cos\omega t'$$

At $t' = 0$, $y = 0 \Rightarrow A = -W_1/k$

At $t' = 0$, $\dot{y} = v_2 \Rightarrow v_2 = B\omega$

Then

$$B = \frac{v_2}{\omega} = \frac{W_1(2gh)^{1/2}}{(W_1 + W_2)[kg/(W_1 + W_2)]^{1/2}}$$

We now have

$$y = -\frac{W_1}{k}\cos\omega t' + W_1\left[\frac{2h}{k(W_1 + W_2)}\right]^{1/2}\sin\omega t' + \frac{W_1}{k}$$

Transforming gives

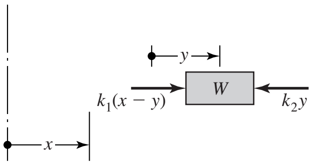
$$y = \frac{W_1}{k}\left(\frac{2hk}{W_1 + W_2} + 1\right)^{1/2}\cos(\omega t' - \phi) + \frac{W_1}{k}$$

where ϕ is a phase angle. The maximum deflection of W_2 and the maximum spring force are thus

$$y_{\max} = \frac{W_1}{k}\left(\frac{2hk}{W_1 + W_2} + 1\right)^{1/2} + \frac{W_1}{k} \quad Ans.$$

$$F_{\max} = ky_{\max} + W_2 = W_1\left(\frac{2hk}{W_1 + W_2} + 1\right)^{1/2} + W_1 + W_2 \quad Ans.$$

4-83 Assume $x > y$ to get a free-body diagram.



Then

$$\frac{W}{g}\ddot{y} = k_1(x - y) - k_2 y$$

A particular solution for $x = a$ is

$$y = \frac{k_1 a}{k_1 + k_2}$$

Then the complementary plus the particular solution is

$$y = A\cos\omega t + B\sin\omega t + \frac{k_1 a}{k_1 + k_2}$$

where $$\omega = \left[\frac{(k_1 + k_2)g}{W}\right]^{1/2}$$

At $t = 0$, $y = 0$, and $\dot{y} = 0$. Therefore $B = 0$ and

$$A = -\frac{k_1 a}{k_1 + k_2}$$

Substituting,

$$y = \frac{k_1 a}{k_1 + k_2}(1 - \cos\omega t)$$

Since y is maximum when the cosine is -1

$$y_{\max} = \frac{2k_1 a}{k_1 + k_2} \quad Ans.$$

Chapter 5

5-1

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma_1 - \sigma_3}$

DE: $n = \dfrac{S_y}{\sigma'}$

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

(a) MSS: $\sigma_1 = 12,\ \sigma_2 = 6,\ \sigma_3 = 0$ kpsi

$$n = \frac{50}{12} = 4.17 \quad \textit{Ans.}$$

DE: $\sigma' = (12^2 - 6(12) + 6^2)^{1/2} = 10.39$ kpsi, $\quad n = \dfrac{50}{10.39} = 4.81 \quad$ *Ans.*

(b) $\sigma_A, \sigma_B = \dfrac{12}{2} \pm \sqrt{\left(\dfrac{12}{2}\right)^2 + (-8)^2} = 16, -4$ kpsi

$\sigma_1 = 16,\ \sigma_2 = 0,\ \sigma_3 = -4$ kpsi

MSS: $n = \dfrac{50}{16 - (-4)} = 2.5 \quad$ *Ans.*

DE: $\sigma' = (12^2 + 3(-8^2))^{1/2} = 18.33$ kpsi, $\quad n = \dfrac{50}{18.33} = 2.73 \quad$ *Ans.*

(c) $\sigma_A, \sigma_B = \dfrac{-6 - 10}{2} \pm \sqrt{\left(\dfrac{-6 + 10}{2}\right)^2 + (-5)^2} = -2.615, -13.385$ kpsi

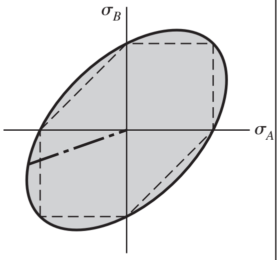
$\sigma_1 = 0,\ \sigma_2 = -2.615,\ \sigma_3 = -13.385$ kpsi

MSS: $n = \dfrac{50}{0 - (-13.385)} = 3.74 \quad$ *Ans.*

DE: $\sigma' = [(-6)^2 - (-6)(-10) + (-10)^2 + 3(-5)^2]^{1/2}$

$= 12.29$ kpsi

$$n = \frac{50}{12.29} = 4.07 \quad \textit{Ans.}$$



(d) $\sigma_A, \sigma_B = \dfrac{12 + 4}{2} \pm \sqrt{\left(\dfrac{12 - 4}{2}\right)^2 + 1^2} = 12.123, 3.877$ kpsi

$\sigma_1 = 12.123,\ \sigma_2 = 3.877,\ \sigma_3 = 0$ kpsi

MSS: $n = \dfrac{50}{12.123 - 0} = 4.12 \quad$ *Ans.*

DE: $\sigma' = [12^2 - 12(4) + 4^2 + 3(1^2)]^{1/2} = 10.72$ kpsi

$$n = \frac{50}{10.72} = 4.66 \quad \textit{Ans.}$$

5-2 $S_y = 50$ kpsi

MSS: $$\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \frac{S_y}{\sigma_1 - \sigma_3}$$

DE: $$\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = S_y/n \quad \Rightarrow \quad n = S_y/\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$$

(a) MSS: $$\sigma_1 = 12 \text{ kpsi}, \sigma_3 = 0, n = \frac{50}{12 - 0} = 4.17 \quad \textit{Ans.}$$

DE: $$n = \frac{50}{[12^2 - (12)(12) + 12^2]^{1/2}} = 4.17 \quad \textit{Ans.}$$

(b) MSS: $$\sigma_1 = 12 \text{ kpsi}, \sigma_3 = 0, n = \frac{50}{12} = 4.17 \quad \textit{Ans.}$$

DE: $$n = \frac{50}{[12^2 - (12)(6) + 6^2]^{1/2}} = 4.81 \quad \textit{Ans.}$$

(c) MSS: $$\sigma_1 = 12 \text{ kpsi}, \sigma_3 = -12 \text{ kpsi}, n = \frac{50}{12 - (-12)} = 2.08 \quad \textit{Ans.}$$

DE: $$n = \frac{50}{[12^2 - (12)(-12) + (-12)^2]^{1/3}} = 2.41 \quad \textit{Ans.}$$

(d) MSS: $$\sigma_1 = 0, \sigma_3 = -12 \text{ kpsi}, n = \frac{50}{-(-12)} = 4.17 \quad \textit{Ans.}$$

DE: $$n = \frac{50}{[(-6)^2 - (-6)(-12) + (-12)^2]^{1/2}} = 4.81$$

5-3 $S_y = 390$ MPa

MSS: $$\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \frac{S_y}{\sigma_1 - \sigma_3}$$

DE: $$\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = S_y/n \quad \Rightarrow \quad n = S_y/\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$$

(a) MSS: $$\sigma_1 = 180 \text{ MPa}, \sigma_3 = 0, n = \frac{390}{180} = 2.17 \quad \textit{Ans.}$$

DE: $$n = \frac{390}{[180^2 - 180(100) + 100^2]^{1/2}} = 2.50 \quad \textit{Ans.}$$

(b) $$\sigma_A, \sigma_B = \frac{180}{2} \pm \sqrt{\left(\frac{180}{2}\right)^2 + 100^2} = 224.5, -44.5 \text{ MPa} = \sigma_1, \sigma_3$$

MSS: $$n = \frac{390}{224.5 - (-44.5)} = 1.45 \quad \textit{Ans.}$$

DE: $$n = \frac{390}{[180^2 + 3(100^2)]^{1/2}} = 1.56 \quad \textit{Ans.}$$

(c) $\sigma_A, \sigma_B = -\dfrac{160}{2} \pm \sqrt{\left(-\dfrac{160}{2}\right)^2 + 100^2} = 48.06, -208.06 \text{ MPa} = \sigma_1, \sigma_3$

MSS: $n = \dfrac{390}{48.06 - (-208.06)} = 1.52$ *Ans.*

DE: $n = \dfrac{390}{[-160^2 + 3(100^2)]^{1/2}} = 1.65$ *Ans.*

(d) $\sigma_A, \sigma_B = 150, -150 \text{ MPa} = \sigma_1, \sigma_3$

MSS: $n = \dfrac{390}{150 - (-150)} = 1.30$ *Ans.*

DE: $n = \dfrac{390}{[3(150)^2]^{1/2}} = 1.50$ *Ans.*

5-4 $S_y = 220$ MPa

(a) $\sigma_1 = 100, \sigma_2 = 80, \sigma_3 = 0$ MPa

MSS: $n = \dfrac{220}{100 - 0} = 2.20$ *Ans.*

DET: $\sigma' = [100^2 - 100(80) + 80^2]^{1/2} = 91.65$ MPa

$n = \dfrac{220}{91.65} = 2.40$ *Ans.*

(b) $\sigma_1 = 100, \sigma_2 = 10, \sigma_3 = 0$ MPa

MSS: $n = \dfrac{220}{100} = 2.20$ *Ans.*

DET: $\sigma' = [100^2 - 100(10) + 10^2]^{1/2} = 95.39$ MPa

$n = \dfrac{220}{95.39} = 2.31$ *Ans.*

(c) $\sigma_1 = 100, \sigma_2 = 0, \sigma_3 = -80$ MPa

MSS: $n = \dfrac{220}{100 - (-80)} = 1.22$ *Ans.*

DE: $\sigma' = [100^2 - 100(-80) + (-80)^2]^{1/2} = 156.2$ MPa

$n = \dfrac{220}{156.2} = 1.41$ *Ans.*

(d) $\sigma_1 = 0, \sigma_2 = -80, \sigma_3 = -100$ MPa

MSS: $n = \dfrac{220}{0 - (-100)} = 2.20$ *Ans.*

DE: $\sigma' = [(-80)^2 - (-80)(-100) + (-100)^2] = 91.65$ MPa

$n = \dfrac{220}{91.65} = 2.40$ *Ans.*

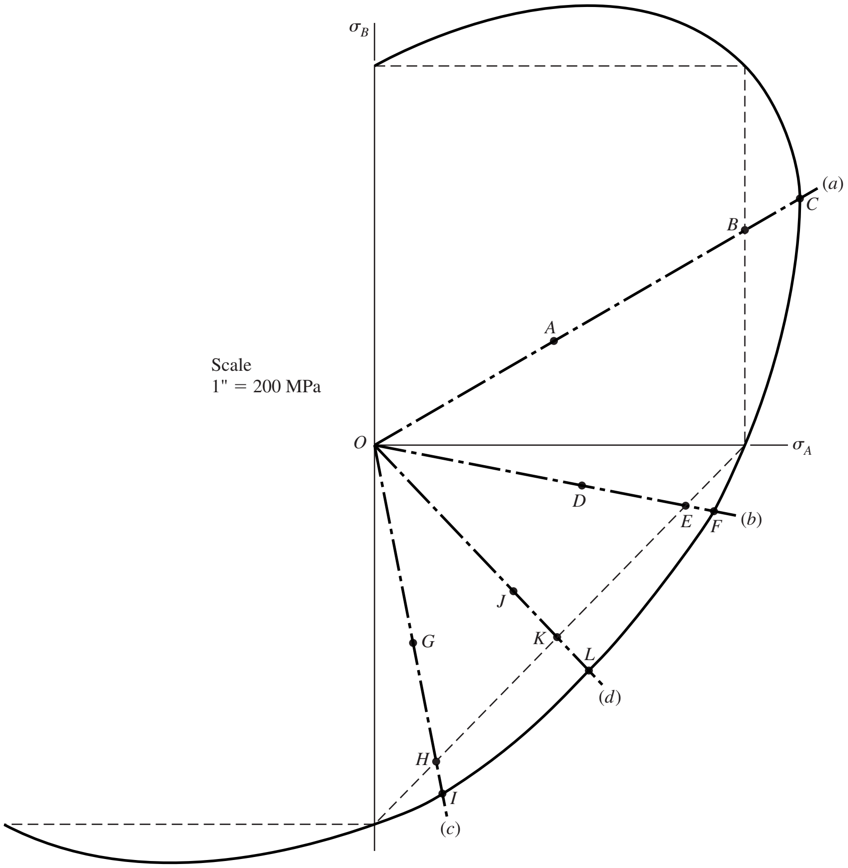
5-5

(a) MSS: $$n = \frac{OB}{OA} = \frac{2.23}{1.08} = 2.1$$

DE: $$n = \frac{OC}{OA} = \frac{2.56}{1.08} = 2.4$$

(b) MSS: $$n = \frac{OE}{OD} = \frac{1.65}{1.10} = 1.5$$

DE: $$n = \frac{OF}{OD} = \frac{1.8}{1.1} = 1.6$$



(c) MSS: $$n = \frac{OH}{OG} = \frac{1.68}{1.05} = 1.6$$

DE: $$n = \frac{OI}{OG} = \frac{1.85}{1.05} = 1.8$$

(d) MSS: $$n = \frac{OK}{OJ} = \frac{1.38}{1.05} = 1.3$$

DE: $$n = \frac{OL}{OJ} = \frac{1.62}{1.05} = 1.5$$

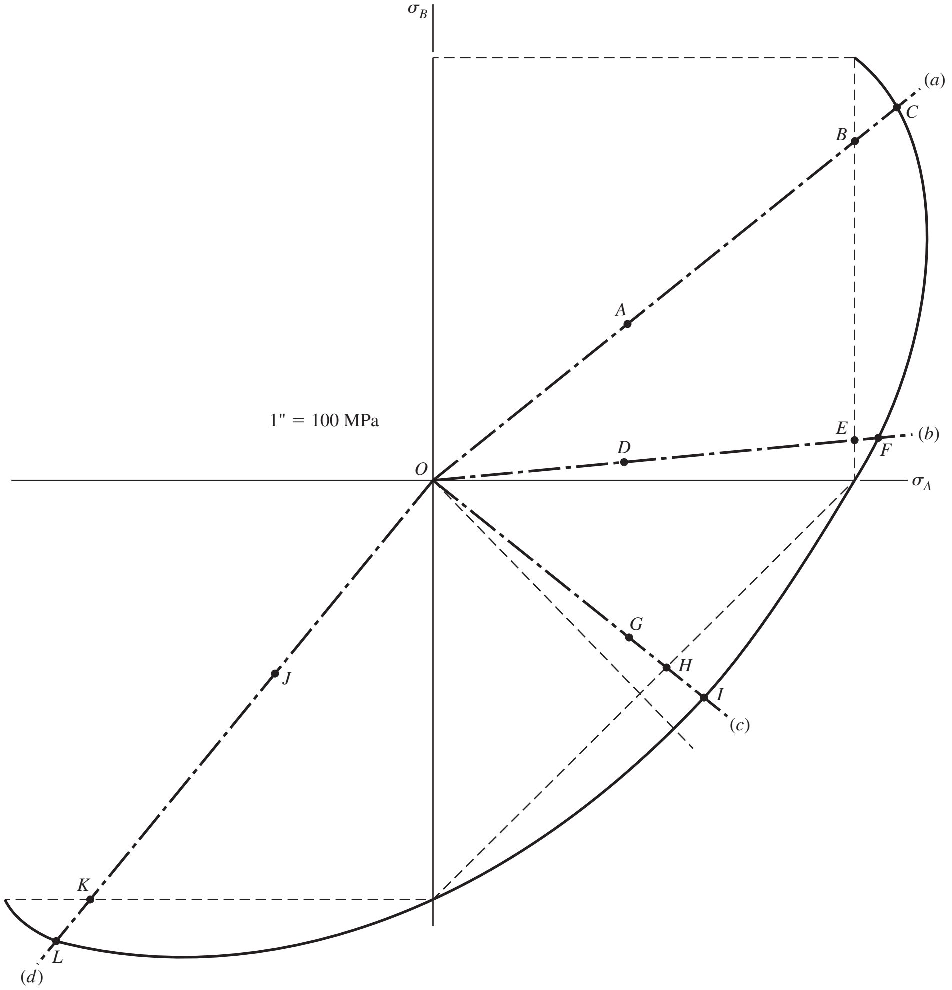
5-6 $S_y = 220$ MPa

(a) MSS: $$n = \frac{OB}{OA} = \frac{2.82}{1.3} = 2.2$$

DE: $$n = \frac{OC}{OA} = \frac{3.1}{1.3} = 2.4$$

(b) MSS: $$n = \frac{OE}{OD} = \frac{2.2}{1} = 2.2$$

DE: $$n = \frac{OF}{OD} = \frac{2.33}{1} = 2.3$$



(c) MSS: $$n = \frac{OH}{OG} = \frac{1.55}{1.3} = 1.2$$

DE: $$n = \frac{OI}{OG} = \frac{1.8}{1.3} = 1.4$$

(d) MSS: $$n = \frac{OK}{OJ} = \frac{2.82}{1.3} = 2.2$$

DE: $$n = \frac{OL}{OJ} = \frac{3.1}{1.3} = 2.4$$

5-7 $S_{ut} = 30$ kpsi, $S_{uc} = 100$ kpsi; $\sigma_A = 20$ kpsi, $\sigma_B = 6$ kpsi

(a) MNS: Eq. (5-30*a*) $n = \dfrac{S_{ut}}{\sigma_x} = \dfrac{30}{20} = 1.5$ *Ans.*

BCM: Eq. (5-31*a*) $n = \dfrac{30}{20} = 1.5$ *Ans.*

MM: Eq. (5-32*a*) $n = \dfrac{30}{20} = 1.5$ *Ans.*

(b) $\sigma_x = 12$ kpsi, $\tau_{xy} = -8$ kpsi

$$\sigma_A, \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 16, -4 \text{ kpsi}$$

MNS: Eq. (5-30*a*) $n = \dfrac{30}{16} = 1.88$ *Ans.*

BCM: Eq. (5-31*b*) $\dfrac{1}{n} = \dfrac{16}{30} - \dfrac{(-4)}{100} \quad \Rightarrow \quad n = 1.74$ *Ans.*

MM: Eq. (5-32*a*) $n = \dfrac{30}{16} = 1.88$ *Ans.*

(c) $\sigma_x = -6$ kpsi, $\sigma_y = -10$ kpsi, $\tau_{xy} = -5$ kpsi

$$\sigma_A, \sigma_B = \frac{-6-10}{2} \pm \sqrt{\left(\frac{-6+10}{2}\right)^2 + (-5)^2} = -2.61, -13.39 \text{ kpsi}$$

MNS: Eq. (5-30*b*) $n = -\dfrac{100}{-13.39} = 7.47$ *Ans.*

BCM: Eq. (5-31*c*) $n = -\dfrac{100}{-13.39} = 7.47$ *Ans.*

MM: Eq. (5-32*c*) $n = -\dfrac{100}{-13.39} = 7.47$ *Ans.*

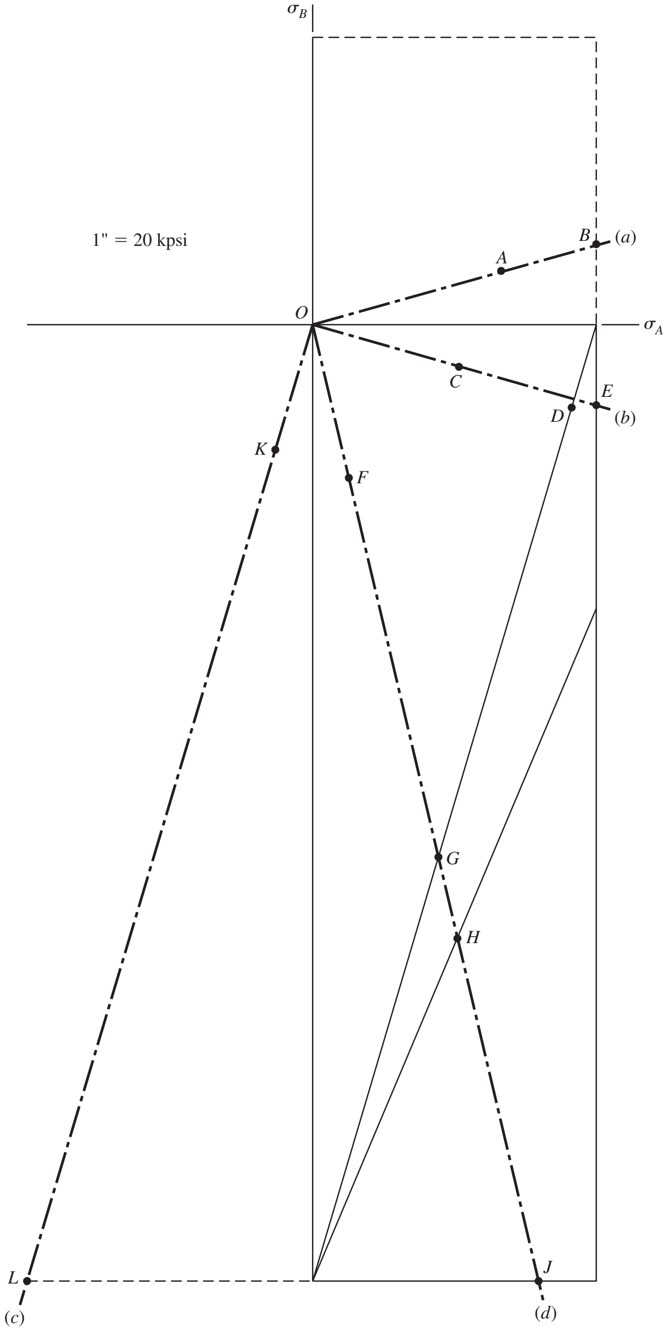
(d) $\sigma_x = -12$ kpsi, $\tau_{xy} = 8$ kpsi

$$\sigma_A, \sigma_B = -\frac{12}{2} \pm \sqrt{\left(-\frac{12}{2}\right)^2 + 8^2} = 4, -16 \text{ kpsi}$$

MNS: Eq. (5-30*b*) $n = \dfrac{-100}{-16} = 6.25$ *Ans.*

BCM: Eq. (5-31*b*) $$\frac{1}{n} = \frac{4}{30} - \frac{(-16)}{100} \quad \Rightarrow \quad n = 3.41 \quad \textit{Ans.}$$

MM: Eq. (5-32*b*) $$\frac{1}{n} = \frac{(100-30)4}{100(30)} - \frac{-16}{100} \quad \Rightarrow \quad n = 3.95 \quad \textit{Ans.}$$



5-8 See Prob. 5-7 for plot.

(a) For all methods: $n = \dfrac{OB}{OA} = \dfrac{1.55}{1.03} = 1.5$

(b) BCM: $n = \dfrac{OD}{OC} = \dfrac{1.4}{0.8} = 1.75$

All other methods: $n = \dfrac{OE}{OC} = \dfrac{1.55}{0.8} = 1.9$

(c) For all methods: $n = \dfrac{OL}{OK} = \dfrac{5.2}{0.68} = 7.6$

(d) MNS: $n = \dfrac{OJ}{OF} = \dfrac{5.12}{0.82} = 6.2$

BCM: $n = \dfrac{OG}{OF} = \dfrac{2.85}{0.82} = 3.5$

MM: $n = \dfrac{OH}{OF} = \dfrac{3.3}{0.82} = 4.0$

5-9 Given: $S_y = 42$ kpsi, $S_{ut} = 66.2$ kpsi, $\varepsilon_f = 0.90$. Since $\varepsilon_f > 0.05$, the material is ductile and thus we may follow convention by setting $S_{yc} = S_{yt}$.

Use DE theory for analytical solution. For σ', use Eq. (5-13) or (5-15) for plane stress and Eq. (5-12) or (5-14) for general 3-D.

(a) $\sigma' = [9^2 - 9(-5) + (-5)^2]^{1/2} = 12.29$ kpsi

$$n = \frac{42}{12.29} = 3.42 \quad \textit{Ans.}$$

(b) $\sigma' = [12^2 + 3(3^2)]^{1/2} = 13.08$ kpsi

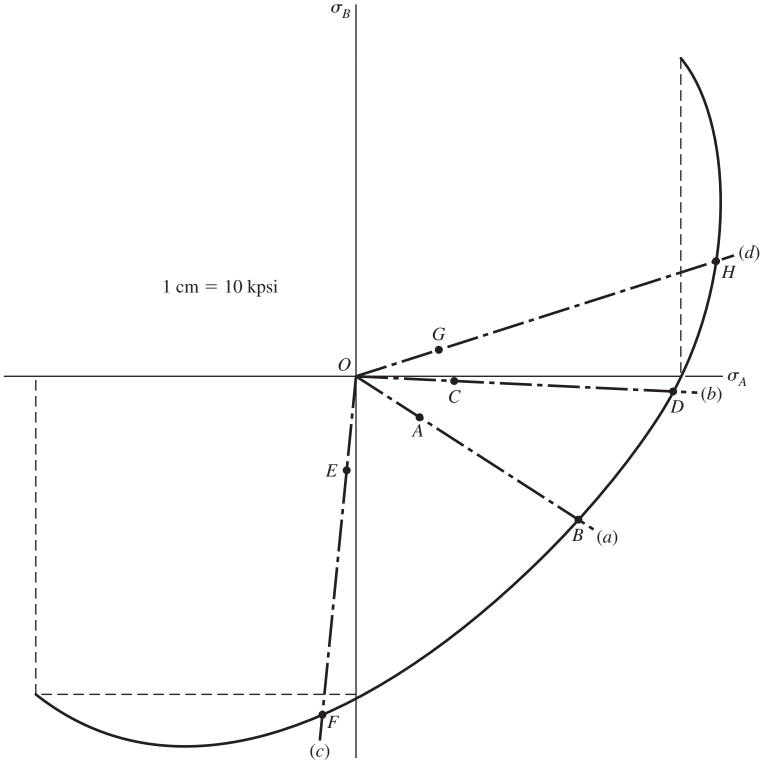
$$n = \frac{42}{13.08} = 3.21 \quad \textit{Ans.}$$

(c) $\sigma' = [(-4)^2 - (-4)(-9) + (-9)^2 + 3(5^2)]^{1/2} = 11.66$ kpsi

$$n = \frac{42}{11.66} = 3.60 \quad \textit{Ans.}$$

(d) $\sigma' = [11^2 - (11)(4) + 4^2 + 3(1^2)]^{1/2} = 9.798$

$$n = \frac{42}{9.798} = 4.29 \quad \textit{Ans.}$$



For graphical solution, plot load lines on DE envelope as shown.

(a) $\sigma_A = 9, \sigma_B = -5$ kpsi

$$n = \frac{OB}{OA} = \frac{3.5}{1} = 3.5 \quad \textit{Ans.}$$

(b) $\sigma_A, \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + 3^2} = 12.7, -0.708$ kpsi

$$n = \frac{OD}{OC} = \frac{4.2}{1.3} = 3.23$$

(c) $\sigma_A, \sigma_B = \frac{-4-9}{2} \pm \sqrt{\left(\frac{4-9}{2}\right)^2 + 5^2} = -0.910, -12.09$ kpsi

$$n = \frac{OF}{OE} = \frac{4.5}{1.25} = 3.6 \quad \textit{Ans.}$$

(d) $\sigma_A, \sigma_B = \frac{11+4}{2} \pm \sqrt{\left(\frac{11-4}{2}\right)^2 + 1^2} = 11.14, 3.86$ kpsi

$$n = \frac{OH}{OG} = \frac{5.0}{1.15} = 4.35 \quad \textit{Ans.}$$

5-10 This heat-treated steel exhibits $S_{yt} = 235$ kpsi, $S_{yc} = 275$ kpsi and $\varepsilon_f = 0.06$. The steel is ductile ($\varepsilon_f > 0.05$) but of unequal yield strengths. The Ductile Coulomb-Mohr hypothesis (DCM) of Fig. 5-19 applies — confine its use to first and fourth quadrants.

(a) $\sigma_x = 90$ kpsi, $\sigma_y = -50$ kpsi, $\sigma_z = 0$ $\therefore$ $\sigma_A = 90$ kpsi and $\sigma_B = -50$ kpsi. For the fourth quadrant, from Eq. (5-31b)

$$n = \frac{1}{(\sigma_A/S_{yt}) - (\sigma_B/S_{uc})} = \frac{1}{(90/235) - (-50/275)} = 1.77 \quad \textit{Ans.}$$

(b) $\sigma_x = 120$ kpsi, $\tau_{xy} = -30$ kpsi ccw. $\sigma_A, \sigma_B = 127.1, -7.08$ kpsi. For the fourth quadrant

$$n = \frac{1}{(127.1/235) - (-7.08/275)} = 1.76 \quad \textit{Ans.}$$

(c) $\sigma_x = -40$ kpsi, $\sigma_y = -90$ kpsi, $\tau_{xy} = 50$ kpsi. $\sigma_A,\ \sigma_B = -9.10,\ -120.9$ kpsi. Although no solution exists for the third quadrant, use

$$n = -\frac{S_{yc}}{\sigma_y} = -\frac{275}{-120.9} = 2.27 \quad \textit{Ans.}$$

(d) $\sigma_x = 110$ kpsi, $\sigma_y = 40$ kpsi, $\tau_{xy} = 10$ kpsi cw. $\sigma_A, \sigma_B = 111.4, 38.6$ kpsi. For the first quadrant

$$n = \frac{S_{yt}}{\sigma_A} = \frac{235}{111.4} = 2.11 \quad \textit{Ans.}$$

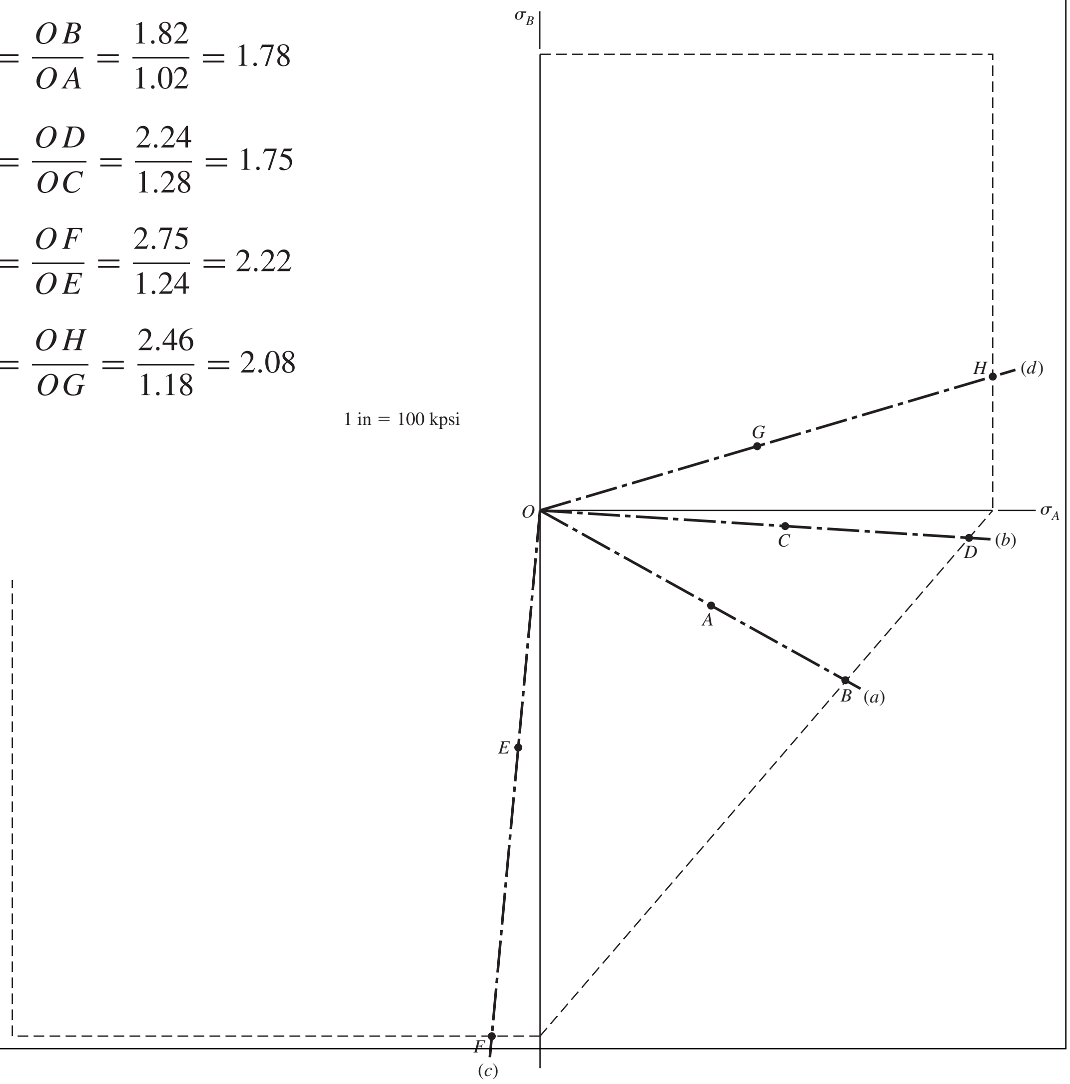
Graphical Solution:

(a) $n = \dfrac{OB}{OA} = \dfrac{1.82}{1.02} = 1.78$

(b) $n = \dfrac{OD}{OC} = \dfrac{2.24}{1.28} = 1.75$

(c) $n = \dfrac{OF}{OE} = \dfrac{2.75}{1.24} = 2.22$

(d) $n = \dfrac{OH}{OG} = \dfrac{2.46}{1.18} = 2.08$



5-11 The material is brittle and exhibits unequal tensile and compressive strengths. *Decision:* Use the Modified Mohr theory.

$$S_{ut} = 22 \text{ kpsi}, \quad S_{uc} = 83 \text{ kpsi}$$

(a) $\sigma_x = 9$ kpsi, $\sigma_y = -5$ kpsi. $\sigma_A, \sigma_B = 9, -5$ kpsi. For the fourth quadrant, $|\frac{\sigma_B}{\sigma_A}| = \frac{5}{9} < 1$, use Eq. (5-32*a*)

$$n = \frac{S_{ut}}{\sigma_A} = \frac{22}{9} = 2.44 \quad Ans.$$

(b) $\sigma_x = 12$ kpsi, $\tau_{xy} = -3$ kpsi ccw. $\sigma_A, \sigma_B = 12.7, -0.708$ kpsi. For the fourth quadrant, $|\frac{\sigma_B}{\sigma_A}| = \frac{0.708}{12.7} < 1$,

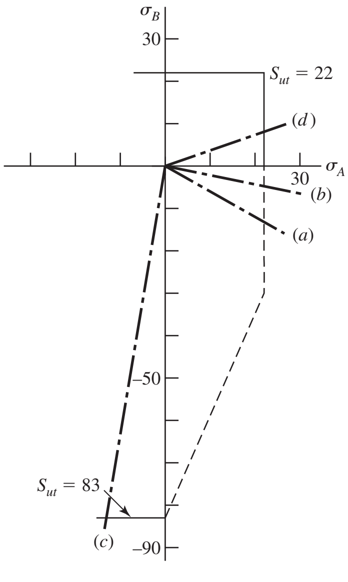
$$n = \frac{S_{ut}}{\sigma_A} = \frac{22}{12.7} = 1.73 \quad Ans.$$

(c) $\sigma_x = -4$ kpsi, $\sigma_y = -9$ kpsi, $\tau_{xy} = 5$ kpsi. $\sigma_A, \sigma_B = -0.910, -12.09$ kpsi. For the third quadrant, no solution exists; however, use Eq. (6-32*c*)

$$n = \frac{-83}{-12.09} = 6.87 \quad Ans.$$

(d) $\sigma_x = 11$ kpsi, $\sigma_y = 4$ kpsi, $\tau_{xy} = 1$ kpsi. $\sigma_A, \sigma_B = 11.14, 3.86$ kpsi. For the first quadrant

$$n = \frac{S_A}{\sigma_A} = \frac{S_{yt}}{\sigma_A} = \frac{22}{11.14} = 1.97 \quad Ans.$$



5-12 Since $\varepsilon_f < 0.05$, the material is brittle. Thus, $S_{ut} \doteq S_{uc}$ and we may use MM which is basically the same as MNS.

(a) $\sigma_A, \sigma_B = 9, -5$ kpsi

$$n = \frac{35}{9} = 3.89 \quad \textit{Ans.}$$

(b) $\sigma_A, \sigma_B = 12.7, -0.708$ kpsi

$$n = \frac{35}{12.7} = 2.76 \quad \textit{Ans.}$$

(c) $\sigma_A, \sigma_B = -0.910, -12.09$ kpsi (3rd quadrant)

$$n = \frac{36}{12.09} = 2.98 \quad \textit{Ans.}$$

(d) $\sigma_A, \sigma_B = 11.14, 3.86$ kpsi

$$n = \frac{35}{11.14} = 3.14 \quad \textit{Ans.}$$

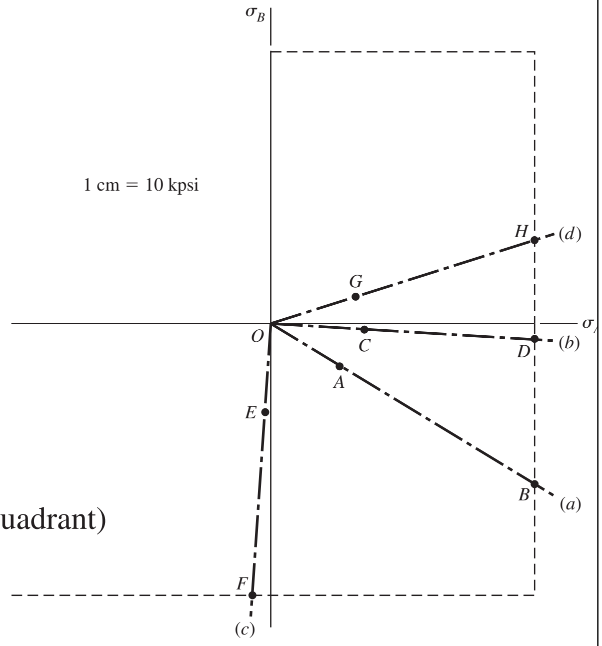
Graphical Solution:

(a) $n = \dfrac{OB}{OA} = \dfrac{4}{1} = 4.0 \quad \textit{Ans.}$

(b) $n = \dfrac{OD}{OC} = \dfrac{3.45}{1.28} = 2.70 \quad \textit{Ans.}$

(c) $n = \dfrac{OF}{OE} = \dfrac{3.7}{1.3} = 2.85 \quad \textit{Ans.}$ (3rd quadrant)

(d) $n = \dfrac{OH}{OG} = \dfrac{3.6}{1.15} = 3.13 \quad \textit{Ans.}$



5-13 $S_{ut} = 30$ kpsi, $S_{uc} = 109$ kpsi

Use MM:

(a) $\sigma_A, \sigma_B = 20, 20$ kpsi

Eq. (5-32*a*): $\quad n = \dfrac{30}{20} = 1.5 \quad \textit{Ans.}$

(b) $\sigma_A, \sigma_B = \pm\sqrt{(15)^2} = 15, -15$ kpsi

Eq. (5-32*a*) $\quad n = \dfrac{30}{15} = 2 \quad \textit{Ans.}$

(c) $\sigma_A, \sigma_B = -80, -80$ kpsi

For the 3rd quadrant, there is no solution but use Eq. (5-32*c*).

Eq. (5-32*c*): $\quad n = -\dfrac{109}{-80} = 1.36 \quad \textit{Ans.}$

(d) $\sigma_A, \sigma_B = 15, -25$ kpsi, $|\sigma_B|\sigma_A| = 25/15 > 1$,

Eq. (5-32*b*):
$$\frac{(109-30)15}{109(30)} - \frac{-25}{109} = \frac{1}{n}$$

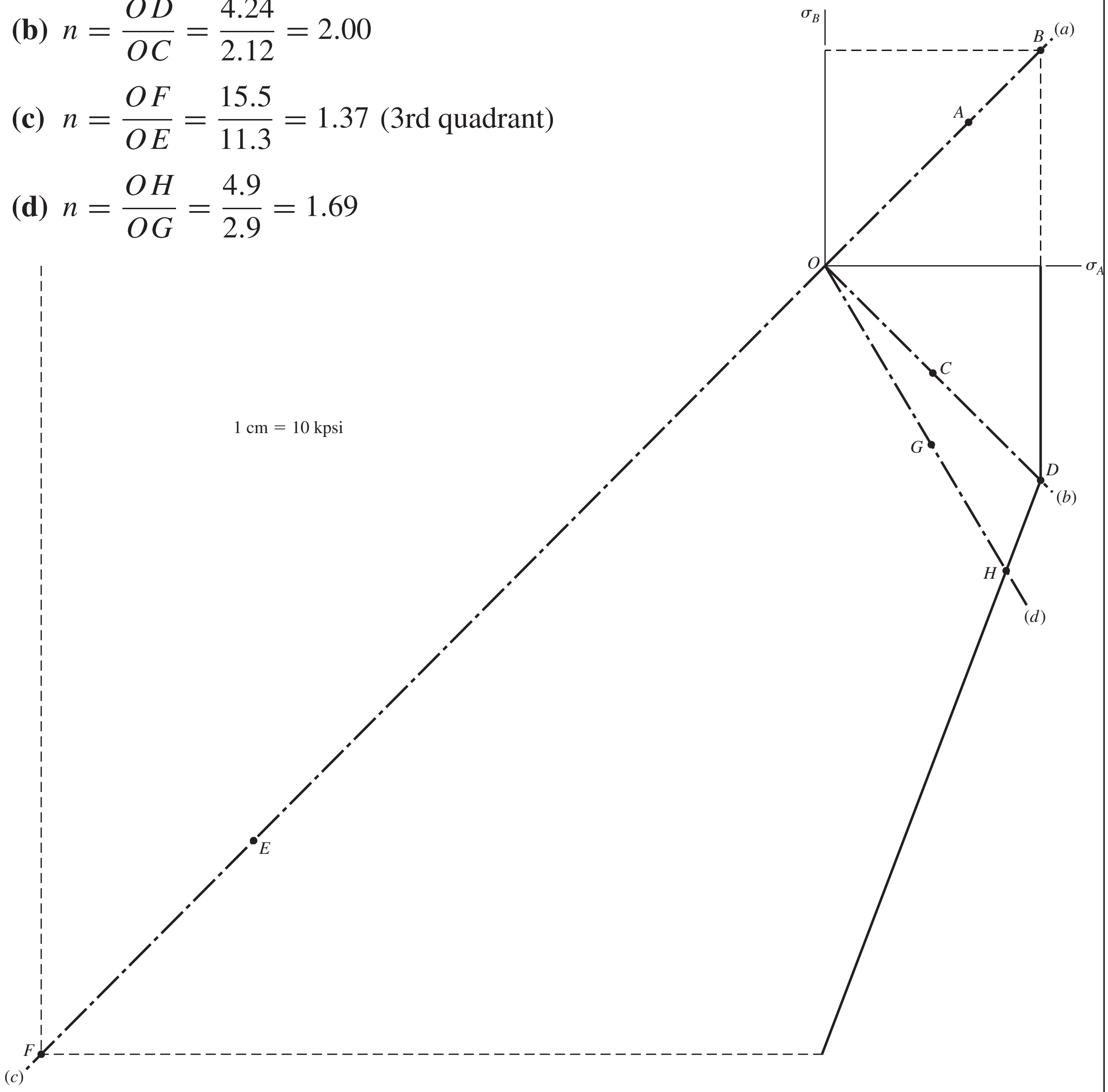
$$n = 1.69 \quad \textit{Ans.}$$

(a) $n = \dfrac{OB}{OA} = \dfrac{4.25}{2.83} = 1.50$

(b) $n = \dfrac{OD}{OC} = \dfrac{4.24}{2.12} = 2.00$

(c) $n = \dfrac{OF}{OE} = \dfrac{15.5}{11.3} = 1.37$ (3rd quadrant)

(d) $n = \dfrac{OH}{OG} = \dfrac{4.9}{2.9} = 1.69$



5-14 Given: AISI 1006 CD steel, $F = 0.55$ N, $P = 8.0$ kN, and $T = 30$ N · m, applying the DE theory to stress elements A and B with $S_y = 280$ MPa

A:
$$\sigma_x = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{32(0.55)(10^3)(0.1)}{\pi(0.020^3)} + \frac{4(8)(10^3)}{\pi(0.020^2)}$$
$$= 95.49(10^6)\text{ Pa} = 95.49\text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(30)}{\pi(0.020^3)} = 19.10(10^6)\text{ Pa} = 19.10\text{ MPa}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = [95.49^2 + 3(19.1)^2]^{1/2} = 101.1\text{ MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{280}{101.1} = 2.77 \quad \textit{Ans.}$$

B:

$$\sigma_x = \frac{4P}{\pi d^3} = \frac{4(8)(10^3)}{\pi(0.020^2)} = 25.47(10^6)\text{ Pa} = 25.47\text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} + \frac{4}{3}\frac{V}{A} = \frac{16(30)}{\pi(0.020^3)} + \frac{4}{3}\left[\frac{0.55(10^3)}{(\pi/4)(0.020^2)}\right]$$

$$= 21.43(10^6)\text{ Pa} = 21.43\text{ MPa}$$

$$\sigma' = [25.47^2 + 3(21.43^2)]^{1/2} = 45.02\text{ MPa}$$

$$n = \frac{280}{45.02} = 6.22 \quad \textit{Ans.}$$

5-15 $S_y = 32\text{ kpsi}$

At A, $M = 6(190) = 1\,140$ lbf·in, $T = 4(190) = 760$ lbf · in.

$$\sigma_x = \frac{32M}{\pi d^3} = \frac{32(1140)}{\pi(3/4)^3} = 27\,520\text{ psi}$$

$$\tau_{zx} = \frac{16T}{\pi d^3} = \frac{16(760)}{\pi(3/4)^3} = 9175\text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{27\,520}{2}\right)^2 + 9175^2} = 16\,540\text{ psi}$$

$$n = \frac{S_y}{2\tau_{\max}} = \frac{32}{2(16.54)} = 0.967 \quad \textit{Ans.}$$

MSS predicts yielding

5-16 From Prob. 4-15, $\sigma_x = 27.52$ kpsi, $\tau_{zx} = 9.175$ kpsi. For Eq. (5-15), adjusted for coordinates,

$$\sigma' = \left[27.52^2 + 3(9.175)^2\right]^{1/2} = 31.78\text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{31.78} = 1.01 \quad \textit{Ans.}$$

DE predicts no yielding, but it is extremely close. Shaft size should be increased.

5-17 Design decisions required:

- Material and condition
- Design factor
- Failure model
- Diameter of pin

Using $F = 416$ lbf from Ex. 5-3

$$\sigma_{\max} = \frac{32M}{\pi d^3}$$

$$d = \left(\frac{32M}{\pi \sigma_{\max}}\right)^{1/3}$$

Decision 1: Select the same material and condition of Ex. 5-3 (AISI 1035 steel, $S_y = 81\,000$).

Decision 2: Since we prefer the pin to yield, set n_d a little larger than 1. Further explanation will follow.

Decision 3: Use the Distortion Energy static failure theory.

Decision 4: Initially set $n_d = 1$

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{S_y}{1} = 81\,000 \text{ psi}$$

$$d = \left[\frac{32(416)(15)}{\pi(81\,000)}\right]^{1/3} = 0.922 \text{ in}$$

Choose preferred size of $d = 1.000$ in

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.274$$

Set design factor to $n_d = 1.274$

Adequacy Assessment:

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{81\,000}{1.274} = 63\,580 \text{ psi}$$

$$d = \left[\frac{32(416)(15)}{\pi(63\,580)}\right]^{1/3} = 1.000 \text{ in} \quad (OK)$$

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.274 \quad \text{(OK)}$$

5-18 For a thin walled cylinder made of AISI 1018 steel, $S_y = 54$ kpsi, $S_{ut} = 64$ kpsi.

The state of stress is

$$\sigma_t = \frac{pd}{4t} = \frac{p(8)}{4(0.05)} = 40p, \quad \sigma_l = \frac{pd}{8t} = 20p, \quad \sigma_r = -p$$

These three are all principal stresses. Therefore,

$$\sigma' = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

$$= \frac{1}{\sqrt{2}}[(40p - 20p)^2 + (20p + p)^2 + (-p - 40p)^2]$$

$$= 35.51p = 54 \quad \Rightarrow \quad p = 1.52 \text{ kpsi} \quad \text{(for yield)} \quad \textit{Ans.}$$

For rupture, $35.51p \doteq 64 \Rightarrow p \doteq 1.80$ kpsi *Ans.*

5-19 For hot-forged AISI steel $w = 0.282$ lbf/in^3, $S_y = 30$ kpsi and $\nu = 0.292$. Then $\rho = w/g = 0.282/386$ lbf $\cdot$ s^2/in; $r_i = 3$ in; $r_o = 5$ in; $r_i^2 = 9$; $r_o^2 = 25$; $3 + \nu = 3.292$; $1 + 3\nu = 1.876$.

Eq. (3-55) for $r = r_i$ becomes

$$\sigma_t = \rho\omega^2 \left(\frac{3+\nu}{8}\right)\left[2r_o^2 + r_i^2\left(1 - \frac{1+3\nu}{3+\nu}\right)\right]$$

Rearranging and substituting the above values:

$$\frac{S_y}{\omega^2} = \frac{0.282}{386}\left(\frac{3.292}{8}\right)\left[50 + 9\left(1 - \frac{1.876}{3.292}\right)\right]$$

$$= 0.016\,19$$

Setting the tangential stress equal to the yield stress,

$$\omega = \left(\frac{30\,000}{0.016\,19}\right)^{1/2} = 1361 \text{ rad/s}$$

or

$$n = 60\omega/2\pi = 60(1361)/(2\pi)$$

$$= 13\,000 \text{ rev/min}$$

Now check the stresses at $r = (r_o r_i)^{1/2}$, or $r = [5(3)]^{1/2} = 3.873$ in

$$\sigma_r = \rho\omega^2\left(\frac{3+\nu}{8}\right)(r_o - r_i)^2$$

$$= \frac{0.282\omega^2}{386}\left(\frac{3.292}{8}\right)(5-3)^2$$

$$= 0.001\,203\omega^2$$

Applying Eq. (3-55) for σ_t

$$\sigma_t = \omega^2\left(\frac{0.282}{386}\right)\left(\frac{3.292}{8}\right)\left[9 + 25 + \frac{9(25)}{15} - \frac{1.876(15)}{3.292}\right]$$

$$= 0.012\,16\omega^2$$

Using the Distortion-Energy theory

$$\sigma' = \left(\sigma_t^2 - \sigma_r\sigma_t + \sigma_r^2\right)^{1/2} = 0.011\,61\omega^2$$

Solving $$\omega = \left(\frac{30\,000}{0.011\,61}\right)^{1/2} = 1607\text{ rad/s}$$

So the inner radius governs and $n = 13\,000$ rev/min *Ans.*

5-20 For a thin-walled pressure vessel,

$$d_i = 3.5 - 2(0.065) = 3.37\text{ in}$$

$$\sigma_t = \frac{p(d_i + t)}{2t}$$

$$\sigma_t = \frac{500(3.37 + 0.065)}{2(0.065)} = 13\,212\text{ psi}$$

$$\sigma_l = \frac{pd_i}{4t} = \frac{500(3.37)}{4(0.065)} = 6481\text{ psi}$$

$$\sigma_r = -p_i = -500\text{ psi}$$

These are all principal stresses, thus,

$$\sigma' = \frac{1}{\sqrt{2}}\{(13\,212 - 6481)^2 + [6481 - (-500)]^2 + (-500 - 13\,212)^2\}^{1/2}$$

$$\sigma' = 11\,876\text{ psi}$$

$$n = \frac{S_y}{\sigma'} = \frac{46\,000}{\sigma'} = \frac{46\,000}{11\,876}$$

$$= 3.87 \quad \textit{Ans.}$$

5-21 Table A-20 gives S_y as 320 MPa. The maximum significant stress condition occurs at r_i where $\sigma_1 = \sigma_r = 0$, $\sigma_2 = 0$, and $\sigma_3 = \sigma_t$. From Eq. (3-49) for $r = r_i$, $p_i = 0$,

$$\sigma_t = -\frac{2r_o^2 p_o}{r_o^2 - r_i^2} = -\frac{2(150^2)p_o}{150^2 - 100^2} = -3.6p_o$$

$$\sigma' = 3.6p_o = S_y = 320$$

$$p_o = \frac{320}{3.6} = 88.9\text{ MPa} \quad \textit{Ans.}$$

5-22 $S_{ut} = 30$ kpsi, $w = 0.260$ lbf/in^3, $\nu = 0.211$, $3 + \nu = 3.211$, $1 + 3\nu = 1.633$. At the inner radius, from Prob. 5-19

$$\frac{\sigma_t}{\omega^2} = \rho\left(\frac{3+\nu}{8}\right)\left(2r_o^2 + r_i^2 - \frac{1+3\nu}{3+\nu}r_i^2\right)$$

Here $r_o^2 = 25, r_i^2 = 9,$ and so

$$\frac{\sigma_t}{\omega^2} = \frac{0.260}{386}\left(\frac{3.211}{8}\right)\left(50 + 9 - \frac{1.633(9)}{3.211}\right) = 0.0147$$

Since σ_r is of the same sign, we use M2M failure criteria in the first quadrant. From Table A-24, $S_{ut} = 31$ kpsi, thus,

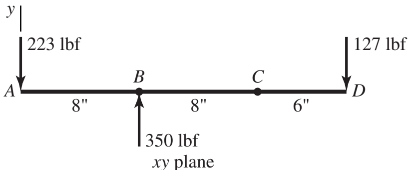
$$\omega = \left(\frac{31\,000}{0.0147}\right)^{1/2} = 1452 \text{ rad/s}$$

$$\text{rpm} = 60\omega/(2\pi) = 60(1452)/(2\pi)$$

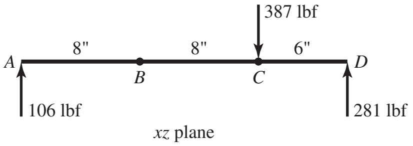
$$= 13\,866 \text{ rev/min}$$

Using the grade number of 30 for $S_{ut} = 30\,000$ kpsi gives a bursting speed of 13640 rev/min.

5-23 $T_C = (360 - 27)(3) = 1000 \text{ lbf} \cdot \text{in}, \quad T_B = (300 - 50)(4) = 1000 \text{ lbf} \cdot \text{in}$



In xy plane, $M_B = 223(8) = 1784 \text{ lbf} \cdot \text{in}$ and $M_C = 127(6) = 762 \text{ lbf} \cdot \text{in}$.



In the xz plane, $M_B = 848 \text{ lbf} \cdot \text{in}$ and $M_C = 1686 \text{ lbf} \cdot \text{in}$. The resultants are

$$M_B = [(1784)^2 + (848)^2]^{1/2} = 1975 \text{ lbf} \cdot \text{in}$$

$$M_C = [(1686)^2 + (762)^2]^{1/2} = 1850 \text{ lbf} \cdot \text{in}$$

So point B governs and the stresses are

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi d^3} = \frac{5093}{d^3} \text{ psi}$$

$$\sigma_x = \frac{32M_B}{\pi d^3} = \frac{32(1975)}{\pi d^3} = \frac{20\,120}{d^3} \text{ psi}$$

Then

$$\sigma_A, \sigma_B = \frac{\sigma_x}{2} \pm \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

$$\sigma_A, \sigma_B = \frac{1}{d^3}\left\{\frac{20.12}{2} \pm \left[\left(\frac{20.12}{2}\right)^2 + (5.09)^2\right]^{1/2}\right\}$$

$$= \frac{(10.06 \pm 11.27)}{d^3} \text{ kpsi} \cdot \text{in}^3$$

Then

$$\sigma_A = \frac{10.06 + 11.27}{d^3} = \frac{21.33}{d^3}\ \text{kpsi}$$

and

$$\sigma_B = \frac{10.06 - 11.27}{d^3} = -\frac{1.21}{d^3}\ \text{kpsi}$$

For this state of stress, use the Brittle-Coulomb-Mohr theory for illustration. Here we use $S_{ut}(\text{min}) = 25\ \text{kpsi}$, $S_{uc}(\text{min}) = 97\ \text{kpsi}$, and Eq. (5-31$b$) to arrive at

$$\frac{21.33}{25d^3} - \frac{-1.21}{97d^3} = \frac{1}{2.8}$$

Solving gives $d = 1.34$ in. So use $d = 1\,3/8$ in *Ans.*

Note that this has been solved as a statics problem. Fatigue will be considered in the next chapter.

5-24 As in Prob. 5-23, we will assume this to be statics problem. Since the proportions are unchanged, the bearing reactions will be the same as in Prob. 5-23. Thus

xy plane: $M_B = 223(4) = 892\ \text{lbf}\cdot\text{in}$

xz plane: $M_B = 106(4) = 424\ \text{lbf}\cdot\text{in}$

So

$$M_{\max} = [(892)^2 + (424)^2]^{1/2} = 988\ \text{lbf}\cdot\text{in}$$

$$\sigma_x = \frac{32M_B}{\pi d^3} = \frac{32(988)}{\pi d^3} = \frac{10\,060}{d^3}\ \text{psi}$$

Since the torsional stress is unchanged,

$$\tau_{xz} = 5.09/d^3\ \text{kpsi}$$

$$\sigma_A, \sigma_B = \frac{1}{d^3}\left\{\left(\frac{10.06}{2}\right) \pm \left[\left(\frac{10.06}{2}\right)^2 + (5.09)^2\right]^{1/2}\right\}$$

$$\sigma_A = 12.19/d^3 \quad \text{and} \quad \sigma_B = -2.13/d^3$$

Using the Brittle-Coulomb-Mohr, as was used in Prob. 5-23, gives

$$\frac{12.19}{25d^3} - \frac{-2.13}{97d^3} = \frac{1}{2.8}$$

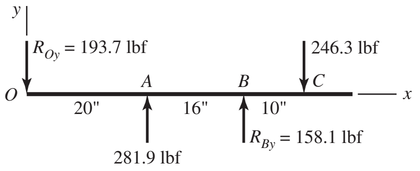
Solving gives $d = 1\,1/8$ in. *Ans.*

5-25

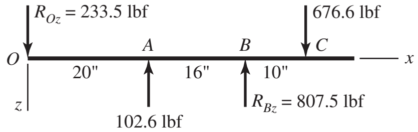
$$(F_A)_t = 300\cos 20 = 281.9\ \text{lbf}, \quad (F_A)_r = 300\sin 20 = 102.6\ \text{lbf}$$

$$T = 281.9(12) = 3383\ \text{lbf}\cdot\text{in}, \quad (F_C)_t = \frac{3383}{5} = 676.6\ \text{lbf}$$

$$(F_C)_r = 676.6\tan 20 = 246.3\ \text{lbf}$$



xy plane



xz plane

$$M_A = 20\sqrt{193.7^2 + 233.5^2} = 6068\ \text{lbf}\cdot\text{in}$$

$$M_B = 10\sqrt{246.3^2 + 676.6^2} = 7200\ \text{lbf}\cdot\text{in} \quad \text{(maximum)}$$

$$\sigma_x = \frac{32(7200)}{\pi d^3} = \frac{73\,340}{d^3}$$

$$\tau_{xy} = \frac{16(3383)}{\pi d^3} = \frac{17\,230}{d^3}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = \frac{S_y}{n}$$

$$\left[\left(\frac{73\,340}{d^3}\right)^2 + 3\left(\frac{17\,230}{d^3}\right)^2\right]^{1/2} = \frac{79\,180}{d^3} = \frac{60\,000}{3.5}$$

$d = 1.665$ in so use a standard diameter size of 1.75 in *Ans.*

5-26 From Prob. 5-25,

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2} = \frac{S_y}{2n}$$

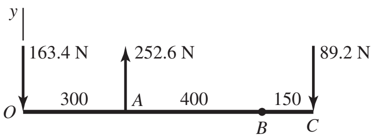
$$\left[\left(\frac{73\,340}{2d^3}\right)^2 + \left(\frac{17\,230}{d^3}\right)^2\right]^{1/2} = \frac{40\,516}{d^3} = \frac{60\,000}{2(3.5)}$$

$d = 1.678$ in so use 1.75 in *Ans.*

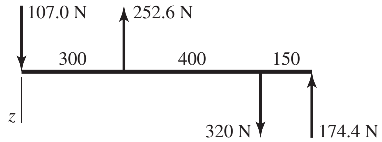
5-27 $T = (270 - 50)(0.150) = 33\ \text{N}\cdot\text{m},\ S_y = 370\ \text{MPa}$

$$(T_1 - 0.15T_1)(0.125) = 33 \quad \Rightarrow \quad T_1 = 310.6\ \text{N}, \quad T_2 = 0.15(310.6) = 46.6\ \text{N}$$

$$(T_1 + T_2)\cos 45 = 252.6\ \text{N}$$



xy plane



xz plane

$$M_A = 0.3\sqrt{163.4^2 + 107^2} = 58.59\ \text{N}\cdot\text{m} \qquad \text{(maximum)}$$

$$M_B = 0.15\sqrt{89.2^2 + 174.4^2} = 29.38\ \text{N}\cdot\text{m}$$

$$\sigma_x = \frac{32(58.59)}{\pi d^3} = \frac{596.8}{d^3}$$

$$\tau_{xy} = \frac{16(33)}{\pi d^3} = \frac{168.1}{d^3}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = \left[\left(\frac{596.8}{d^3}\right)^2 + 3\left(\frac{168.1}{d^3}\right)^2\right]^{1/2} = \frac{664.0}{d^3} = \frac{370(10^6)}{3.0}$$

$d = 17.5(10^{-3})$ m $= 17.5$ mm, so use 18 mm *Ans.*

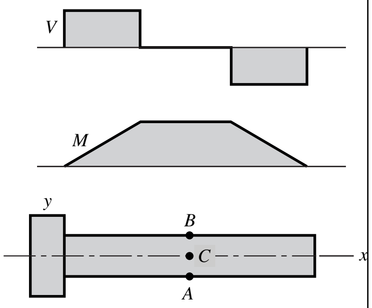
5-28 From Prob. 5-27,

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2} = \frac{S_y}{2n}$$

$$\left[\left(\frac{596.8}{2d^3}\right)^2 + \left(\frac{168.1}{d^3}\right)^2\right]^{1/2} = \frac{342.5}{d^3} = \frac{370(10^6)}{2(3.0)}$$

$d = 17.7(10^{-3})$ m $= 17.7$ mm, so use 18 mm *Ans.*

5-29 For the loading scheme shown in Figure (*c*),



$$M_{\max} = \frac{F}{2}\left(\frac{a}{2} + \frac{b}{4}\right) = \frac{4.4}{2}(6 + 4.5)$$
$$= 23.1\ \text{N}\cdot\text{m}$$

For a stress element at *A*:

$$\sigma_x = \frac{32M}{\pi d^3} = \frac{32(23.1)(10^3)}{\pi(12)^3} = 136.2\ \text{MPa}$$

The shear at *C* is

$$\tau_{xy} = \frac{4(F/2)}{3\pi d^2/4} = \frac{4(4.4/2)(10^3)}{3\pi(12)^2/4} = 25.94\ \text{MPa}$$

$$\tau_{\max} = \left[\left(\frac{136.2}{2}\right)^2\right]^{1/2} = 68.1\ \text{MPa}$$

Since $S_y = 220$ MPa, $S_{sy} = 220/2 = 110$ MPa, and

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{110}{68.1} = 1.62 \quad \textit{Ans.}$$

For the loading scheme depicted in Figure (*d*)

$$M_{\max} = \frac{F}{2}\left(\frac{a+b}{2}\right) - \frac{F}{2}\left(\frac{1}{2}\right)\left(\frac{b}{2}\right)^2 = \frac{F}{2}\left(\frac{a}{2}+\frac{b}{4}\right)$$

This result is the same as that obtained for Figure (*c*). At point *B*, we also have a surface compression of

$$\sigma_y = \frac{-F}{A} = \frac{-F}{bd} - \frac{-4.4(10^3)}{18(12)} = -20.4\text{ MPa}$$

With $\sigma_x = -136.2$ MPa. From a Mohrs circle diagram, $\tau_{\max} = 136.2/2 = 68.1$ MPa.

$$n = \frac{110}{68.1} = 1.62\text{ MPa}\quad Ans.$$

5-30 Based on Figure (*c*) and using Eq. (5-15)

$$\sigma' = \left(\sigma_x^2\right)^{1/2}$$

$$= (136.2^2)^{1/2} = 136.2\,\text{MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{220}{136.2} = 1.62\quad Ans.$$

Based on Figure (*d*) and using Eq. (5-15) and the solution of Prob. 5-29,

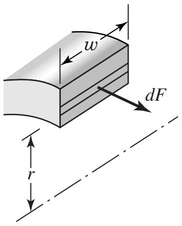
$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2\right)^{1/2}$$

$$= [(-136.2)^2 - (-136.2)(-20.4) + (-20.4)^2]^{1/2}$$

$$= 127.2\,\text{MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{220}{127.2} = 1.73\quad Ans.$$

5-31



When the ring is set, the hoop tension in the ring is equal to the screw tension.

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

We have the hoop tension at any radius. The differential hoop tension dF is

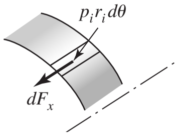
$$dF = w\sigma_t\,dr$$

$$F = \int_{r_i}^{r_o} w\sigma_t\,dr = \frac{wr_i^2 p_i}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(1 + \frac{r_o^2}{r^2}\right)dr = wr_i\,p_i \qquad (1)$$

The screw equation is

$$F_i = \frac{T}{0.2d} \tag{2}$$

From Eqs. (1) and (2)



$$p_i = \frac{F}{wr_i} = \frac{T}{0.2dwr_i}$$

$$dF_x = fp_i r_i \, d\theta$$

$$F_x = \int_o^{2\pi} fp_i wr_i \, d\theta = \frac{fTw}{0.2dwr_i} r_i \int_o^{2\pi} d\theta$$

$$= \frac{2\pi fT}{0.2d} \quad Ans.$$

5-32

(a) From Prob. 5-31, $T = 0.2F_i d$

$$F_i = \frac{T}{0.2d} = \frac{190}{0.2(0.25)} = 3800 \text{ lbf} \quad Ans.$$

(b) From Prob. 5-31, $F = wr_i p_i$

$$p_i = \frac{F}{wr_i} = \frac{F_i}{wr_i} = \frac{3800}{0.5(0.5)} = 15\,200 \text{ psi} \quad Ans.$$

(c)

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r}\right)_{r=r_i} = \frac{p_i\left(r_i^2 + r_o^2\right)}{r_o^2 - r_i^2}$$

$$= \frac{15\,200(0.5^2 + 1^2)}{1^2 - 0.5^2} = 25\,333 \text{ psi} \quad Ans.$$

$$\sigma_r = -p_i = -15\,200 \text{ psi}$$

(d)

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_t - \sigma_r}{2}$$

$$= \frac{25\,333 - (-15\,200)}{2} = 20\,267 \text{ psi} \quad Ans.$$

$$\sigma' = \left(\sigma_A^2 + \sigma_B^2 - \sigma_A\sigma_B\right)^{1/2}$$

$$= [25\,333^2 + (-15\,200)^2 - 25\,333(-15\,200)]^{1/2}$$

$$= 35\,466 \text{ psi} \quad Ans.$$

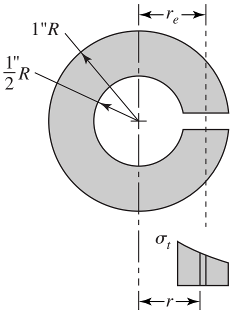
(e) Maximum Shear hypothesis

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{0.5S_y}{\tau_{\max}} = \frac{0.5(63)}{20.267} = 1.55 \quad Ans.$$

Distortion Energy theory

$$n = \frac{S_y}{\sigma'} = \frac{63}{35\,466} = 1.78 \quad Ans.$$

5-33



The moment about the center caused by force F is Fr_e where r_e is the effective radius. This is balanced by the moment about the center caused by the tangential (hoop) stress.

$$
\begin{aligned}
Fr_e &= \int_{r_i}^{r_o} r\sigma_t w\,dr \\
&= \frac{wp_i r_i^2}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(r + \frac{r_o^2}{r}\right)dr \\
r_e &= \frac{wp_i r_i^2}{F\left(r_o^2 - r_i^2\right)}\left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln\frac{r_o}{r_i}\right)
\end{aligned}
$$

From Prob. 5-31, $F = wr_i p_i$. Therefore,

$$r_e = \frac{r_i}{r_o^2 - r_i^2}\left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln\frac{r_o}{r_i}\right)$$

For the conditions of Prob. 5-31, $r_i = 0.5$ and $r_o = 1$ in

$$r_e = \frac{0.5}{1^2 - 0.5^2}\left(\frac{1^2 - 0.5^2}{2} + 1^2 \ln\frac{1}{0.5}\right) = 0.712 \text{ in}$$

5-34 $\delta_{\text{nom}} = 0.0005$ in

(a) From Eq. (3-57)

$$p = \frac{30(10^6)(0.0005)}{(1^3)}\left[\frac{(1.5^2 - 1^2)(1^2 - 0.5^2)}{2(1.5^2 - 0.5^2)}\right] = 3516 \text{ psi} \quad \textit{Ans.}$$

Inner member:

Eq. (3-58) $$(\sigma_t)_i = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -3516\left(\frac{1^2 + 0.5^2}{1^2 - 0.5^2}\right) = -5860 \text{ psi}$$

$$(\sigma_r)_i = -p = -3516 \text{ psi}$$

Eq. (5-13)
$$
\begin{aligned}
\sigma_i' &= \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} \\
&= [(-5860)^2 - (-5860)(-3516) + (-3516)^2]^{1/2} \\
&= 5110 \text{ psi} \quad \textit{Ans.}
\end{aligned}
$$

Outer member:

Eq. (3-59) $$(\sigma_t)_o = 3516\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 9142 \text{ psi}$$

$$(\sigma_r)_o = -p = -3516 \text{ psi}$$

Eq. (5-13)
$$
\begin{aligned}
\sigma_o' &= [9142^2 - 9142(-3516) + (-3516)^2]^{1/2} \\
&= 11\,320 \text{ psi} \quad \textit{Ans.}
\end{aligned}
$$

(b) For a solid inner tube,

$$p = \frac{30(10^6)(0.0005)}{1}\left[\frac{(1.5^2 - 1^2)(1^2)}{2(1^2)(1.5^2)}\right] = 4167 \text{ psi} \quad \textit{Ans.}$$

$$(\sigma_t)_i = -p = -4167 \text{ psi}, \quad (\sigma_r)_i = -4167 \text{ psi}$$

$$\sigma_i' = [(-4167)^2 - (-4167)(-4167) + (-4167)^2]^{1/2} = 4167 \text{ psi} \quad \textit{Ans.}$$

$$(\sigma_t)_o = 4167\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 10\,830 \text{ psi}, \quad (\sigma_r)_o = -4167 \text{ psi}$$

$$\sigma_o' = [10\,830^2 - 10\,830(-4167) + (-4167)^2]^{1/2} = 13\,410 \text{ psi} \quad \textit{Ans.}$$

5-35 Using Eq. (3-57) with diametral values,

$$p = \frac{207(10^3)(0.02)}{(50^3)}\left[\frac{(75^2 - 50^2)(50^2 - 25^2)}{2(75^2 - 25^2)}\right] = 19.41 \text{ MPa} \quad \textit{Ans.}$$

Eq. (3-58) $$(\sigma_t)_i = -19.41\left(\frac{50^2 + 25^2}{50^2 - 25^2}\right) = -32.35 \text{ MPa}$$

$$(\sigma_r)_i = -19.41 \text{ MPa}$$

Eq. (5-13) $$\sigma_i' = [(-32.35)^2 - (-32.35)(-19.41) + (-19.41)^2]^{1/2}$$

$$= 28.20 \text{ MPa} \quad \textit{Ans.}$$

Eq. (3-59) $$(\sigma_t)_o = 19.41\left(\frac{75^2 + 50^2}{75^2 - 50^2}\right) = 50.47 \text{ MPa},$$

$$(\sigma_r)_o = -19.41 \text{ MPa}$$

$$\sigma_o' = [50.47^2 - 50.47(-19.41) + (-19.41)^2]^{1/2} = 62.48 \text{ MPa} \quad \textit{Ans.}$$

5-36 Max. shrink-fit conditions: Diametral interference $\delta_d = 50.01 - 49.97 = 0.04$ mm. Equation (3-57) using diametral values:

$$p = \frac{207(10^3)0.04}{50^3}\left[\frac{(75^2 - 50^2)(50^2 - 25^2)}{2(75^2 - 25^2)}\right] = 38.81 \text{ MPa} \qquad \textit{Ans.}$$

Eq. (3-58): $$(\sigma_t)_i = -38.81\left(\frac{50^2 + 25^2}{50^2 - 25^2}\right) = -64.68 \text{ MPa}$$

$$(\sigma_r)_i = -38.81 \text{ MPa}$$

Eq. (5-13):

$$\sigma_i' = \left[(-64.68)^2 - (-64.68)(-38.81) + (-38.81)^2\right]^{1/2}$$

$$= 56.39 \text{ MPa} \qquad \textit{Ans.}$$

5-37

$$\delta = \frac{1.9998}{2} - \frac{1.999}{2} = 0.0004 \text{ in}$$

Eq. (3-56)

$$0.0004 = \frac{p(1)}{14.5(10^6)}\left[\frac{2^2+1^2}{2^2-1^2} + 0.211\right] + \frac{p(1)}{30(10^6)}\left[\frac{1^2+0}{1^2-0} - 0.292\right]$$

$$p = 2613 \text{ psi}$$

Applying Eq. (4-58) at *R*,

$$(\sigma_t)_o = 2613\left(\frac{2^2+1^2}{2^2-1^2}\right) = 4355 \text{ psi}$$

$$(\sigma_r)_o = -2613 \text{ psi}, \quad S_{ut} = 20 \text{ kpsi}, \quad S_{uc} = 83 \text{ kpsi}$$

$$\left|\frac{\sigma_o}{\sigma_A}\right| = \frac{2613}{4355} < 1, \ \therefore \text{ use Eq. (5-32a)}$$

$$h = S_{ut}/\sigma_A = 20/4.355 = 4.59 \quad \textit{Ans.}$$

5-38 $E = 30(10^6)$ psi, $\nu = 0.292$, $I = (\pi/64)(2^4 - 1.5^4) = 0.5369 \text{ in}^4$

Eq. (3-57) can be written in terms of diameters,

$$p = \frac{E\delta_d}{D}\left[\frac{\left(d_o^2 - D^2\right)\left(D^2 - d_i^2\right)}{2D^2\left(d_o^2 - d_i^2\right)}\right] = \frac{30(10^6)}{1.75}(0.002\,46)\left[\frac{(2^2-1.75^2)(1.75^2-1.5^2)}{2(1.75^2)(2^2-1.5^2)}\right]$$

$$= 2997 \text{ psi} = 2.997 \text{ kpsi}$$

Outer member:

Outer radius: $$(\sigma_t)_o = \frac{1.75^2(2.997)}{2^2-1.75^2}(2) = 19.58 \text{ kpsi}, (\sigma_r)_o = 0$$

Inner radius: $$(\sigma_t)_i = \frac{1.75^2(2.997)}{2^2-1.75^2}\left(1 + \frac{2^2}{1.75^2}\right) = 22.58 \text{ kpsi}, (\sigma_r)_i = -2.997 \text{ kpsi}$$

Bending:

r_o: $$(\sigma_x)_o = \frac{6.000(2/2)}{0.5369} = 11.18 \text{ kpsi}$$

r_i: $$(\sigma_x)_i = \frac{6.000(1.75/2)}{0.5369} = 9.78 \text{ kpsi}$$

Torsion: $$J = 2I = 1.0738 \text{ in}^4$$

r_o: $$(\tau_{xy})_o = \frac{8.000(2/2)}{1.0738} = 7.45 \text{ kpsi}$$

r_i: $$(\tau_{xy})_i = \frac{8.000(1.75/2)}{1.0738} = 6.52 \text{ kpsi}$$

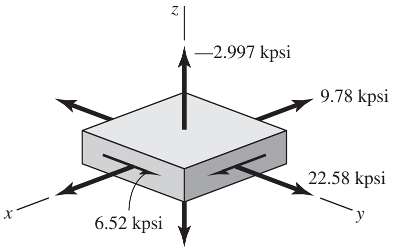
Outer radius is plane stress

$$\sigma_x = 11.18 \text{ kpsi}, \quad \sigma_y = 19.58 \text{ kpsi}, \quad \tau_{xy} = 7.45 \text{ kpsi}$$

Eq. (5-15) $\sigma' = [11.18^2 - (11.18)(19.58) + 19.58^2 + 3(7.45^2)]^{1/2} = \dfrac{S_y}{n_o} = \dfrac{60}{n_o}$

$$21.35 = \frac{60}{n_o} \quad \Rightarrow \quad n_o = 2.81 \quad \textit{Ans.}$$

Inner radius, 3D state of stress



From Eq. (5-14) with $\tau_{yz} = \tau_{zx} = 0$

$$\sigma' = \frac{1}{\sqrt{2}}[(9.78 - 22.58)^2 + (22.58 + 2.997)^2 + (-2.997 - 9.78)^2 + 6(6.52)^2]^{1/2} = \frac{60}{n_i}$$

$$24.86 = \frac{60}{n_i} \quad \Rightarrow \quad n_i = 2.41 \quad \textit{Ans.}$$

5-39 From Prob. 5-38: $p = 2.997$ kpsi, $I = 0.5369 \text{ in}^4$, $J = 1.0738 \text{ in}^4$

Inner member:

Outer radius: $$(\sigma_t)_o = -2.997\left[\frac{(0.875^2 + 0.75^2)}{(0.875^2 - 0.75^2)}\right] = -19.60 \text{ kpsi}$$

$$(\sigma_r)_o = -2.997 \text{ kpsi}$$

Inner radius: $$(\sigma_t)_i = -\frac{2(2.997)(0.875^2)}{0.875^2 - 0.75^2} = -22.59 \text{ kpsi}$$

$$(\sigma_r)_i = 0$$

Bending:

r_o: $$(\sigma_x)_o = \frac{6(0.875)}{0.5369} = 9.78 \text{ kpsi}$$

r_i: $$(\sigma_x)_i = \frac{6(0.75)}{0.5369} = 8.38 \text{ kpsi}$$

Torsion:

r_o: $$(\tau_{xy})_o = \frac{8(0.875)}{1.0738} = 6.52 \text{ kpsi}$$

r_i: $$(\tau_{xy})_i = \frac{8(0.75)}{1.0738} = 5.59 \text{ kpsi}$$

The inner radius is in plane stress: $\sigma_x = 8.38$ kpsi, $\sigma_y = -22.59$ kpsi, $\tau_{xy} = 5.59$ kpsi

$$\sigma_i' = [8.38^2 - (8.38)(-22.59) + (-22.59)^2 + 3(5.59^2)]^{1/2} = 29.4 \text{ kpsi}$$

$$n_i = \frac{S_y}{\sigma_i'} = \frac{60}{29.4} = 2.04 \quad \textit{Ans.}$$

Outer radius experiences a radial stress, σ_r

$$\sigma_o' = \frac{1}{\sqrt{2}}\left[(-19.60 + 2.997)^2 + (-2.997 - 9.78)^2 + (9.78 + 19.60)^2 + 6(6.52)^2\right]^{1/2}$$
$$= 27.9 \text{ kpsi}$$

$$n_o = \frac{60}{27.9} = 2.15 \quad \textit{Ans.}$$

5-40

$$\sigma_p = \frac{1}{2}\left(2\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\right) \pm \left[\left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\sin\frac{3\theta}{2}\right)^2 + \left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)^2\right]^{1/2}$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left[\cos\frac{\theta}{2} \pm \left(\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\sin^2\frac{3\theta}{2} + \sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\cos^2\frac{3\theta}{2}\right)^{1/2}\right]$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left(\cos\frac{\theta}{2} \pm \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right) = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 \pm \sin\frac{\theta}{2}\right)$$

Plane stress: The third principal stress is zero and

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right), \quad \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\right), \quad \sigma_3 = 0 \quad \textit{Ans.}$$

Plane strain: σ_1 and σ_2 equations still valid however,

$$\sigma_3 = \nu(\sigma_x + \sigma_y) = 2\nu\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2} \quad \textit{Ans.}$$

5-41 For $\theta = 0$ and plane strain, the principal stress equations of Prob. 5-40 give

$$\sigma_1 = \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}, \quad \sigma_3 = 2\nu\frac{K_I}{\sqrt{2\pi r}} = 2\nu\sigma_1$$

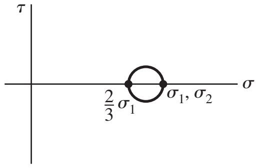
(a) DE:
$$\frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_1)^2 + (\sigma_1 - 2\nu\sigma_1)^2 + (2\nu\sigma_1 - \sigma_1)^2]^{1/2} = S_y$$
$$\sigma_1 - 2\nu\sigma_1 = S_y$$

For $\nu = \frac{1}{3}$, $\left[1 - 2\left(\frac{1}{3}\right)\right]\sigma_1 = S_y \quad \Rightarrow \quad \sigma_1 = 3S_y \quad \textit{Ans.}$

(b) MSS: $\sigma_1 - \sigma_3 = S_y \quad \Rightarrow \quad \sigma_1 - 2\nu\sigma_1 = S_y$

$$\nu = \frac{1}{3} \quad \Rightarrow \quad \sigma_1 = 3S_y \quad \textit{Ans.}$$

$$\sigma_3 = \frac{2}{3}\sigma_1$$



Radius of largest circle

$$R = \frac{1}{2}\left[\sigma_1 - \frac{2}{3}\sigma_1\right] = \frac{\sigma_1}{6}$$

5-42 **(a)** Ignoring stress concentration

$$F = S_y A = 160(4)(0.5) = 320 \text{ kips} \quad \textit{Ans.}$$

(b) From Fig. 6-36: $h/b = 1$, $a/b = 0.625/4 = 0.1563$, $\beta = 1.3$

Eq. (6-51) $$70 = 1.3\frac{F}{4(0.5)}\sqrt{\pi(0.625)}$$

$$F = 76.9 \text{ kips} \quad \textit{Ans.}$$

5-43 Given: $a = 12.5$ mm, $K_{Ic} = 80 \text{ MPa} \cdot \sqrt{m}$, $S_y = 1200$ MPa, $S_{ut} = 1350$ MPa

$$r_o = \frac{350}{2} = 175 \text{ mm}, \quad r_i = \frac{350 - 50}{2} = 150 \text{ mm}$$

$$a/(r_o - r_i) = \frac{12.5}{175 - 150} = 0.5$$

$$r_i/r_o = \frac{150}{175} = 0.857$$

Fig. 5-30: $\beta \doteq 2.5$

Eq. (5-37): $K_{Ic} = \beta\sigma\sqrt{\pi a}$

$$80 = 2.5\sigma\sqrt{\pi(0.0125)}$$

$$\sigma = 161.5 \text{ MPa}$$

Eq. (3-50) at $r = r_o$:

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}(2)$$

$$161.5 = \frac{150^2 p_i(2)}{175^2 - 150^2}$$

$$p_i = 29.2 \text{ MPa} \quad \textit{Ans.}$$

5-44

(a) First convert the data to radial dimensions to agree with the formulations of Fig. 3-33. Thus

$$r_o = 0.5625 \pm 0.001\,\text{in}$$
$$r_i = 0.1875 \pm 0.001\,\text{in}$$
$$R_o = 0.375 \pm 0.0002\,\text{in}$$
$$R_i = 0.376 \pm 0.0002\,\text{in}$$

The stochastic nature of the dimensions affects the $\boldsymbol{\delta} = |\mathbf{R}_i| - |\mathbf{R}_o|$ relation in Eq. (3-57) but not the others. Set $R = (1/2)(R_i + R_o) = 0.3755$. From Eq. (3-57)

$$\mathbf{p} = \frac{E\boldsymbol{\delta}}{R}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{2R^2\left(r_o^2 - r_i^2\right)}\right]$$

Substituting and solving with $E = 30$ Mpsi gives

$$\mathbf{p} = 18.70(10^6)\,\boldsymbol{\delta}$$

Since $\boldsymbol{\delta} = \mathbf{R}_i - \mathbf{R}_o$

$$\bar{\delta} = \bar{R}_i - \bar{R}_o = 0.376 - 0.375 = 0.001\text{ in}$$

and

$$\hat{\sigma}_\delta = \left[\left(\frac{0.0002}{4}\right)^2 + \left(\frac{0.0002}{4}\right)^2\right]^{1/2}$$
$$= 0.000\,070\,7\text{ in}$$

Then

$$C_\delta = \frac{\hat{\sigma}_\delta}{\bar{\delta}} = \frac{0.000\,070\,7}{0.001} = 0.0707$$

The tangential inner-cylinder stress at the shrink-fit surface is given by

$$\boldsymbol{\sigma}_{it} = -\mathbf{p}\frac{\bar{R}^2 + \bar{r}_i^2}{\bar{R}^2 - \bar{r}_i^2}$$
$$= -18.70(10^6)\,\boldsymbol{\delta}\left(\frac{0.3755^2 + 0.1875^2}{0.3755^2 - 0.1875^2}\right)$$
$$= -31.1(10^6)\,\boldsymbol{\delta}$$
$$\bar{\sigma}_{it} = -31.1(10^6)\,\bar{\delta} = -31.1(10^6)(0.001)$$
$$= -31.1(10^3)\text{ psi}$$

Also

$$\hat{\sigma}_{\sigma_{it}} = |C_\delta \bar{\sigma}_{it}| = 0.0707(-31.1)10^3$$
$$= 2899\text{ psi}$$
$$\boldsymbol{\sigma}_{it} = \mathbf{N}(-31\,100,\,2899)\text{ psi} \quad \textit{Ans.}$$

(b) The tangential stress for the outer cylinder at the shrink-fit surface is given by

$$\begin{aligned}
\boldsymbol{\sigma}_{ot} &= \mathbf{p}\left(\frac{\bar{r}_o^2+\bar{R}^2}{\bar{r}_o^2-\bar{R}^2}\right)\\
&= 18.70(10^6)\,\boldsymbol{\delta}\left(\frac{0.5625^2+0.3755^2}{0.5625^2-0.3755^2}\right)\\
&= 48.76(10^6)\,\boldsymbol{\delta}\ \text{psi}\\
\bar{\sigma}_{ot} &= 48.76(10^6)(0.001) = 48.76(10^3)\ \text{psi}\\
\hat{\sigma}_{\sigma_{ot}} &= C_\delta \bar{\sigma}_{ot} = 0.0707(48.76)(10^3) = 34.45\ \text{psi}\\
\therefore \boldsymbol{\sigma}_{ot} &= \mathbf{N}(48\,760,\,3445)\ \text{psi} \quad \textit{Ans.}
\end{aligned}$$

5-45 From Prob. 5-44, at the fit surface $\boldsymbol{\sigma}_{ot} = \mathbf{N}(48.8, 3.45)$ kpsi. The radial stress is the fit pressure which was found to be

$$\begin{aligned}
\mathbf{p} &= 18.70(10^6)\,\boldsymbol{\delta}\\
\bar{p} &= 18.70(10^6)(0.001) = 18.7(10^3)\ \text{psi}\\
\hat{\sigma}_p &= C_\delta \bar{p} = 0.0707(18.70)(10^3)\\
&= 1322\ \text{psi}
\end{aligned}$$

and so

$$\mathbf{p} = \mathbf{N}(18.7,\,1.32)\ \text{kpsi}$$

and

$$\boldsymbol{\sigma}_{or} = -\mathbf{N}(18.7,\,1.32)\ \text{kpsi}$$

These represent the principal stresses. The von Mises stress is next assessed.

$$\begin{aligned}
\bar{\sigma}_A &= 48.8\ \text{kpsi}, \quad \bar{\sigma}_B = -18.7\ \text{kpsi}\\
k &= \bar{\sigma}_B/\bar{\sigma}_A = -18.7/48.8 = -0.383\\
\bar{\sigma}' &= \bar{\sigma}_A(1-k+k^2)^{1/2}\\
&= 48.8[1-(-0.383)+(-0.383)^2]^{1/2}\\
&= 60.4\ \text{kpsi}\\
\hat{\sigma}_{\sigma'} &= C_p\bar{\sigma}' = 0.0707(60.4) = 4.27\ \text{kpsi}
\end{aligned}$$

Using the interference equation

$$\begin{aligned}
z &= -\frac{\bar{S}-\bar{\sigma}'}{\left(\hat{\sigma}_S^2+\hat{\sigma}_{\sigma'}^2\right)^{1/2}}\\
&= -\frac{95.5-60.4}{[(6.59)^2+(4.27)^2]^{1/2}} = -4.5\\
p_f &= \alpha = 0.000\,003\,40,
\end{aligned}$$

or about 3 chances in a million. *Ans.*

5-46

$$\boldsymbol{\sigma}_t = \frac{\mathbf{p}d}{2t} = \frac{6000\mathbf{N}(1,\ 0.083\,33)(0.75)}{2(0.125)}$$
$$= 18\mathbf{N}(1,\ 0.083\,33)\ \text{kpsi}$$

$$\boldsymbol{\sigma}_l = \frac{\mathbf{p}d}{4t} = \frac{6000\mathbf{N}(1,\ 0.083\,33)(0.75)}{4(0.125)}$$
$$= 9\mathbf{N}(1,\ 0.083\,33)\ \text{kpsi}$$

$$\boldsymbol{\sigma}_r = -\mathbf{p} = -6000\mathbf{N}(1,\ 0.083\,33)\ \text{kpsi}$$

These three stresses are principal stresses whose variability is due to the loading. From Eq. (5-12), we find the von Mises stress to be

$$\sigma' = \left\{\frac{(18-9)^2 + [9-(-6)]^2 + (-6-18)^2}{2}\right\}^{1/2}$$
$$= 21.0\ \text{kpsi}$$
$$\hat{\sigma}_{\sigma'} = C_p\bar{\sigma}' = 0.083\,33(21.0) = 1.75\ \text{kpsi}$$
$$z = -\frac{\bar{S} - \bar{\sigma}'}{\left(\hat{\sigma}_S^2 + \hat{\sigma}_{\sigma'}^2\right)^{1/2}}$$
$$= \frac{50 - 21.0}{(4.1^2 + 1.75^2)^{1/2}} = -6.5$$

The reliability is very high

$$R = 1 - \Phi(6.5) = 1 - 4.02(10^{-11}) \doteq 1 \quad \textit{Ans.}$$

Chapter 6

Note to the instructor: Many of the problems in this chapter are carried over from the previous edition. The solutions have changed slightly due to some minor changes. First, the calculation of the endurance limit of a rotating-beam specimen S'_e is given by $S'_e = 0.5S_{ut}$ instead of $S'_e = 0.504S_{ut}$. Second, when the fatigue stress calculation is made for deterministic problems, only one approach is given, which uses the notch sensitivity factor, q, together with Eq. (6-32). *Neuber's* equation, Eq. (6-33), is simply another form of this. These changes were made to hopefully make the calculations less confusing, and diminish the idea that stress life calculations are precise.

6-1 $H_B = 490$

Eq. (2-17): $S_{ut} = 0.495(490) = 242.6 \text{ kpsi} > 212 \text{ kpsi}$

Eq. (6-8): $S'_e = 100 \text{ kpsi}$

Table 6-2: $a = 1.34, \quad b = -0.085$

Eq. (6-19): $k_a = 1.34(242.6)^{-0.085} = 0.840$

Eq. (6-20): $k_b = \left(\dfrac{1/4}{0.3}\right)^{-0.107} = 1.02$

Eq. (6-18): $S_e = k_a k_b S'_e = 0.840(1.02)(100) = 85.7 \text{ kpsi}$ *Ans.*

6-2

(a) $S_{ut} = 68$ kpsi, $S'_e = 0.5(68) = 34$ kpsi *Ans.*

(b) $S_{ut} = 112$ kpsi, $S'_e = 0.5(112) = 56$ kpsi *Ans.*

(c) 2024T3 has no endurance limit *Ans.*

(d) Eq. (6-8): $S'_e = 100$ kpsi *Ans.*

6-3

Eq. (2-11): $\sigma'_F = \sigma_0 \varepsilon^m = 115(0.90)^{0.22} = 112.4 \text{ kpsi}$

Eq. (6-8): $S'_e = 0.5(66.2) = 33.1 \text{ kpsi}$

Eq. (6-12): $b = -\dfrac{\log(112.4/33.1)}{\log(2 \cdot 10^6)} = -0.084\,26$

Eq. (6-10): $f = \dfrac{112.4}{66.2}(2 \cdot 10^3)^{-0.084\,26} = 0.8949$

Eq. (6-14): $a = \dfrac{[0.8949(66.2)]^2}{33.1} = 106.0 \text{ kpsi}$

Eq. (6-13): $S_f = aN^b = 106.0(12\,500)^{-0.084\,26} = 47.9 \text{ kpsi}$ *Ans.*

Eq. (6-16): $N = \left(\dfrac{\sigma_a}{a}\right)^{1/b} = \left(\dfrac{36}{106.0}\right)^{-1/0.084\,26} = 368\,250 \text{ cycles}$ *Ans.*

6-4 From $S_f = aN^b$

$$\log S_f = \log a + b \log N$$

Substituting $(1, S_{ut})$

$$\log S_{ut} = \log a + b \log (1)$$

From which $$a = S_{ut}$$

Substituting $(10^3, fS_{ut})$ and $a = S_{ut}$

$$\log fS_{ut} = \log S_{ut} + b \log 10^3$$

From which

$$b = \frac{1}{3} \log f$$

$$\therefore S_f = S_{ut} N^{(\log f)/3} \qquad 1 \le N \le 10^3$$

For 500 cycles as in Prob. 6-3

$$S_f \ge 66.2(500)^{(\log 0.8949)/3} = 59.9 \text{ kpsi} \quad Ans.$$

6-5 Read from graph: $(10^3, 90)$ and $(10^6, 50)$. From $S = aN^b$

$$\log S_1 = \log a + b \log N_1$$

$$\log S_2 = \log a + b \log N_2$$

From which

$$\log a = \frac{\log S_1 \log N_2 - \log S_2 \log N_1}{\log N_2/N_1}$$

$$= \frac{\log 90 \log 10^6 - \log 50 \log 10^3}{\log 10^6/10^3}$$

$$= 2.2095$$

$$a = 10^{\log a} = 10^{2.2095} = 162.0$$

$$b = \frac{\log 50/90}{3} = -0.085\,09$$

$$(S_f)_{ax} = 162^{-0.085\,09} \qquad 10^3 \le N \le 10^6 \text{ in kpsi} \quad Ans.$$

Check:

$$10^3(S_f)_{ax} = 162(10^3)^{-0.085\,09} = 90 \text{ kpsi}$$

$$10^6(S_f)_{ax} = 162(10^6)^{-0.085\,09} = 50 \text{ kpsi}$$

The end points agree.

6-6

Eq. (6-8): $S'_e = 0.5(710) = 355$ MPa

Table 6-2: $a = 4.51, \quad b = -0.265$

Eq. (6-19): $k_a = 4.51(710)^{-0.265} = 0.792$

Eq. (6-20): $k_b = \left(\dfrac{d}{7.62}\right)^{-0.107} = \left(\dfrac{32}{7.62}\right)^{-0.107} = 0.858$

Eq. (6-18): $S_e = k_a k_b S'_e = 0.792(0.858)(355) = 241$ MPa *Ans.*

6-7 For AISI 4340 as forged steel,

Eq. (6-8): $S_e = 100$ kpsi

Table 6-2: $a = 39.9, \quad b = -0.995$

Eq. (6-19): $k_a = 39.9(260)^{-0.995} = 0.158$

Eq. (6-20): $k_b = \left(\dfrac{0.75}{0.30}\right)^{-0.107} = 0.907$

Each of the other Marin factors is unity.

$$S_e = 0.158(0.907)(100) = 14.3 \text{ kpsi}$$

For AISI 1040:

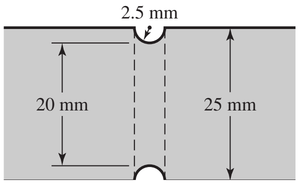
$$S'_e = 0.5(113) = 56.5 \text{ kpsi}$$
$$k_a = 39.9(113)^{-0.995} = 0.362$$
$$k_b = 0.907 \text{ (same as 4340)}$$

Each of the other Marin factors is unity.

$$S_e = 0.362(0.907)(56.5) = 18.6 \text{ kpsi}$$

Not only is AISI 1040 steel a contender, it has a superior endurance strength. Can you see why?

6-8



(a) For an AISI 1018 CD-machined steel, the strengths are

Eq. (2-17): $S_{ut} = 440 \text{ MPa} \quad \Rightarrow \quad H_B = \dfrac{440}{3.41} = 129$

$$S_y = 370 \text{ MPa}$$
$$S_{su} = 0.67(440) = 295 \text{ MPa}$$

Fig. A-15-15: $\dfrac{r}{d} = \dfrac{2.5}{20} = 0.125, \quad \dfrac{D}{d} = \dfrac{25}{20} = 1.25, \quad K_{ts} = 1.4$

Fig. 6-21: $q_s = 0.94$

Eq. (6-32): $K_{fs} = 1 + 0.94(1.4 - 1) = 1.376$

For a purely reversing torque of 200 N · m

$$\tau_{\max} = \frac{K_{fs} 16T}{\pi d^3} = \frac{1.376(16)(200 \times 10^3 \text{ N} \cdot \text{mm})}{\pi (20 \text{ mm})^3}$$
$$\tau_{\max} = 175.2 \text{ MPa} = \tau_a$$

$$S'_e = 0.5(440) = 220 \text{ MPa}$$

The Marin factors are

$$k_a = 4.51(440)^{-0.265} = 0.899$$

$$k_b = \left(\frac{20}{7.62}\right)^{-0.107} = 0.902$$

$$k_c = 0.59, \quad k_d = 1, \quad k_e = 1$$

Eq. (6-18): $$S_e = 0.899(0.902)(0.59)(220) = 105.3 \text{ MPa}$$

Eq. (6-14): $$a = \frac{[0.9(295)]^2}{105.3} = 669.4$$

Eq. (6-15): $$b = -\frac{1}{3}\log\frac{0.9(295)}{105.3} = -0.133\,88$$

Eq. (6-16): $$N = \left(\frac{175.2}{669.4}\right)^{1/-0.133\,88}$$

$$N = 22\,300 \text{ cycles} \quad \textit{Ans.}$$

(b) For an operating temperature of 450°C, the temperature modification factor, from Table 6-4, is

$$k_d = 0.843$$

Thus $$S_e = 0.899(0.902)(0.59)(0.843)(220) = 88.7 \text{ MPa}$$

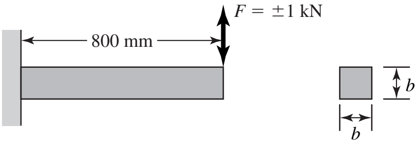
$$a = \frac{[0.9(295)]^2}{88.7} = 794.7$$

$$b = -\frac{1}{3}\log\frac{0.9(295)}{88.7} = -0.158\,71$$

$$N = \left(\frac{175.2}{794.7}\right)^{1/-0.15871}$$

$$N = 13\,700 \text{ cycles} \quad \textit{Ans.}$$

6-9



$$f = 0.9$$

$$n = 1.5$$

$$N = 10^4 \text{ cycles}$$

For AISI 1045 HR steel, $S_{ut} = 570$ MPa and $S_y = 310$ MPa

$$S_e' = 0.5(570 \text{ MPa}) = 285 \text{ MPa}$$

Find an initial guess based on yielding:

$$\sigma_a = \sigma_{\max} = \frac{Mc}{I} = \frac{M(b/2)}{b(b^3)/12} = \frac{6M}{b^3}$$

$$M_{\max} = (1 \text{ kN})(800 \text{ mm}) = 800 \text{ N} \cdot \text{m}$$

$$\sigma_{\max} = \frac{S_y}{n} \quad \Rightarrow \quad \frac{6(800 \times 10^3 \text{ N} \cdot \text{mm})}{b^3} = \frac{310 \text{ N/mm}^2}{1.5}$$

$$b = 28.5 \text{ mm}$$

Eq. (6-25): $d_e = 0.808b$

Eq. (6-20): $k_b = \left(\dfrac{0.808b}{7.62}\right)^{-0.107} = 1.2714b^{-0.107}$

$$k_b = 0.888$$

The remaining Marin factors are

$$k_a = 57.7(570)^{-0.718} = 0.606$$

$$k_c = k_d = k_e = k_f = 1$$

Eq. (6-18): $S_e = 0.606(0.888)(285) = 153.4 \text{ MPa}$

Eq. (6-14): $a = \dfrac{[0.9(570)]^2}{153.4} = 1715.6$

Eq. (6-15): $b = -\dfrac{1}{3} \log \dfrac{0.9(570)}{153.4} = -0.174\,76$

Eq. (6-13): $S_f = aN^b = 1715.6[(10^4)^{-0.174\,76}] = 343.1 \text{ MPa}$

$$n = \frac{S_f}{\sigma_a} \quad \text{or} \quad \sigma_a = \frac{S_f}{n}$$

$$\frac{6(800 \times 10^3)}{b^3} = \frac{343.1}{1.5} \quad \Rightarrow \quad b = 27.6 \text{ mm}$$

Check values for k_b, S_e, etc.

$$k_b = 1.2714(27.6)^{-0.107} = 0.891$$

$$S_e = 0.606(0.891)(285) = 153.9 \text{ MPa}$$

$$a = \frac{[0.9(570)]^2}{153.9} = 1710$$

$$b = -\frac{1}{3} \log \frac{0.9(570)}{153.9} = -0.174\,29$$

$$S_f = 1710[(10^4)^{-0.174\,29}] = 343.4 \text{ MPa}$$

$$\frac{6(800 \times 10^3)}{b^3} = \frac{343.4}{1.5}$$

$b = 27.6$ mm *Ans.*

6-10

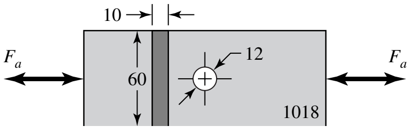


Table A-20: $S_{ut} = 440$ MPa, $S_y = 370$ MPa

$$S'_e = 0.5(440) = 220 \text{ MPa}$$

Table 6-2: $k_a = 4.51(440)^{-0.265} = 0.899$

$$k_b = 1 \quad \text{(axial loading)}$$

Eq. (6-26): $k_c = 0.85$

$$S_e = 0.899(1)(0.85)(220) = 168.1 \text{ MPa}$$

Table A-15-1: $d/w = 12/60 = 0.2, \quad K_t = 2.5$

From Fig. 6-20, $q \doteq 0.82$

Eq. (6-32): $K_f = 1 + 0.82(2.5 - 1) = 2.23$

$$\sigma_a = K_f \frac{F_a}{A} \quad \Rightarrow \quad \frac{S_e}{n_f} = \frac{2.23F_a}{10(60-12)} = \frac{168.1}{1.8}$$

$$F_a = 20\,100 \text{ N} = 20.1 \text{ kN} \quad \textit{Ans.}$$

$$\frac{F_a}{A} = \frac{S_y}{n_y} \quad \Rightarrow \quad \frac{F_a}{10(60-12)} = \frac{370}{1.8}$$

$$F_a = 98\,700 \text{ N} = 98.7 \text{ kN} \quad \textit{Ans.}$$

Largest force amplitude is 20.1 kN. *Ans.*

6-11 A priori design decisions:

The design decision will be: d

Material and condition: 1095 HR and from Table A-20 $S_{ut} = 120$, $S_y = 66$ kpsi.

Design factor: $n_f = 1.6$ per problem statement.

Life: $(1150)(3) = 3450$ cycles

Function: carry 10 000 lbf load

Preliminaries to iterative solution:

$$S'_e = 0.5(120) = 60 \text{ kpsi}$$

$$k_a = 2.70(120)^{-0.265} = 0.759$$

$$\frac{I}{c} = \frac{\pi d^3}{32} = 0.098\,17d^3$$

$$M(\text{crit.}) = \left(\frac{6}{24}\right)(10\,000)(12) = 30\,000 \text{ lbf} \cdot \text{in}$$

The critical location is in the middle of the shaft at the shoulder. From Fig. A-15-9: $D/d = 1.5$, $r/d = 0.10$, and $K_t = 1.68$. With no direct information concerning f, use $f = 0.9$.

For an initial trial, set $d = 2.00$ in

$$k_b = \left(\frac{2.00}{0.30}\right)^{-0.107} = 0.816$$

$$S_e = 0.759(0.816)(60) = 37.2 \text{ kpsi}$$

$$a = \frac{[0.9(120)]^2}{37.2} = 313.5$$

$$b = -\frac{1}{3}\log\frac{0.9(120)}{37.2} = -0.15429$$

$$S_f = 313.5(3450)^{-0.15429} = 89.2 \text{ kpsi}$$

$$\sigma_0 = \frac{M}{I/c} = \frac{30}{0.098\,17d^3} = \frac{305.6}{d^3}$$

$$= \frac{305.6}{2^3} = 38.2 \text{ kpsi}$$

$$r = \frac{d}{10} = \frac{2}{10} = 0.2$$

Fig. 6-20: $q \doteq 0.87$

Eq. (6-32): $K_f \doteq 1 + 0.87(1.68 - 1) = 1.59$

$$\sigma_a = K_f\sigma_0 = 1.59(38.2) = 60.7 \text{ kpsi}$$

$$n_f = \frac{S_f}{\sigma_a} = \frac{89.2}{60.7} = 1.47$$

Design is adequate unless more uncertainty prevails.

Choose $d = 2.00$ in *Ans.*

6-12

Yield: $\sigma'_{\max} = [172^2 + 3(103^2)]^{1/2} = 247.8$ kpsi

$$n_y = S_y/\sigma'_{\max} = 413/247.8 = 1.67 \quad \textit{Ans.}$$

$$\sigma'_a = 172\,\text{MPa} \quad \sigma'_m = \sqrt{3}\tau_m = \sqrt{3}(103) = 178.4\text{ MPa}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(172/276) + (178.4/551)} = 1.06 \quad \textit{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{551}{178.4}\right)^2\left(\frac{172}{276}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(178.4)(276)}{551(172)}\right]^2}\right\} = 1.31 \quad \textit{Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \left[\frac{1}{(172/276)^2 + (178.4/413)^2}\right]^{1/2} = 1.32 \quad \textit{Ans.}$$

6-13

Yield: $\sigma'_{\max} = [69^2 + 3(138)^2]^{1/2} = 248.8 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{413}{248.8} = 1.66 \quad \textit{Ans.}$$

$$\sigma'_a = 69 \text{ MPa}, \quad \sigma'_m = \sqrt{3}(138) = 239 \text{ MPa}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(69/276) + (239/551)} = 1.46 \quad \textit{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{551}{239}\right)^2\left(\frac{69}{276}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(239)(276)}{551(69)}\right]^2}\right\} = 1.73 \quad \textit{Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \left[\frac{1}{(69/276)^2 + (239/413)^2}\right]^{1/2} = 1.59 \quad \textit{Ans.}$$

6-14

Yield: $\sigma'_{\max} = [83^2 + 3(103 + 69)^2]^{1/2} = 309.2 \text{ MPa}$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{413}{309.3} = 1.34 \quad \textit{Ans.}$$

$$\sigma'_a = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{83^2 + 3(69^2)} = 145.5 \text{ MPa}, \quad \sigma'_m = \sqrt{3}(103) = 178.4 \text{ MPa}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(145.5/276) + (178.4/551)} = 1.18 \quad \textit{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{551}{178.4}\right)^2\left(\frac{145.5}{276}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(178.4)(276)}{551(145.5)}\right]^2}\right\} = 1.47 \quad \textit{Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \left[\frac{1}{(145.5/276)^2 + (178.4/413)^2}\right]^{1/2} = 1.47 \quad \textit{Ans.}$$

6-15

$$\sigma'_{\max} = \sigma'_a = \sqrt{3}(207) = 358.5 \text{ MPa}, \qquad \sigma'_m = 0$$

Yield:
$$358.5 = \frac{413}{n_y} \quad \Rightarrow \quad n_y = 1.15 \quad Ans.$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(358.5/276)} = 0.77 \quad Ans.$$

(b) Gerber criterion of Table 6-7 does not work; therefore use Eq. (6-47).

$$n_f \frac{\sigma_a}{S_e} = 1 \quad \Rightarrow \quad n_f = \frac{S_e}{\sigma_a} = \frac{276}{358.5} = 0.77 \quad Ans.$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\left(\frac{1}{358.5/276}\right)^2} = 0.77 \quad Ans.$$

Let $f = 0.9$ to assess the cycles to failure by fatigue

Eq. (6-14):
$$a = \frac{[0.9(551)]^2}{276} = 891.0 \text{ MPa}$$

Eq. (6-15):
$$b = -\frac{1}{3} \log \frac{0.9(551)}{276} = -0.084\,828$$

Eq. (6-16):
$$N = \left(\frac{358.5}{891.0}\right)^{-1/0.084\,828} = 45\,800 \text{ cycles} \quad Ans.$$

6-16

$$\sigma'_{\max} = [103^2 + 3(103)^2]^{1/2} = 206 \text{ MPa}$$
$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{413}{206} = 2.00 \quad Ans.$$
$$\sigma'_a = \sqrt{3}(103) = 178.4 \text{ MPa}, \quad \sigma'_m = 103 \text{ MPa}$$

(a) Modified Goodman, Table 7-9

$$n_f = \frac{1}{(178.4/276) + (103/551)} = 1.20 \quad Ans.$$

(b) Gerber, Table 7-10

$$n_f = \frac{1}{2}\left(\frac{551}{103}\right)^2 \left(\frac{178.4}{276}\right) \left\{-1 + \sqrt{1 + \left[\frac{2(103)(276)}{551(178.4)}\right]^2}\right\} = 1.44 \quad Ans.$$

(c) ASME-Elliptic, Table 7-11

$$n_f = \left[\frac{1}{(178.4/276)^2 + (103/413)^2}\right]^{1/2} = 1.44 \quad Ans.$$

6-17 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

$$A = 0.375(1 - 0.25) = 0.2813 \text{ in}^2$$

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{3000}{0.2813}(10^{-3}) = 10.67 \text{ kpsi}$$

$$n_y = \frac{54}{10.67} = 5.06 \quad \textit{Ans.}$$

$$S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$k_a = 2.70(64)^{-0.265} = 0.897$$

$$k_b = 1, \qquad k_c = 0.85$$

$$S_e = 0.897(1)(0.85)(32) = 24.4 \text{ kpsi}$$

Table A-15-1: $w = 1$ in, $d = 1/4$ in, $d/w = 0.25$ $\therefore K_t = 2.45$.

Fig. 6-20, with $r = 0.125$ in, $q \doteq 0.8$

Eq. (6-32): $K_f = 1 + 0.8(2.45 - 1) = 2.16$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right|$$

$$= 2.16 \left| \frac{3.000 - 0.800}{2(0.2813)} \right| = 8.45 \text{ kpsi}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A}$$

$$= 2.16 \left[\frac{3.000 + 0.800}{2(0.2813)} \right] = 14.6 \text{ kpsi}$$

(a) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{64}{14.6}\right)^2 \left(\frac{8.45}{24.4}\right) \left[-1 + \sqrt{1 + \left(\frac{2(14.6)(24.4)}{8.45(64)}\right)^2} \right]$$

$$= 2.17 \quad \textit{Ans.}$$

(b) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(8.45/24.4)^2 + (14.6/54)^2}} = 2.28 \quad \textit{Ans.}$$

6-18 Referring to the solution of Prob. 6-17, for load fluctuations of −800 to 3000 lbf

$$\sigma_a = 2.16 \left| \frac{3.000 - (-0.800)}{2(0.2813)} \right| = 14.59 \text{ kpsi}$$

$$\sigma_m = 2.16 \left| \frac{3.000 + (-0.800)}{2(0.2813)} \right| = 8.45 \text{ kpsi}$$

(a) Table 6-7, DE-Gerber

$$n_f = \frac{1}{2}\left(\frac{64}{8.45}\right)^2\left(\frac{14.59}{24.4}\right)\left[-1+\sqrt{1+\left(\frac{2(8.45)(24.4)}{64(14.59)}\right)^2}\,\right] = 1.60 \quad Ans.$$

(b) Table 6-8, DE-Elliptic

$$n_f = \sqrt{\frac{1}{(14.59/24.4)^2 + (8.45/54)^2}} = 1.62 \quad Ans.$$

6-19 Referring to the solution of Prob. 6-17, for load fluctuations of 800 to −3000 lbf

$$\sigma_a = 2.16\left|\frac{0.800-(-3.000)}{2(0.2813)}\right| = 14.59 \text{ kpsi}$$

$$\sigma_m = 2.16\left[\frac{0.800+(-3.000)}{2(0.2813)}\right] = -8.45 \text{ kpsi}$$

(a) We have a compressive midrange stress for which the failure locus is horizontal at the S_e level.

$$n_f = \frac{S_e}{\sigma_a} = \frac{24.4}{14.59} = 1.67 \quad Ans.$$

(b) Same as (a)

$$n_f = \frac{S_e}{\sigma_a} = \frac{24.4}{14.59} = 1.67 \quad Ans.$$

6-20

$$S_{ut} = 0.495(380) = 188.1 \text{ kpsi}$$
$$S'_e = 0.5(188.1) = 94.05 \text{ kpsi}$$
$$k_a = 14.4(188.1)^{-0.718} = 0.335$$

For a non-rotating round bar in bending, Eq. (6-24) gives: $d_e = 0.370d = 0.370(3/8) =$ 0.1388 in

$$k_b = \left(\frac{0.1388}{0.3}\right)^{-0.107} = 1.086$$
$$S_e = 0.335(1.086)(94.05) = 34.22 \text{ kpsi}$$
$$F_a = \frac{30-15}{2} = 7.5 \text{ lbf}, \quad F_m = \frac{30+15}{2} = 22.5 \text{ lbf}$$
$$\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(22.5)(16)}{\pi(0.375^3)}(10^{-3}) = 69.54 \text{ kpsi}$$

$$\sigma_a = \frac{32(7.5)(16)}{\pi(0.375^3)}(10^{-3}) = 23.18 \text{ kpsi}$$

$$r = \frac{23.18}{69.54} = 0.333$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(23.18/34.22) + (69.54/188.1)} = 0.955$$

Since finite failure is predicted, proceed to calculate N

From Fig. 6-18, for $S_{ut} = 188.1$ kpsi, $f = 0.778$

Eq. (6-14): $$a = \frac{[0.7781(188.1)]^2}{34.22} = 625.8 \text{ kpsi}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\frac{0.778(188.1)}{34.22} = -0.210\,36$$

$$\frac{\sigma_a}{S_f} + \frac{\sigma_m}{S_{ut}} = 1 \quad \Rightarrow \quad S_f = \frac{\sigma_a}{1-(\sigma_m/S_{ut})} = \frac{23.18}{1-(69.54/188.1)} = 36.78 \text{ kpsi}$$

Eq. (7-15) with $\sigma_a = S_f$

$$N = \left(\frac{36.78}{625.8}\right)^{1/-0.210\,36} = 710\,000 \text{ cycles} \quad Ans.$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{188.1}{69.54}\right)^2\left(\frac{23.18}{34.22}\right)\left\{-1+\sqrt{1+\left[\frac{2(69.54)(34.22)}{188.1(23.18)}\right]^2}\right\}$$

$= 1.20$ Thus, infinite life is predicted ($N \geq 10^6$ cycles). *Ans.*

6-21

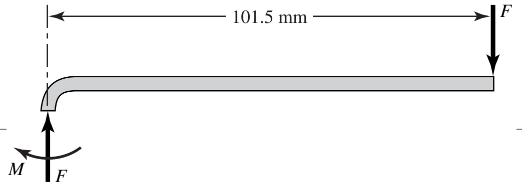
(a) $$I = \frac{1}{12}(18)(3^3) = 40.5 \text{ mm}^4$$

$$y = \frac{Fl^3}{3EI} \quad \Rightarrow \quad F = \frac{3EIy}{l^3}$$

$$F_{\min} = \frac{3(207)(10^9)(40.5)(10^{-12})(2)(10^{-3})}{(100^3)(10^{-9})} = 50.3 \text{ N} \quad Ans.$$

$$F_{\max} = \frac{6}{2}(50.3) = 150.9 \text{ N} \quad Ans.$$

(b)



$$M = 0.1015F \text{ N}\cdot\text{m}$$
$$A = 3(18) = 54 \text{ mm}^2$$

Curved beam: $r_n = \dfrac{h}{\ln(r_o/r_i)} = \dfrac{3}{\ln(6/3)} = 4.3281 \text{ mm}$

$$r_c = 4.5 \text{ mm}, \quad e = r_c - r_n = 4.5 - 4.3281 = 0.1719 \text{ mm}$$

$$\sigma_i = -\frac{Mc_i}{Aer_i} - \frac{F}{A} = -\frac{(0.1015F)(1.5 - 0.1719)}{54(0.1719)(3)(10^{-3})} - \frac{F}{54} = -4.859F \text{ MPa}$$

$$\sigma_o = \frac{Mc_o}{Aer_o} - \frac{F}{A} = \frac{(0.1015F)(1.5 + 0.1719)}{54(0.1719)(6)(10^{-3})} - \frac{F}{54} = 3.028F \text{ MPa}$$

$$(\sigma_i)_{\min} = -4.859(150.9) = -733.2 \text{ MPa}$$

$$(\sigma_i)_{\max} = -4.859(50.3) = -244.4 \text{ MPa}$$

$$(\sigma_o)_{\max} = 3.028(150.9) = 456.9 \text{ MPa}$$

$$(\sigma_o)_{\min} = 3.028(50.3) = 152.3 \text{ MPa}$$

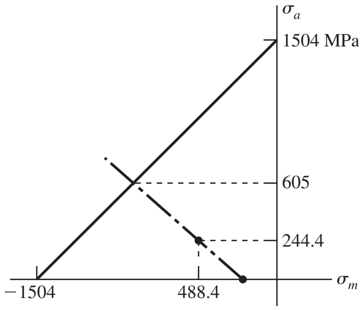
Eq. (2-17) $S_{ut} = 3.41(490) = 1671 \text{ MPa}$

Per the problem statement, estimate the yield as $S_y = 0.9S_{ut} = 0.9(1671) = 1504$ MPa. Then from Eq. (6-8), $S'_e = 700$ MPa; Eq. (6-19), $k_a = 1.58(1671)^{-0.085} = 0.841$; Eq. (6-25) $d_e = 0.808[18(3)]^{1/2} = 5.938$ mm; and Eq. (6-20), $k_b = (5.938/7.62)^{-0.107} = 1.027$.

$$S_e = 0.841(1.027)(700) = 605 \text{ MPa}$$

At Inner Radius $(\sigma_i)_a = \left|\dfrac{-733.2 + 244.4}{2}\right| = 244.4 \text{ MPa}$

$$(\sigma_i)_m = \frac{-733.2 - 244.4}{2} = -488.8 \text{ MPa}$$



Load line: $\sigma_m = -244.4 - \sigma_a$

Langer (yield) line: $\sigma_m = \sigma_a - 1504 = -244.4 - \sigma_a$

Intersection: $\sigma_a = 629.8$ MPa, $\sigma_m = -874.2$ MPa

(Note that σ_a is more than 605 MPa)

Yield: $n_y = \dfrac{629.8}{244.4} = 2.58$

Fatigue: $n_f = \dfrac{605}{244.4} = 2.48$ Thus, the spring is likely to fail in fatigue at the inner radius. *Ans.*

At Outer Radius

$$(\sigma_o)_a = \frac{456.9 - 152.3}{2} = 152.3 \text{ MPa}$$

$$(\sigma_o)_m = \frac{456.9 + 152.3}{2} = 304.6 \text{ MPa}$$

Yield load line: $\sigma_m = 152.3 + \sigma_a$

Langer line: $\sigma_m = 1504 - \sigma_a = 152.3 + \sigma_a$

Intersection: $\sigma_a = 675.9 \text{ MPa}, \quad \sigma_m = 828.2 \text{ MPa}$

$$n_y = \frac{675.9}{152.3} = 4.44$$

Fatigue line: $\sigma_a = [1 - (\sigma_m/S_{ut})^2]S_e = \sigma_m - 152.3$

$$605\left[1 - \left(\frac{\sigma_m}{1671}\right)^2\right] = \sigma_m - 152.3$$

$$\sigma_m^2 + 4615.3\sigma_m - 3.4951(10^6) = 0$$

$$\sigma_m = \frac{-4615.3 + \sqrt{4615.3^2 + 4(3.4951)(10^6)}}{2} = 662.2 \text{ MPa}$$

$$\sigma_a = 662.2 - 152.3 = 509.9 \text{ MPa}$$

$$n_f = \frac{509.9}{152.3} = 3.35$$

Thus, the spring is not likely to fail in fatigue at the outer radius. *Ans.*

6-22 The solution at the inner radius is the same as in Prob. 6-21. At the outer radius, the yield solution is the same.

Fatigue line: $\sigma_a = \left(1 - \dfrac{\sigma_m}{S_{ut}}\right)S_e = \sigma_m - 152.3$

$$605\left(1 - \frac{\sigma_m}{1671}\right) = \sigma_m - 152.3$$

$$1.362\sigma_m = 757.3 \quad \Rightarrow \quad \sigma_m = 556.0 \text{ MPa}$$

$$\sigma_a = 556.0 - 152.3 = 403.7 \text{ MPa}$$

$$n_f = \frac{403.7}{152.3} = 2.65 \quad \textit{Ans.}$$

6-23 Preliminaries:

Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

$$S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$k_a = 2.70(64)^{-0.265} = 0.897$$

$$k_b = 1$$

$$k_c = 0.85$$

$$S_e = 0.897(1)(0.85)(32) = 24.4 \text{ kpsi}$$

Fillet:

Fig. A-15-5: $D = 3.75$ in, $d = 2.5$ in, $D/d = 3.75/2.5 = 1.5$, and $r/d = 0.25/2.5 = 0.10$
$\therefore K_t = 2.1$. Fig. 6-20 with $r = 0.25$ in, $q \doteq 0.82$

Eq. (6-32): $K_f = 1 + 0.82(2.1 - 1) = 1.90$

$$\sigma_{\max} = \frac{4}{2.5(0.5)} = 3.2 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-16}{2.5(0.5)} = -12.8 \text{ kpsi}$$

$$\sigma_a = 1.90\left|\frac{3.2 - (-12.8)}{2}\right| = 15.2 \text{ kpsi}$$

$$\sigma_m = 1.90\left[\frac{3.2 + (-12.8)}{2}\right] = -9.12 \text{ kpsi}$$

$$n_y = \left|\frac{S_y}{\sigma_{\min}}\right| = \left|\frac{54}{-12.8}\right| = 4.22$$

Since the midrange stress is negative,

$$S_a = S_e = 24.4 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{24.4}{15.2} = 1.61$$

Hole:

Fig. A-15-1: $d/w = 0.75/3.75 = 0.20$, $K_t = 2.5$. Fig. 6-20, with $r = 0.375$ in, $q \doteq 0.85$

Eq. (6-32): $K_f = 1 + 0.85(2.5 - 1) = 2.28$

$$\sigma_{\max} = \frac{4}{0.5(3.75 - 0.75)} = 2.67 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-16}{0.5(3.75 - 0.75)} = -10.67 \text{ kpsi}$$

$$\sigma_a = 2.28\left|\frac{2.67 - (-10.67)}{2}\right| = 15.2 \text{ kpsi}$$

$$\sigma_m = 2.28\,\frac{2.67 + (-10.67)}{2} = -9.12 \text{ kpsi}$$

Since the midrange stress is negative,

$$n_y = \left|\frac{S_y}{\sigma_{\min}}\right| = \left|\frac{54}{-10.67}\right| = 5.06$$

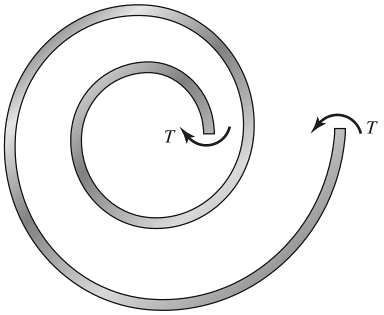
$$S_a = S_e = 24.4 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{24.4}{15.2} = 1.61$$

Thus the design is controlled by the threat of fatigue equally at the fillet and the hole; the minimum factor of safety is $n_f = 1.61$. *Ans.*

6-24

(a)



Curved beam in pure bending where $M = -T$ throughout. The maximum stress will occur at the inner fiber where $r_c = 20$ mm, but will be compressive. The maximum tensile stress will occur at the outer fiber where $r_c = 60$ mm. Why?

Inner fiber where $r_c = 20$ *mm*

$$r_n = \frac{h}{\ln(r_o/r_i)} = \frac{5}{\ln(22.5/17.5)} = 19.8954 \text{ mm}$$

$$e = 20 - 19.8954 = 0.1046 \text{ mm}$$

$$c_i = 19.8954 - 17.5 = 2.395 \text{ mm}$$

$$A = 25 \text{ mm}^2$$

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-T(2.395)10^{-3}}{25(10^{-6})0.1046(10^{-3})17.5(10^{-3})}(10^{-6}) = -52.34\,T \qquad (1)$$

where T is in N · m, and σ_i is in MPa.

$$\sigma_m = \frac{1}{2}(-52.34T) = -26.17T, \qquad \sigma_a = 26.17T$$

For the endurance limit, $S'_e = 0.5(770) = 385$ MPa

$$k_a = 4.51(770)^{-0.265} = 0.775$$

$$d_e = 0.808[5(5)]^{1/2} = 4.04 \text{ mm}$$

$$k_b = (4.04/7.62)^{-0.107} = 1.07$$

$$S_e = 0.775(1.07)385 = 319.3 \text{ MPa}$$

For a compressive midrange component, $\sigma_a = S_e/n_f$. Thus,

$$26.17T = 319.3/3 \Rightarrow T = 4.07 \text{ N} \cdot \text{m}$$

Outer fiber where $r_c = 60$ *mm*

$$r_n = \frac{5}{\ln(62.5/57.5)} = 59.96526 \text{ mm}$$

$$e = 60 - 59.96526 = 0.03474 \text{ mm}$$

$$c_o = 62.5 - 59.96526 = 2.535 \text{ mm}$$

$$\sigma_o = -\frac{Mc_i}{Aer_i} = -\frac{-T(2.535)10^{-3}}{25(10^{-6})0.03474(10^{-3})62.5(10^{-3})}(10^{-6}) = 46.7\ T$$

Comparing this with Eq. (1), we see that it is less in magnitude, but the midrange component is *tension.*

$$\sigma_a = \sigma_m = \frac{1}{2}(46.7T) = 23.35T$$

Using Eq. (6-46), for modified Goodman, we have

$$\frac{23.35T}{319.3} + \frac{23.35T}{770} = \frac{1}{3} \Rightarrow T = 3.22\,\text{N}\cdot\text{m} \quad \textit{Ans.}$$

(b) Gerber, Eq. (6-47), at the outer fiber,

$$\frac{3(23.35T)}{319.3} + \left[\frac{3(23.35T)}{770}\right]^2 = 1$$

reduces to $T^2 + 26.51T - 120.83 = 0$

$$T = \frac{1}{2}\left(-26.51 + \sqrt{26.51^2 + 4(120.83)}\right) = 3.96\,\text{N}\cdot\text{m} \quad \textit{Ans.}$$

(c) To guard against yield, use T of part (b) and the inner stress.

$$n_y = \frac{420}{52.34(3.96)} = 2.03 \quad \textit{Ans.}$$

6-25 From Prob. 6-24, $S_e = 319.3$ MPa, $S_y = 420$ MPa, and $S_{ut} = 770$ MPa

(a) Assuming the beam is straight,

$$\sigma_{\max} = \frac{6M}{bh^2} = \frac{6T}{5^3[(10^{-3})^3]} = 48(10^6)T$$

Goodman: $$\frac{24T}{319.3} + \frac{24T}{770} = \frac{1}{3} \quad \Rightarrow \quad T = 3.13\,\text{N}\cdot\text{m} \quad \textit{Ans.}$$

(b) Gerber: $$\frac{3(24)T}{319.3} + \left[\frac{3(24)T}{770}\right]^2 = 1$$

$$T^2 + 25.79T - 114.37 = 1$$

$$T = \frac{1}{2}\left[-25.79 + \sqrt{25.79^2 + 4(114.37)}\right] = 3.86\,\text{N}\cdot\text{m} \quad \textit{Ans.}$$

(c) Using $\sigma_{\max} = 52.34(10^6)T$ from Prob. 6-24,

$$n_y = \frac{420}{52.34(3.86)} = 2.08 \quad Ans.$$

6-26

(a)

$$\tau_{\max} = \frac{16K_{fs}T_{\max}}{\pi d^3}$$

Fig. 6-21 for $H_B > 200, r = 3\text{ mm}, q_s \doteq 1$

$$K_{fs} = 1 + q_s(K_{ts} - 1)$$

$$K_{fs} = 1 + 1(1.6 - 1) = 1.6$$

$$T_{\max} = 2000(0.05) = 100\text{ N}\cdot\text{m}, \quad T_{\min} = \frac{500}{2000}(100) = 25\text{ N}\cdot\text{m}$$

$$\tau_{\max} = \frac{16(1.6)(100)(10^{-6})}{\pi(0.02)^3} = 101.9\text{ MPa}$$

$$\tau_{\min} = \frac{500}{2000}(101.9) = 25.46\text{ MPa}$$

$$\tau_m = \frac{1}{2}(101.9 + 25.46) = 63.68\text{ MPa}$$

$$\tau_a = \frac{1}{2}(101.9 - 25.46) = 38.22\text{ MPa}$$

$$S_{su} = 0.67S_{ut} = 0.67(320) = 214.4\text{ MPa}$$

$$S_{sy} = 0.577S_y = 0.577(180) = 103.9\text{ MPa}$$

$$S'_e = 0.5(320) = 160\text{ MPa}$$

$$k_a = 57.7(320)^{-0.718} = 0.917$$

$$d_e = 0.370(20) = 7.4\text{ mm}$$

$$k_b = \left(\frac{7.4}{7.62}\right)^{-0.107} = 1.003$$

$$k_c = 0.59$$

$$S_e = 0.917(1.003)(0.59)(160) = 86.8\text{ MPa}$$

Modified Goodman, Table 6-6

$$n_f = \frac{1}{(\tau_a/S_e) + (\tau_m/S_{su})} = \frac{1}{(38.22/86.8) + (63.68/214.4)} = 1.36 \quad Ans.$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_e}{S_{su}\tau_a}\right)^2}\,\right]$$

$$= \frac{1}{2}\left(\frac{214.4}{63.68}\right)^2 \frac{38.22}{86.8}\left\{-1 + \sqrt{1 + \left[\frac{2(63.68)(86.8)}{214.4(38.22)}\right]^2}\,\right\} = 1.70 \quad Ans.$$

6-27 $S_y = 800$ MPa, $S_{ut} = 1000$ MPa

(a) From Fig. 6-20, for a notch radius of 3 mm and $S_{ut} = 1$ GPa, $q \doteq 0.92$.

$$K_f = 1 + q(K_t - 1) = 1 + 0.92(3 - 1) = 2.84$$

$$\sigma_{\max} = -K_f \frac{4P}{\pi d^2} = -\frac{2.84(4)P}{\pi(0.030)^2} = -4018P$$

$$\sigma_m = -\sigma_a = \frac{1}{2}(-4018P) = -2009P$$

$$T = fP\left(\frac{D+d}{4}\right)$$

$$T_{\max} = 0.3P\left(\frac{0.150 + 0.03}{4}\right) = 0.0135P$$

From Fig. 6-21, $q_s \doteq 0.95$. Also, K_{ts} is given as 1.8. Thus,

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.95(1.8 - 1) = 1.76$$

$$\tau_{\max} = \frac{16K_{fs}T}{\pi d^3} = \frac{16(1.76)(0.0135P)}{\pi(0.03)^3} = 4482P$$

$$\tau_a = \tau_m = \frac{1}{2}(4482P) = 2241P$$

Eqs. (6-55) and (6-56):

$$\sigma'_a = \sigma'_m = \left[(\sigma_a/0.85)^2 + 3\tau_a^2\right]^{1/2} = \left[(-2009P/0.85)^2 + 3(2241P)^2\right]^{1/2} = 4545P$$

$$S'_e = 0.5(1000) = 500 \text{ MPa}$$

$$k_a = 4.51(1000)^{-0.265} = 0.723$$

$$k_b = \left(\frac{30}{7.62}\right)^{-0.107} = 0.864$$

$$S_e = 0.723(0.864)(500) = 312.3 \text{ MPa}$$

Modified Goodman: $$\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{1}{n}$$

$$\frac{4545P}{312.3(10^6)} + \frac{4545P}{1000(10^6)} = \frac{1}{3} \quad \Rightarrow \quad P = 17.5(10^3)\text{ N} = 16.1 \text{ kN} \quad \textit{Ans.}$$

Yield (conservative): $n_y = \dfrac{S_y}{\sigma'_a + \sigma'_m}$

$$n_y = \frac{800(10^6)}{2(4545)(17.5)(10^3)} = 5.03 \quad \textit{Ans.}$$

(actual): $\sigma'_{\max} = \left(\sigma^2_{\max} + 3\tau^2_{\max}\right)^{1/2} = \left[(-4018P)^2 + 3(4482P)^2\right]^{1/2}$

$$= 8741P$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{800(10^6)}{8741(17.5)10^3} = 5.22$$

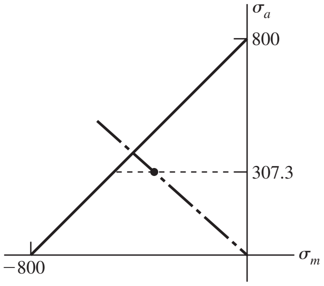
(b) If the shaft is not rotating, $\tau_m = \tau_a = 0$.

$$\sigma_m = \sigma_a = -2009P$$

$$k_b = 1 \qquad \text{(axial)}$$

$$k_c = 0.85 \qquad \text{(Since there is no tension, } k_c = 1 \text{ might be more appropriate.)}$$

$$S_e = 0.723(1)(0.85)(500) = 307.3 \text{ MPa}$$



$$n_f = \frac{307.3(10^6)}{2009P} \quad \Rightarrow \quad P = \frac{307.3(10^6)}{3(2009)} = 51.0(10^3) \text{ N}$$

$$= 51.0 \text{ kN} \quad \textit{Ans.}$$

Yield: $$n_y = \frac{800(10^6)}{2(2009)(51.0)(10^3)} = 3.90 \quad \textit{Ans.}$$

6-28 From Prob. 6-27, $K_f = 2.84$, $K_{fs} = 1.76$, $S_e = 312.3$ MPa

$$\sigma_{\max} = -K_f \frac{4P_{\max}}{\pi d^2} = -2.84\left[\frac{(4)(80)(10^{-3})}{\pi(0.030)^2}\right] = -321.4 \text{ MPa}$$

$$\sigma_{\min} = \frac{20}{80}(-321.4) = -80.4 \text{ MPa}$$

$$T_{\max} = fP_{\max}\left(\frac{D+d}{4}\right) = 0.3(80)(10^3)\left(\frac{0.150+0.03}{4}\right) = 1080 \text{ N}\cdot\text{m}$$

$$T_{\min} = \frac{20}{80}(1080) = 270 \text{ N}\cdot\text{m}$$

$$\tau_{\max} = K_{fs}\frac{16T_{\max}}{\pi d^3} = 1.76\left[\frac{16(1080)}{\pi(0.030)^3}(10^{-6})\right] = 358.5 \text{ MPa}$$

$$\tau_{\min} = \frac{20}{80}(358.5) = 89.6 \text{ MPa}$$

$$\sigma_a = \frac{321.4 - 80.4}{2} = 120.5 \text{ MPa}$$

$$\sigma_m = \frac{-321.4 - 80.4}{2} = -200.9 \text{ MPa}$$

$$\tau_a = \frac{358.5 - 89.6}{2} = 134.5 \text{ MPa}$$

$$\tau_m = \frac{358.5 + 89.6}{2} = 224.1 \text{ MPa}$$

Eqs. (6-55) and (6-56):

$$\sigma_a' = \left[(\sigma_a/0.85)^2 + 3\tau_a^2\right]^{1/2} = \left[(120.5/0.85)^2 + 3(134.5)^2\right]^{1/2} = 272.7\text{ MPa}$$

$$\sigma_m' = \left[(-200.9/0.85)^2 + 3(224.1)^2\right]^{1/2} = 454.5\text{ MPa}$$

Goodman:

$$(\sigma_a)_e = \frac{\sigma_a'}{1 - \sigma_m'/S_{ut}} = \frac{272.7}{1 - 454.5/1000} = 499.9\text{ MPa}$$

Let $f = 0.9$

$$a = \frac{[0.9(1000)]^2}{312.3} = 2594\text{ MPa}$$

$$b = -\frac{1}{3}\log\left[\frac{0.9(1000)}{312.3}\right] = -0.1532$$

$$N = \left[\frac{(\sigma_a)_e}{a}\right]^{1/b} = \left[\frac{499.9}{2594}\right]^{1/-0.1532} = 46\,520\text{ cycles} \quad \textit{Ans.}$$

6-29

$$S_y = 490\text{ MPa}, \quad S_{ut} = 590\text{ MPa}, \quad S_e = 200\text{ MPa}$$

$$\sigma_m = \frac{420 + 140}{2} = 280\text{ MPa}, \quad \sigma_a = \frac{420 - 140}{2} = 140\text{ MPa}$$

Goodman:

$$(\sigma_a)_e = \frac{\sigma_a}{1 - \sigma_m/S_{ut}} = \frac{140}{1 - (280/590)} = 266.5\text{ MPa} > S_e \;\therefore \text{finite life}$$

$$a = \frac{[0.9(590)]^2}{200} = 1409.8\text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9(590)}{200} = -0.141\,355$$

$$N = \left(\frac{266.5}{1409.8}\right)^{-1/0.143\,55} = 131\,200\text{ cycles}$$

$$N_{\text{remaining}} = 131\,200 - 50\,000 = 81\,200\text{ cycles}$$

Second loading:

$$(\sigma_m)_2 = \frac{350 + (-200)}{2} = 75\text{ MPa}$$

$$(\sigma_a)_2 = \frac{350 - (-200)}{2} = 275\text{ MPa}$$

$$(\sigma_a)_{e2} = \frac{275}{1 - (75/590)} = 315.0\text{ MPa}$$

(a) Miner's method

$$N_2 = \left(\frac{315}{1409.8}\right)^{-1/0.141\,355} = 40\,200 \text{ cycles}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \quad \Rightarrow \quad \frac{50\,000}{131\,200} + \frac{n_2}{40\,200} = 1$$

$$n_2 = 24\,880 \text{ cycles} \quad \textit{Ans.}$$

(b) Manson's method

Two data points: 0.9(590 MPa), 10^3 cycles
266.5 MPa, 81 200 cycles

$$\frac{0.9(590)}{266.5} = \frac{a_2(10^3)^{b_2}}{a_2(81\,200)^{b_2}}$$

$$1.9925 = (0.012\,315)^{b_2}$$

$$b_2 = \frac{\log 1.9925}{\log 0.012\,315} = -0.156\,789$$

$$a_2 = \frac{266.5}{(81\,200)^{-0.156\,789}} = 1568.4 \text{ MPa}$$

$$n_2 = \left(\frac{315}{1568.4}\right)^{1/-0.156\,789} = 27\,950 \text{ cycles} \quad \textit{Ans.}$$

6-30 **(a)** Miner's method

$$a = \frac{[0.9(76)]^2}{30} = 155.95 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.9(76)}{30} = -0.119\,31$$

$$\sigma_1 = 48 \text{ kpsi}, \quad N_1 = \left(\frac{48}{155.95}\right)^{1/-0.119\,31} = 19\,460 \text{ cycles}$$

$$\sigma_2 = 38 \text{ kpsi}, \quad N_2 = \left(\frac{38}{155.95}\right)^{1/-0.119\,31} = 137\,880 \text{ cycles}$$

$$\sigma_3 = 32 \text{ kpsi}, \quad N_3 = \left(\frac{32}{155.95}\right)^{1/-0.119\,31} = 582\,150 \text{ cycles}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

$$\frac{4000}{19\,460} + \frac{60\,000}{137\,880} + \frac{n_3}{582\,150} = 1 \quad \Rightarrow \quad n_3 = 209\,160 \text{ cycles} \quad \textit{Ans.}$$

(b) Manson's method

The life remaining after the first cycle is $N_{R_1} = 19\,460 - 4000 = 15\,460$ cycles. The two data points required to define $S'_{e,1}$ are $[0.9(76), 10^3]$ and (48, 15 460).

$$\frac{0.9(76)}{48} = \frac{a_2(10^3)^{b_2}}{a_2(15\,460)} \quad \Rightarrow \quad 1.425 = (0.064\,683)^{b_2}$$

$$b_2 = \frac{\log(1.425)}{\log(0.064\,683)} = -0.129\,342$$

$$a_2 = \frac{48}{(15\,460)^{-0.129\,342}} = 167.14 \text{ kpsi}$$

$$N_2 = \left(\frac{38}{167.14}\right)^{-1/0.129\,342} = 94\,110 \text{ cycles}$$

$$N_{R_2} = 94\,110 - 60\,000 = 34\,110 \text{ cycles}$$

$$\frac{0.9(76)}{38} = \frac{a_3(10^3)^{b_3}}{a_3(34\,110)^{b_3}} \quad \Rightarrow \quad 1.8 = (0.029\,317)^{b_3}$$

$$b_3 = \frac{\log 1.8}{\log(0.029\,317)} = -0.166\,531, \quad a_3 = \frac{38}{(34\,110)^{-0.166\,531}} = 216.10 \text{ kpsi}$$

$$N_3 = \left(\frac{32}{216.1}\right)^{-1/0.166\,531} = 95\,740 \text{ cycles} \quad \textit{Ans.}$$

6-31 Using Miner's method

$$a = \frac{[0.9(100)]^2}{50} = 162 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.9(100)}{50} = -0.085\,091$$

$$\sigma_1 = 70 \text{ kpsi}, \quad N_1 = \left(\frac{70}{162}\right)^{1/-0.085\,091} = 19\,170 \text{ cycles}$$

$$\sigma_2 = 55 \text{ kpsi}, \quad N_2 = \left(\frac{55}{162}\right)^{1/-0.085\,091} = 326\,250 \text{ cycles}$$

$$\sigma_3 = 40 \text{ kpsi}, \quad N_3 \to \infty$$

$$\frac{0.2N}{19\,170} + \frac{0.5N}{326\,250} + \frac{0.3N}{\infty} = 1$$

$$N = 83\,570 \text{ cycles} \quad \textit{Ans.}$$

6-32 Given $\mathbf{S}_{ut} = 245\mathbf{LN}(1, 0.0508)$ kpsi

From Table 7-13: $a = 1.34, b = -0.086, C = 0.12$

$$\mathbf{k}_a = 1.34\bar{S}_{ut}^{-0.086}\mathbf{LN}(1, 0.120)$$
$$= 1.34(245)^{-0.086}\mathbf{LN}(1, 0.12)$$
$$= 0.835\mathbf{LN}(1, 0.12)$$

$$k_b = 1.02 \quad \text{(as in Prob. 6-1)}$$

Eq. (6-70) $\mathbf{S}_e = 0.835(1.02)\mathbf{LN}(1, 0.12)[107\mathbf{LN}(1, 0.139)]$

$$\bar{S}_e = 0.835(1.02)(107) = 91.1 \text{ kpsi}$$

Now

$$C_{Se} \doteq (0.12^2 + 0.139^2)^{1/2} = 0.184$$

$$\mathbf{S}_e = 91.1\mathbf{LN}(1, 0.184)\text{ kpsi} \quad \textit{Ans.}$$

6-33 A Priori Decisions:

- Material and condition: 1018 CD, $\mathbf{S}_{ut} = 440\mathbf{LN}(1, 0.03)$, and $\mathbf{S}_y = 370\mathbf{LN}(1, 0.061)$ MPa
- Reliability goal: $R = 0.999 \quad (z = -3.09)$
- Function:

  Critical location—hole

- Variabilities:

$$C_{ka} = 0.058$$

$$C_{kc} = 0.125$$

$$C_\phi = 0.138$$

$$C_{Se} = \left(C_{ka}^2 + C_{kc}^2 + C_\phi^2\right)^{1/2} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$

$$C_{kc} = 0.10$$

$$C_{Fa} = 0.20$$

$$C_{\sigma a} = (0.10^2 + 0.20^2)^{1/2} = 0.234$$

$$C_n = \sqrt{\frac{C_{Se}^2 + C_{\sigma a}^2}{1 + C_{\sigma a}^2}} = \sqrt{\frac{0.195^2 + 0.234^2}{1 + 0.234^2}} = 0.297$$

Resulting in a design factor n_f of,

Eq. (6-88): $n_f = \exp[-(-3.09)\sqrt{\ln(1 + 0.297^2)} + \ln\sqrt{1 + 0.297^2}] = 2.56$

- Decision: Set $n_f = 2.56$

Now proceed deterministically using the mean values:

Table 6-10: $\bar{k}_a = 4.45(440)^{-0.265} = 0.887$

$k_b = 1$

Table 6-11: $\bar{k}_c = 1.43(440)^{-0.0778} = 0.891$

Eq. (6-70): $\bar{S}'_e = 0.506(440) = 222.6$ MPa

Eq. (6-71): $\bar{S}_e = 0.887(1)0.891(222.6) = 175.9$ MPa

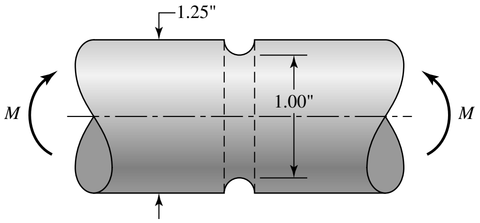
From Prob. 6-10, $K_f = 2.23$. Thus,

$$\bar{\sigma}_a = \bar{K}_f \frac{\bar{F}_a}{A} = \bar{K}_f \frac{\bar{F}_a}{t(60-12)} = \frac{\bar{S}_e}{\bar{n}_f}$$

$$\text{and, } t = \frac{\bar{n}_f \bar{K}_f \bar{F}_a}{48\bar{S}_e} = \frac{2.56(2.23)15(10^3)}{48(175.9)} = 10.14\text{ mm}$$

Decision: Depending on availability, (1) select $t = 10$ mm, recalculate n_f and R, and determine whether the reduced reliability is acceptable, or, (2) select $t = 11$ mm or larger, and determine whether the increase in cost and weight is acceptable. *Ans.*

6-34



Rotation is presumed. M and S_{ut} are given as deterministic, but notice that $\boldsymbol{\sigma}$ is not; therefore, a reliability estimation can be made.

From Eq. (6-70):

$$\begin{aligned}\mathbf{S}'_e &= 0.506(110)\mathbf{LN}(1, 0.138)\\ &= 55.7\mathbf{LN}(1, 0.138)\text{ kpsi}\end{aligned}$$

Table 6-10:

$$\begin{aligned}\mathbf{k}_a &= 2.67(110)^{-0.265}\mathbf{LN}(1, 0.058)\\ &= 0.768\mathbf{LN}(1, 0.058)\end{aligned}$$

Based on $d = 1$ in, Eq. (6-20) gives

$$k_b = \left(\frac{1}{0.30}\right)^{-0.107} = 0.879$$

Conservatism is not necessary

$$\begin{aligned}\mathbf{S}_e &= 0.768[\mathbf{LN}(1, 0.058)](0.879)(55.7)[\mathbf{LN}(1, 0.138)]\\ \bar{S}_e &= 37.6\text{ kpsi}\\ C_{Se} &= (0.058^2 + 0.138^2)^{1/2} = 0.150\\ \mathbf{S}_e &= 37.6\mathbf{LN}(1, 0.150)\end{aligned}$$

Fig. A-15-14: $D/d = 1.25$, $r/d = 0.125$. Thus $K_t = 1.70$ and Eqs. (6-78), (6-79) and Table 6-15 give

$$\begin{aligned}\mathbf{K}_f &= \frac{1.70\mathbf{LN}(1, 0.15)}{1 + \left(2/\sqrt{0.125}\right)[(1.70 - 1)/(1.70)](3/110)}\\ &= 1.598\mathbf{LN}(1, 0.15)\\ \boldsymbol{\sigma} &= \mathbf{K}_f \frac{32M}{\pi d^3} = 1.598[\mathbf{LN}(1 - 0.15)]\left[\frac{32(1400)}{\pi(1)^3}\right]\\ &= 22.8\mathbf{LN}(1, 0.15)\text{ kpsi}\end{aligned}$$

From Eq. (5-43), p. 242:

$$z = -\frac{\ln\left[(37.6/22.8)\sqrt{(1 + 0.15^2)/(1 + 0.15^2)}\right]}{\sqrt{\ln[(1 + 0.15^2)(1 + 0.15^2)]}} = -2.37$$

From Table A-10, $p_f = 0.008\,89$

$$\therefore R = 1 - 0.008\,89 = 0.991 \quad Ans.$$

Note: The correlation method uses only the mean of $\mathbf{S}_{ut}$; its variability is already included in the 0.138. When a deterministic load, in this case M, is used in a reliability estimate, engineers state, "For a *Design* Load of M, the reliability is 0.991." They are in fact referring to a Deterministic Design Load.

6-35 For completely reversed torsion, $\mathbf{k}_a$ and $\mathbf{k}_b$ of Prob. 6-34 apply, but $\mathbf{k}_c$ must also be considered.

Eq. 6-74:
$$\mathbf{k}_c = 0.328(110)^{0.125}\mathbf{LN}(1, 0.125)$$
$$= 0.590\mathbf{LN}(1, 0.125)$$

Note 0.590 is close to 0.577.

$$\mathbf{S}_{Se} = \mathbf{k}_a\, k_b\, \mathbf{k}_c\, \mathbf{S}'_e$$
$$= 0.768[\mathbf{LN}(1, 0.058)](0.878)[0.590\mathbf{LN}(1, 0.125)][55.7\mathbf{LN}(1, 0.138)]$$
$$\bar{S}_{Se} = 0.768(0.878)(0.590)(55.7) = 22.2 \text{ kpsi}$$
$$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$
$$\mathbf{S}_{Se} = 22.2\mathbf{LN}(1, 0.195) \text{ kpsi}$$

Fig. A-15-15: $D/d = 1.25$, $r/d = 0.125$, then $K_{ts} = 1.40$. From Eqs. (6-78), (6-79) and Table 6-15

$$\mathbf{K}_{ts} = \frac{1.40\mathbf{LN}(1, 0.15)}{1 + \left(2/\sqrt{0.125}\right)[(1.4-1)/1.4](3/110)} = 1.34\mathbf{LN}(1, 0.15)$$

$$\tau = \mathbf{K}_{ts}\,\frac{16T}{\pi d^3}$$
$$\tau = 1.34[\mathbf{LN}(1, 0.15)]\left[\frac{16(1.4)}{\pi(1)^3}\right]$$
$$= 9.55\mathbf{LN}(1, 0.15) \text{ kpsi}$$

From Eq. (5-43), p. 242:

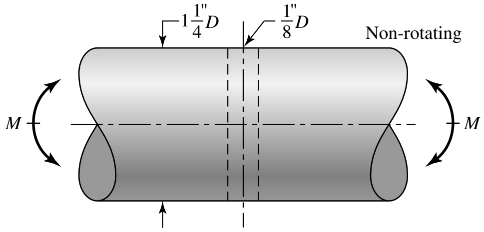
$$z = -\frac{\ln\left[(22.2/9.55)\sqrt{(1+0.15^2)/(1+0.195^2)}\right]}{\sqrt{\ln[(1+0.195^2)(1+0.15^2)]}} = -3.43$$

From Table A-10, $p_f = 0.0003$

$$R = 1 - p_f = 1 - 0.0003 = 0.9997 \quad Ans.$$

For a design with completely-reversed torsion of 1400 lbf · in, the reliability is 0.9997. The improvement comes from a smaller stress-concentration factor in torsion. See the note at the end of the solution of Prob. 6-34 for the reason for the phraseology.

6-36



$$S_{ut} = 58 \text{ kpsi}$$

$$\mathbf{S}'_e = 0.506(58)\mathbf{LN}(1, 0.138)$$

$$= 29.3\mathbf{LN}(1, 0.138) \text{ kpsi}$$

Table 6-10: $$\mathbf{k}_a = 14.5(58)^{-0.719}\mathbf{LN}(1, 0.11)$$

$$= 0.782\mathbf{LN}(1, 0.11)$$

Eq. (6-24):

$$d_e = 0.37(1.25) = 0.463 \text{ in}$$

$$k_b = \left(\frac{0.463}{0.30}\right)^{-0.107} = 0.955$$

$$\mathbf{S}_e = 0.782[\mathbf{LN}(1, 0.11)](0.955)[29.3\mathbf{LN}(1, 0.138)]$$

$$\bar{S}_e = 0.782(0.955)(29.3) = 21.9 \text{ kpsi}$$

$$C_{Se} = (0.11^2 + 0.138^2)^{1/2} = 0.150$$

Table A-16: $d/D = 0, a/D = 0.1, A = 0.83 \therefore K_t = 2.27.$

From Eqs. (6-78) and (6-79) and Table 6-15

$$\mathbf{K}_f = \frac{2.27\mathbf{LN}(1, 0.10)}{1 + \left(2/\sqrt{0.125}\right)[(2.27 - 1)/2.27](5/58)} = 1.783\mathbf{LN}(1, 0.10)$$

Table A-16:

$$Z = \frac{\pi A D^3}{3^2} = \frac{\pi(0.83)(1.25^3)}{32} = 0.159 \text{ in}^3$$

$$\boldsymbol{\sigma} = \mathbf{K}_f \frac{M}{Z} = 1.783\mathbf{LN}(1, 0.10)\left(\frac{1.6}{0.159}\right)$$

$$= 17.95\mathbf{LN}(1, 0.10) \text{ kpsi}$$

$$\bar{\sigma} = 17.95 \text{ kpsi}$$

$$C_\sigma = 0.10$$

Eq. (5-43), p. 242: $$z = -\frac{\ln\left[(21.9/17.95)\sqrt{(1 + 0.10^2)/(1 + 0.15^2)}\right]}{\sqrt{\ln[(1 + 0.15^2)(1 + 0.10^2)]}} = -1.07$$

Table A-10: $$p_f = 0.1423$$

$$R = 1 - p_f = 1 - 0.1423 = 0.858 \quad \textit{Ans.}$$

For a completely-reversed design load M_a of 1400 lbf · in, the reliability estimate is 0.858.

6-37 For a non-rotating bar subjected to completely reversed torsion of $T_a = 2400$ lbf · in

From Prob. 6-36:

$$S'_e = 29.3\mathbf{LN}(1, 0.138) \text{ kpsi}$$
$$\mathbf{k}_a = 0.782\mathbf{LN}(1, 0.11)$$
$$k_b = 0.955$$

For $\mathbf{k}_c$ use Eq. (6-74):

$$\mathbf{k}_c = 0.328(58)^{0.125}\mathbf{LN}(1, 0.125)$$
$$= 0.545\mathbf{LN}(1, 0.125)$$
$$\mathbf{S}_{Se} = 0.782[\mathbf{LN}(1, 0.11)](0.955)[0.545\mathbf{LN}(1, 0.125)][29.3\mathbf{LN}(1, 0.138)]$$
$$\bar{S}_{Se} = 0.782(0.955)(0.545)(29.3) = 11.9 \text{ kpsi}$$
$$C_{Se} = (0.11^2 + 0.125^2 + 0.138^2)^{1/2} = 0.216$$

Table A-16: $d/D = 0, a/D = 0.1, A = 0.92, K_{ts} = 1.68$

From Eqs. (6-78), (6-79), Table 6-15

$$\mathbf{K}_{fs} = \frac{1.68\mathbf{LN}(1, 0.10)}{1 + \left(2/\sqrt{0.125}\right)[(1.68 - 1)/1.68](5/58)}$$
$$= 1.403\mathbf{LN}(1, 0.10)$$

Table A-16:

$$J_{\text{net}} = \frac{\pi A D^4}{32} = \frac{\pi(0.92)(1.25^4)}{32} = 0.2201$$
$$\tau_a = \mathbf{K}_{fs}\,\frac{T_a c}{J_{\text{net}}}$$
$$= 1.403[\mathbf{LN}(1, 0.10)]\left[\frac{2.4(1.25/2)}{0.2201}\right]$$
$$= 9.56\mathbf{LN}(1, 0.10) \text{ kpsi}$$

From Eq. (5-43), p. 242:

$$z = -\frac{\ln\left[(11.9/9.56)\sqrt{(1 + 0.10^2)/(1 + 0.216^2)}\right]}{\sqrt{\ln[(1 + 0.10^2)(1 + 0.216^2)]}} = -0.85$$

Table A-10, $p_f = 0.1977$

$$R = 1 - p_f = 1 - 0.1977 = 0.80 \quad \textit{Ans.}$$

6-38 This is a very important task for the student to attempt before starting Part 3. It illustrates the drawback of the deterministic factor of safety method. It also identifies the a priori decisions and their consequences.

The range of force fluctuation in Prob. 6-23 is −16 to +4 kip, or 20 kip. Repeatedly-applied F_a is 10 kip. The stochastic properties of this heat of AISI 1018 CD are given.

Function	Consequences
Axial	$F_a = 10$ kip
Fatigue load	$C_{Fa} = 0$
	$C_{kc} = 0.125$
Overall reliability $R \geq 0.998$;	$z = -3.09$
with twin fillets	$C_{Kf} = 0.11$
$R \geq \sqrt{0.998} \geq 0.999$	
Cold rolled or machined surfaces	$C_{ka} = 0.058$
Ambient temperature	$C_{kd} = 0$
Use correlation method	$C_\phi = 0.138$
Stress amplitude	$C_{Kf} = 0.11$
	$C_{\sigma a} = 0.11$
Significant strength S_e	$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2}$
	$= 0.195$

Choose the mean design factor which will meet the reliability goal

$$C_n = \sqrt{\frac{0.195^2 + 0.11^2}{1 + 0.11^2}} = 0.223$$

$$\bar{n} = \exp\left[-(-3.09)\sqrt{\ln(1 + 0.223^2)} + \ln\sqrt{1 + 0.223^2}\right]$$

$$\bar{n} = 2.02$$

Review the number and quantitative consequences of the designer's a priori decisions to accomplish this. The operative equation is the definition of the design factor

$$\boldsymbol{\sigma}_a = \frac{\mathbf{S}_e}{\mathbf{n}}$$

$$\bar{\sigma}_a = \frac{\bar{S}_e}{\bar{n}} \Rightarrow \frac{\bar{K}_f F_a}{w_2 h} = \frac{\bar{S}_e}{\bar{n}}$$

Solve for thickness h. To do so we need

$$\bar{k}_a = 2.67\bar{S}_{ut}^{-0.265} = 2.67(64)^{-0.265} = 0.887$$

$$k_b = 1$$

$$\bar{k}_c = 1.23\bar{S}_{ut}^{-0.078} = 1.23(64)^{-0.078} = 0.889$$

$$\bar{k}_d = \bar{k}_e = 1$$

$$\bar{S}_e = 0.887(1)(0.889)(1)(1)(0.506)(64) = 25.5 \text{ kpsi}$$

Fig. A-15-5: $D = 3.75$ in, $d = 2.5$ in, $D/d = 3.75/2.5 = 1.5$, $r/d = 0.25/2.5 = 0.10$

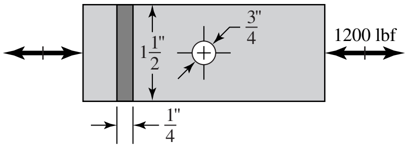
$\therefore K_t = 2.1$

$$\bar{K}_f = \frac{2.1}{1 + \left(2/\sqrt{0.25}\right)[(2.1 - 1)/(2.1)](4/64)} = 1.857$$

$$h = \frac{\bar{K}_f \bar{n} F_a}{w_2 \bar{S}_e} = \frac{1.857(2.02)(10)}{2.5(25.5)} = 0.667 \quad \textit{Ans.}$$

This thickness separates $\bar{S}_e$ and $\bar{\sigma}_a$ so as to realize the reliability goal of 0.999 at each shoulder. The design decision is to make t the next available thickness of 1018 CD steel strap from the same heat. This eliminates machining to the desired thickness and the extra cost of thicker work stock will be less than machining the fares. Ask your steel supplier what is available *in this heat*.

6-39



$$F_a = 1200 \text{ lbf}$$
$$S_{ut} = 80 \text{ kpsi}$$

(a) *Strength*

$$\begin{aligned}
\mathbf{k}_a &= 2.67(80)^{-0.265}\mathbf{LN}(1, 0.058)\\
&= 0.836\mathbf{LN}(1, 0.058)\\
k_b &= 1\\
\mathbf{k}_c &= 1.23(80)^{-0.078}\mathbf{LN}(1, 0.125)\\
&= 0.874\mathbf{LN}(1, 0.125)\\
\mathbf{S}'_a &= 0.506(80)\mathbf{LN}(1, 0.138)\\
&= 40.5\mathbf{LN}(1, 0.138) \text{ kpsi}
\end{aligned}$$

$$\mathbf{S}_e = 0.836[\mathbf{LN}(1, 0.058)](1)[0.874\mathbf{LN}(1, 0.125)][40.5\mathbf{LN}(1, 0.138)]$$

$$\bar{S}_e = 0.836(1)(0.874)(40.5) = 29.6 \text{ kpsi}$$

$$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$

Stress: Fig. A-15-1; $d/w = 0.75/1.5 = 0.5$, $K_t = 2.17$. From Eqs. (6-78), (6-79) and Table 6-15

$$\begin{aligned}
\mathbf{K}_f &= \frac{2.17\mathbf{LN}(1, 0.10)}{1 + \left(2/\sqrt{0.375}\right)[(2.17 - 1)/2.17](5/80)}\\
&= 1.95\mathbf{LN}(1, 0.10)
\end{aligned}$$

$$\boldsymbol{\sigma}_a = \frac{\mathbf{K}_f F_a}{(w - d)t}, \qquad C_\sigma = 0.10$$

$$\bar{\sigma}_a = \frac{\bar{K}_f F_a}{(w - d)t} = \frac{1.95(1.2)}{(1.5 - 0.75)(0.25)} = 12.48 \text{ kpsi}$$

$$\bar{S}_a = \bar{S}_e = 29.6 \text{ kpsi}$$

$$z = -\frac{\ln(\bar{S}_a/\bar{\sigma}_a)\sqrt{\left(1 + C_\sigma^2\right)/\left(1 + C_S^2\right)}}{\sqrt{\ln\left(1 + C_\sigma^2\right)\left(1 + C_S^2\right)}}$$

$$= -\frac{\ln\left[(29.6/12.48)\sqrt{(1 + 0.10^2)/(1 + 0.195^2)}\right]}{\sqrt{\ln(1 + 0.10^2)(1 + 0.195^2)}} = -3.9$$

From Table A-20

$$p_f = 4.481(10^{-5})$$

$$R = 1 - 4.481(10^{-5}) = 0.999\,955 \quad \textit{Ans.}$$

(b) All computer programs will differ in detail.

6-40 Each computer program will differ in detail. When the programs are working, the experience should reinforce that the decision regarding $\bar{n}_f$ is independent of mean values of strength, stress or associated geometry. The reliability goal can be realized by noting the impact of all those a priori decisions.

6-41 Such subprograms allow a simple call when the information is needed. The calling program is often named an executive routine (executives tend to delegate chores to others and only want the answers).

6-42 This task is similar to Prob. 6-41.

6-43 Again, a similar task.

6-44 The results of Probs. 6-41 to 6-44 will be the basis of a class computer aid for fatigue problems. The codes should be made available to the class through the library of the computer network or main frame available to your students.

6-45 Peterson's notch sensitivity q has very little statistical basis. This subroutine can be used to show the variation in $\mathbf{q}$, which is not apparent to those who embrace a deterministic q.

6-46 An additional program which is useful.

Chapter 7

7-1 **(a)** DE-Gerber, Eq. (7-10):

$$A = \left\{4[2.2(600)]^2 + 3[1.8(400)]^2\right\}^{1/2} = 2920 \text{ lbf} \cdot \text{in}$$

$$B = \left\{4[2.2(500)]^2 + 3[1.8(300)]^2\right\}^{1/2} = 2391 \text{ lbf} \cdot \text{in}$$

$$d = \left\{\frac{8(2)(2920)}{\pi(30\,000)}\left[1 + \left(1 + \left[\frac{2(2391)(30\,000)}{2920(100\,000)}\right]^2\right)^{1/2}\right]\right\}^{1/3}$$

$$= 1.016 \text{ in} \quad \textit{Ans.}$$

(b) DE-elliptic, Eq. (7-12) can be shown to be

$$d = \left(\frac{16n}{\pi}\sqrt{\frac{A^2}{S_e^2} + \frac{B^2}{S_y^2}}\right)^{1/3}$$

$$= \left[\frac{16(2)}{\pi}\sqrt{\left(\frac{2920}{30\,000}\right)^2 + \left(\frac{2391}{80\,000}\right)^2}\right]^{1/3} = 1.012 \text{ in} \quad \textit{Ans.}$$

(c) DE-Soderberg, Eq. (7-14) can be shown to be

$$d = \left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{S_y}\right)\right]^{1/3}$$

$$= \left[\frac{16(2)}{\pi}\left(\frac{2920}{30\,000} + \frac{2391}{80\,000}\right)\right]^{1/3}$$

$$= 1.090 \text{ in} \quad \textit{Ans.}$$

(d) DE-Goodman: Eq. (7-8) can be shown to be

$$d = \left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)\right]^{1/3}$$

$$= \left[\frac{16(2)}{\pi}\left(\frac{2920}{30\,000} + \frac{2391}{100\,000}\right)\right]^{1/3} = 1.073 \text{ in} \quad \textit{Ans.}$$

Criterion	d(in)	Compared to DE-Gerber	
DE-Gerber	1.016		
DE-elliptic	1.012	0.4% lower	less conservative
DE-Soderberg	1.090	7.3% higher	more conservative
DE-Goodman	1.073	5.6% higher	more conservative

7-2 This problem has to be done by successive trials, since S_e is a function of shaft size. The material is SAE 2340 for which $S_{ut} = 1226$ MPa, $S_y = 1130$ MPa, and $H_B \geq 368$.

Eq. (6-19): $$k_a = 4.51(1226)^{-0.265} = 0.685$$

Trial #1: Choose $d_r = 22$ mm

Eq. (6-20): $$k_b = \left(\frac{22}{7.62}\right)^{-0.107} = 0.893$$

Eq. (6-18): $$S_e = 0.685(0.893)(0.5)(1226) = 375 \text{ MPa}$$

$$d_r = d - 2r = 0.75D - 2D/20 = 0.65D$$

$$D = \frac{d_r}{0.65} = \frac{22}{0.65} = 33.8 \text{ mm}$$

$$r = \frac{D}{20} = \frac{33.8}{20} = 1.69 \text{ mm}$$

Fig. A-15-14:

$$d = d_r + 2r = 22 + 2(1.69) = 25.4 \text{ mm}$$

$$\frac{d}{d_r} = \frac{25.4}{22} = 1.15$$

$$\frac{r}{d_r} = \frac{1.69}{22} = 0.077$$

$$K_t = 1.9$$

Fig. A-15-15: $$K_{ts} = 1.5$$

Fig. 6-20: $$r = 1.69 \text{ mm}, \quad q = 0.90$$

Fig. 6-21: $$r = 1.69 \text{ mm}, \quad q_s = 0.97$$

Eq. (6-32): $$K_f = 1 + 0.90(1.9 - 1) = 1.81$$

$$K_{fs} = 1 + 0.97(1.5 - 1) = 1.49$$

We select the DE-ASME Elliptic failure criteria.

Eq. (7-12) with d as d_r, and $M_m = T_a = 0$,

$$d_r = \left\{\frac{16(2.5)}{\pi}\left[4\left(\frac{1.81(70)(10^3)}{375}\right)^2 + 3\left(\frac{1.49(45)(10^3)}{1130}\right)^2\right]^{1/2}\right\}^{1/3}$$

$$= 20.6 \text{ mm}$$

Trial #2: Choose $d_r = 20.6$ mm

$$k_b = \left(\frac{20.6}{7.62}\right)^{-0.107} = 0.899$$

$$S_e = 0.685(0.899)(0.5)(1226) = 377.5 \text{ MPa}$$

$$D = \frac{d_r}{0.65} = \frac{20.6}{0.65} = 31.7 \text{ mm}$$

$$r = \frac{D}{20} = \frac{31.7}{20} = 1.59 \text{ mm}$$

Figs. A-15-14 and A-15-15:

$$d = d_r + 2r = 20.6 + 2(1.59) = 23.8 \text{ mm}$$

$$\frac{d}{d_r} = \frac{23.8}{20.6} = 1.16$$

$$\frac{r}{d_r} = \frac{1.59}{20.6} = 0.077$$

We are at the limit of readability of the figures so

$$K_t = 1.9, \quad K_{ts} = 1.5 \qquad q = 0.9, \quad q_s = 0.97$$

$$\therefore K_f = 1.81 \qquad K_{fs} = 1.49$$

Using Eq. (7-12) produces $d_r = 20.5$ mm. Further iteration produces no change.

Decisions:

$$d_r = 20.5 \text{ mm}$$

$$D = \frac{20.5}{0.65} = 31.5 \text{ mm}, \quad d = 0.75(31.5) = 23.6 \text{ mm}$$

Use $D = 32$ mm, $d = 24$ mm, $r = 1.6$ mm *Ans.*

7-3 $F \cos 20°(d/2) = T, \quad F = 2T/(d \cos 20°) = 2(3000)/(6 \cos 20°) = 1064 \text{ lbf}$

$$M_C = 1064(4) = 4257 \text{ lbf} \cdot \text{in}$$

For sharp fillet radii at the shoulders, from Table 7-1, $K_t = 2.7$, and $K_{ts} = 2.2$. Examining Figs. 6-20 and 6-21, with $S_{ut} = 80$ kpsi, conservatively estimate $q = 0.8$ and $q_s = 0.9$. These estimates can be checked once a specific fillet radius is determined.

Eq. (6-32): $K_f = 1 + (0.8)(2.7 - 1) = 2.4$

$$K_{fs} = 1 + (0.9)(2.2 - 1) = 2.1$$

(a) Static analysis using fatigue stress concentration factors:

From Eq. (7-15) with $M = M_m$, $T = T_m$, and $M_a = T_a = 0$,

$$\sigma'_{\max} = \left[\left(\frac{32K_f M}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T}{\pi d^3}\right)^2\right]^{1/2}$$

Eq. (7-16):
$$n = \frac{S_y}{\sigma'_{\max}} = \frac{S_y}{\left[\left(\frac{32K_f M}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T}{\pi d^3}\right)^2\right]^{1/2}}$$

Solving for d,

$$d = \left\{\frac{16n}{\pi S_y}\left[4(K_f M)^2 + 3(K_{fs}T)^2\right]^{1/2}\right\}^{1/3}$$
$$= \left\{\frac{16(2.5)}{\pi(60\,000)}\left[4(2.4)(4257)^2 + 3(2.1)(3000)^2\right]^{1/2}\right\}^{1/3}$$
$$= 1.700 \text{ in} \quad \textit{Ans.}$$

(b)
$$k_a = 2.70(80)^{-0.265} = 0.845$$

Assume $d = 2.00$ in to estimate the size factor,

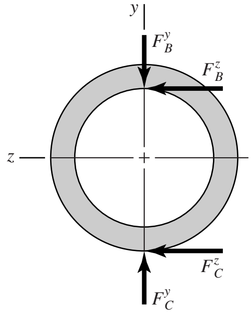
$$k_b = \left(\frac{2}{0.3}\right)^{-0.107} = 0.816$$
$$S_e = 0.845(0.816)(0.5)(80) = 27.6 \text{ kpsi}$$

Selecting the DE-ASME Elliptic criteria, use Eq. (7-12) with $M_m = T_a = 0$.

$$d = \left\{\frac{16(2.5)}{\pi}\left[4\left(\frac{2.4(4257)}{27\,600}\right)^2 + 3\left(\frac{2.1(3000)}{60\,000}\right)^2\right]^{1/2}\right\}^{1/3} = 2.133 \text{ in}$$

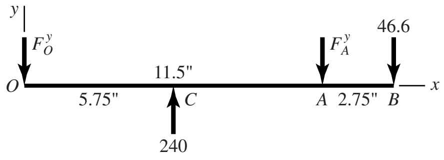
Revising k_b results in $d = 2.138$ in *Ans.*

7-4 We have a design task of identifying bending moment and torsion diagrams which are preliminary to an industrial roller shaft design.



$$F_C^y = 30(8) = 240 \text{ lbf}$$
$$F_C^z = 0.4(240) = 96 \text{ lbf}$$
$$T = F_C^z(2) = 96(2) = 192 \text{ lbf} \cdot \text{in}$$
$$F_B^z = \frac{T}{1.5} = \frac{192}{1.5} = 128 \text{ lbf}$$
$$F_B^y = F_B^z \tan 20° = 128 \tan 20° = 46.6 \text{ lbf}$$

(a) *xy-plane*



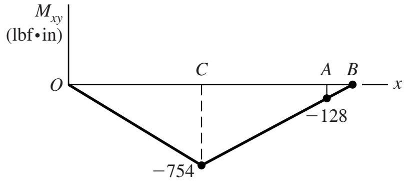
$$\sum M_O = 240(5.75) - F_A^y(11.5) - 46.6(14.25) = 0$$

$$F_A^y = \frac{240(5.75) - 46.6(14.25)}{11.5} = 62.3\,\text{lbf}$$

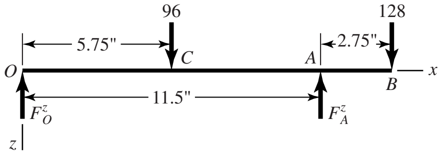
$$\sum M_A = F_O^y(11.5) - 46.6(2.75) - 240(5.75) = 0$$

$$F_O^y = \frac{240(5.75) + 46.6(2.75)}{11.5} = 131.1\,\text{lbf}$$

Bending moment diagram



xz-plane



$$\sum M_O = 0$$

$$= 96(5.75) - F_A^z(11.5) + 128(14.25)$$

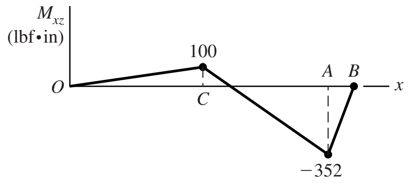
$$F_A^z = \frac{96(5.75) + 128(14.25)}{11.5} = 206.6\,\text{lbf}$$

$$\sum M_A = 0$$

$$= F_O^z(11.5) + 128(2.75) - 96(5.75)$$

$$F_O^z = \frac{96(5.75) - 128(2.75)}{11.5} = 17.4\,\text{lbf}$$

Bending moment diagram:

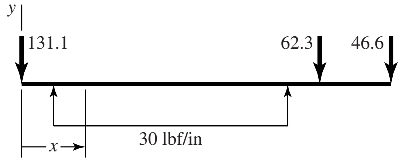


$$M_C = \sqrt{100^2 + (-754)^2} = 761\,\text{lbf}\cdot\text{in}$$

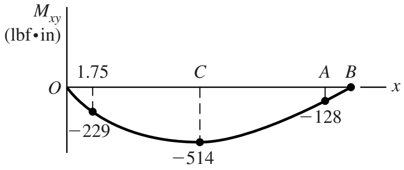
$$M_A = \sqrt{(-128)^2 + (-352)^2} = 375\,\text{lbf}\cdot\text{in}$$

This approach over-estimates the bending moment at *C*, but not at *A*.

(b) *xy-plane*



$$M_{xy} = -131.1x + 15\langle x - 1.75\rangle^2 - 15\langle x - 9.75\rangle^2 - 62.3\langle x - 11.5\rangle^1$$



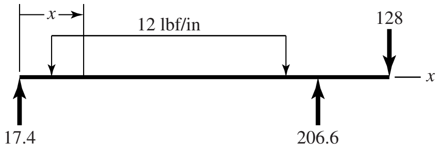
$$M_{\max} \text{ occurs at } 6.12 \text{ in}$$

$$M_{\max} = -516 \,\text{lbf} \cdot \text{in}$$

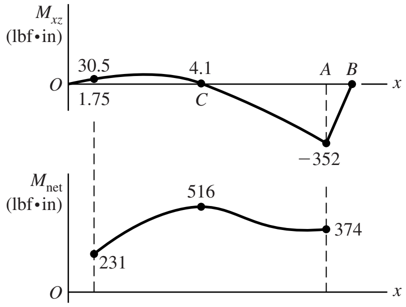
$$M_C = 131.1(5.75) - 15(5.75 - 1.75)^2 = 514$$

Reduced from 754 lbf · in. The maximum occurs at $x = 6.12$ in rather than C, but it is close enough.

xz-plane



$$M_{xz} = 17.4x - 6\langle x - 1.75\rangle^2 + 6\langle x - 9.75\rangle^2 + 206.6\langle x - 11.5\rangle^1$$



Let $M_{\text{net}} = \sqrt{M_{xy}^2 + M_{xz}^2}$

Plot $M_{\text{net}}(x)$

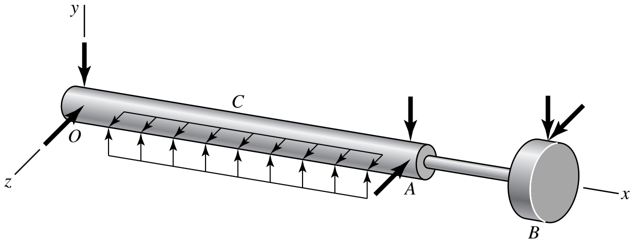
$1.75 \le x \le 11.5$ in

$M_{\max} = 516 \,\text{lbf} \cdot \text{in}$
at $x = 6.25$ in

Torque: In both cases the torque rises from 0 to 192 lbf · in linearly across the roller and is steady until the coupling keyway is encountered; then it falls linearly to 0 across the key. *Ans.*

7-5 This is a design problem, which can have many acceptable designs. See the solution for Problem 7-7 for an example of the design process.

7-6 If students have access to finite element or beam analysis software, have them model the shaft to check deflections. If not, solve a simpler version of shaft. The 1" diameter sections will not affect the results much, so model the 1" diameter as 1.25". Also, ignore the step in *AB*.



From Prob. 18-10, integrate M_{xy} and M_{xz}

xy plane, with $dy/dx = y'$

$$EIy' = -\frac{131.1}{2}(x^2) + 5\langle x - 1.75\rangle^3 - 5\langle x - 9.75\rangle^3 - \frac{62.3}{2}\langle x - 11.5\rangle^2 + C_1 \qquad (1)$$

$$EIy = -\frac{131.1}{6}(x^3) + \frac{5}{4}\langle x - 1.75\rangle^4 - \frac{5}{4}\langle x - 9.75\rangle^4 - \frac{62.3}{6}\langle x - 11.5\rangle^3 + C_1 x + C_2$$

$$y = 0 \text{ at } x = 0 \quad \Rightarrow \quad C_2 = 0$$

$$y = 0 \text{ at } x = 11.5 \quad \Rightarrow \quad C_1 = 1908.4 \text{ lbf} \cdot \text{in}^3$$

From (1)

$$x = 0: \quad EIy' = 1908.4$$

$$x = 11.5: \quad EIy' = -2153.1$$

xz plane (treating $z \uparrow +$)

$$EIz' = \frac{17.4}{2}(x^2) - 2\langle x - 1.75\rangle^3 + 2\langle x - 9.75\rangle^3 + \frac{206.6}{2}\langle x - 11.5\rangle^2 + C_3 \qquad (2)$$

$$EIz = \frac{17.4}{6}(x^3) - \frac{1}{2}\langle x - 1.75\rangle^4 + \frac{1}{2}\langle x - 9.75\rangle^4 + \frac{206.6}{6}\langle x - 11.5\rangle^3 + C_3 x + C_4$$

$$z = 0 \text{ at } x = 0 \quad \Rightarrow \quad C_4 = 0$$

$$z = 0 \text{ at } x = 11.5 \quad \Rightarrow \quad C_3 = 8.975 \text{ lbf} \cdot \text{in}^3$$

From (2)

$$x = 0: \quad EIz' = 8.975$$

$$x = 11.5: \quad EIz' = -683.5$$

At *O*: $\quad EI\theta = \sqrt{1908.4^2 + 8.975^2} = 1908.4 \text{ lbf} \cdot \text{in}^3$

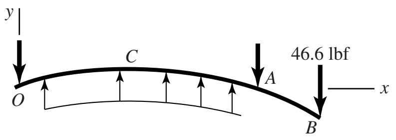
A: $\quad EI\theta = \sqrt{(-2153.1)^2 + (-683.5)^2} = 2259 \text{ lbf} \cdot \text{in}^3$ (dictates size)

$$\theta = \frac{2259}{30(10^6)(\pi/64)(1.25^4)} = 0.000\,628 \text{ rad}$$

$$n = \frac{0.001}{0.000\,628} = 1.59$$

At gear mesh, B

xy plane



With $I = I_1$ in section *OCA*,

$$y'_A = -2153.1/EI_1$$

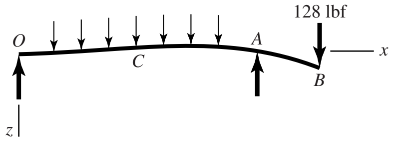
Since $y'_{B/A}$ is a cantilever, from Table A-9-1, with $I = I_2$ in section *AB*

$$y'_{B/A} = \frac{Fx(x-2l)}{2EI_2} = \frac{46.6}{2EI_2}(2.75)[2.75 - 2(2.75)] = -176.2/EI_2$$

$$\therefore y'_B = y'_A + y'_{B/A} = -\frac{2153.1}{30(10^6)(\pi/64)(1.25^4)} - \frac{176.2}{30(10^6)(\pi/64)(0.875^4)}$$

$$= -0.000\,803 \text{ rad} \quad \text{(magnitude greater than 0.0005 rad)}$$

xz plane



$$z'_A = -\frac{683.5}{EI_1}, \quad z'_{B/A} = -\frac{128(2.75^2)}{2EI_2} = -\frac{484}{EI_2}$$

$$z'_B = -\frac{683.5}{30(10^6)(\pi/64)(1.25^4)} - \frac{484}{30(10^6)(\pi/64)(0.875^4)} = -0.000\,751 \text{ rad}$$

$$\theta_B = \sqrt{(-0.000\,803)^2 + (0.000\,751)^2} = 0.001\,10 \text{ rad}$$

Crowned teeth must be used.

Finite element results:	Error in simplified model
$\theta_O = 5.47(10^{-4})$ rad	3.0%
$\theta_A = 7.09(10^{-4})$ rad	11.4%
$\theta_B = 1.10(10^{-3})$ rad	0.0%

The simplified model yielded reasonable results.

Strength $\quad S_{ut} = 72$ kpsi, $\quad S_y = 39.5$ kpsi

At the shoulder at A, $x = 10.75$ in. From Prob. 7-4,

$$M_{xy} = -209.3 \text{ lbf}\cdot\text{in}, \quad M_{xz} = -293.0 \text{ lbf}\cdot\text{in}, \quad T = 192 \text{ lbf}\cdot\text{in}$$

$$M = \sqrt{(-209.3)^2 + (-293)^2} = 360.0 \text{ lbf}\cdot\text{in}$$

$$S'_e = 0.5(72) = 36 \text{ kpsi}$$

$$k_a = 2.70(72)^{-0.265} = 0.869$$

$$k_b = \left(\frac{1}{0.3}\right)^{-0.107} = 0.879$$

$$k_c = k_d = k_e = k_f = 1$$

$$S_e = 0.869(0.879)(36) = 27.5 \text{ kpsi}$$

From Fig. A-15-8 with $D/d = 1.25$ and $r/d = 0.03$, $K_{ts} = 1.8$.

From Fig. A-15-9 with $D/d = 1.25$ and $r/d = 0.03$, $K_t = 2.3$

From Fig. 6-20 with $r = 0.03$ in, $q = 0.65$.

From Fig. 6-21 with $r = 0.03$ in, $q_s = 0.83$

Eq. (6-31): $K_f = 1 + 0.65(2.3 - 1) = 1.85$

$$K_{fs} = 1 + 0.83(1.8 - 1) = 1.66$$

Using DE-elliptic, Eq. (7-11) with $M_m = T_a = 0$,

$$\frac{1}{n} = \frac{16}{\pi(1^3)}\left\{4\left[\frac{1.85(360)}{27\,500}\right]^2 + 3\left[\frac{1.66(192)}{39\,500}\right]^2\right\}^{1/2}$$

$$n = 3.89$$

Perform a similar analysis at the profile keyway under the gear.

The main problem with the design is the undersized shaft overhang with excessive slope at the gear. The use of crowned-teeth in the gears will eliminate this problem.

7-7 **(a)** One possible shaft layout is shown. Both bearings and the gear will be located against shoulders. The gear and the motor will transmit the torque through keys. The bearings can be lightly pressed onto the shaft. The left bearing will locate the shaft in the housing, while the right bearing will float in the housing.

(b) From summing moments around the shaft axis, the tangential transmitted load through the gear will be

$$W_t = T/(d/2) = 2500/(4/2) = 1250 \text{ lbf}$$

The radial component of gear force is related by the pressure angle.

$$W_r = W_t \tan\phi = 1250 \tan 20^\circ = 455 \text{ lbf}$$

$$W = [W_r^2 + W_t^2]^{1/2} = (455^2 + 1250^2)^{1/2} = 1330 \text{ lbf}$$

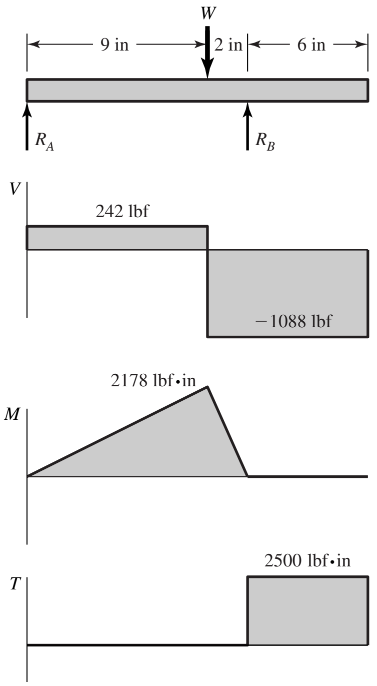
Reactions R_A and R_B, and the load W are all in the same plane. From force and moment balance,

$$R_A = 1330(2/11) = 242 \text{ lbf}$$

$$R_B = 1330(9/11) = 1088 \text{ lbf}$$

$$M_{\max} = R_A(9) = (242)(9) = 2178 \text{ lbf} \cdot \text{in}$$

Shear force, bending moment, and torque diagrams can now be obtained.



Ans.

(c) Potential critical locations occur at each stress concentration (shoulders and keyways). To be thorough, the stress at each potentially critical location should be evaluated. For now, we will choose the most likely critical location, by observation of the loading situation, to be in the keyway for the gear. At this point there is a large stress concentration, a large bending moment, and the torque is present. The other locations either have small bending moments, or no torque. The stress concentration for the keyway is highest at the ends. For simplicity, and to be conservative, we will use the maximum bending moment, even though it will have dropped off a little at the end of the keyway.

(d) At the gear keyway, approximately 9 in from the left end of the shaft, the bending is completely reversed and the torque is steady.

$$M_a = 2178 \text{ lbf} \cdot \text{in} \qquad T_m = 2500 \text{ lbf} \cdot \text{in} \qquad M_m = T_a = 0$$

From Table 7-1, estimate stress concentrations for the end-milled keyseat to be $K_t = 2.2$ and $K_{ts} = 3.0$. For the relatively low strength steel specified (AISI 1020 CD), estimate notch sensitivities of $q = 0.75$ and $q_s = 0.9$, obtained by observation of Figs. 6-20 and 6-21. Assuming a typical radius at the bottom of the keyseat of $r/d = 0.02$ (p. 361), these estimates for notch sensitivity are good for up to about 3 in shaft diameter.

Eq. (6-32): $K_f = 1 + 0.75(2.2 - 1) = 1.9$

$K_{fs} = 1 + 0.9(3.0 - 1) = 2.8$

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

For estimating k_b, guess $d = 2$ in.

$k_b = (2/0.3)^{-0.107} = 0.816$

$S_e = (0.883)(0.816)(0.5)(68) = 24.5$ kpsi

Selecting the DE-Goodman criteria for a conservative first design,

Eq. (7-8): $$d = \left[\frac{16n}{\pi} \left\{ \frac{\left[4(K_f M_a)^2\right]^{1/2}}{S_e} + \frac{\left[3(K_{fs} T_m)^2\right]^{1/2}}{S_{ut}} \right\} \right]^{1/3}$$

$$d = \left[\frac{16n}{\pi} \left\{ \frac{\left[4(1.9 \cdot 2178)^2\right]^{1/2}}{24\,500} + \frac{\left[3(2.8 \cdot 2500)^2\right]^{1/2}}{68\,000} \right\} \right]^{1/3}$$

$$d = 1.58 \text{ in} \quad \textit{Ans.}$$

With this diameter, the estimates for notch sensitivity and size factor were conservative, but close enough for a first iteration until deflections are checked.

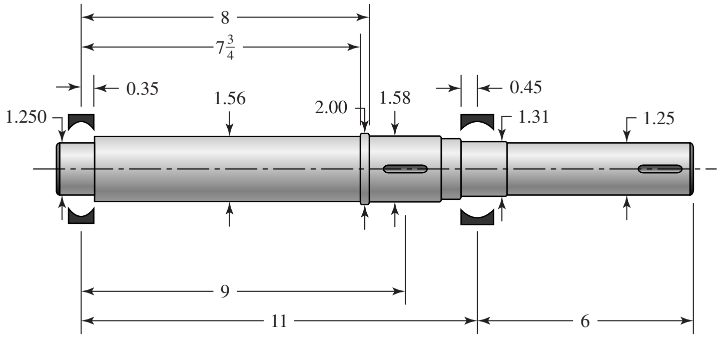
Check for static failure.

Eq. (7-15): $$\sigma'_{\max} = \left[\left(\frac{32 K_f M_a}{\pi d^3} \right)^2 + 3 \left(\frac{16 K_{fs} T_m}{\pi d^3} \right)^2 \right]^{1/2}$$

$$\sigma'_{\max} = \left[\left(\frac{32(1.9)(2178)}{\pi (1.58)^3} \right)^2 + 3 \left(\frac{16(2.8)(2500)}{\pi (1.58)^3} \right)^2 \right]^{1/2} = 19.0 \text{ kpsi}$$

$$n_y = S_y / \sigma'_{\max} = 57/19.0 = 3.0 \quad \textit{Ans.}$$

(e) Now estimate other diameters to provide typical shoulder supports for the gear and bearings (p. 360). Also, estimate the gear and bearing widths.



(f) Entering this shaft geometry into beam analysis software (or Finite Element software), the following deflections are determined:

Left bearing slope:	0.000532 rad
Right bearing slope:	−0.000850 rad
Gear slope:	−0.000545 rad
Right end of shaft slope:	−0.000850 rad
Gear deflection:	−0.00145 in
Right end of shaft deflection:	0.00510 in

Comparing these deflections to the recommendations in Table 7-2, everything is within typical range except the gear slope is a little high for an uncrowned gear.

(g) To use a non-crowned gear, the gear slope is recommended to be less than 0.0005 rad. Since all other deflections are acceptable, we will target an increase in diameter only for the long section between the left bearing and the gear. Increasing this diameter from the proposed 1.56 in to 1.75 in, produces a gear slope of –0.000401 rad. All other deflections are improved as well.

7-8 **(a)** Use the distortion-energy elliptic failure locus. The torque and moment loadings on the shaft are shown in the solution to Prob. 7-7.

Candidate critical locations for strength:

- Pinion seat keyway
- Right bearing shoulder
- Coupling keyway

Table A-20 for 1030 HR: $S_{ut} = 68$ kpsi, $S_y = 37.5$ kpsi, $H_B = 137$

Eq. (6-8): $S'_e = 0.5(68) = 34.0\,\text{kpsi}$

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

$$k_c = k_d = k_e = 1$$

Pinion seat keyway

See Table 7-1 for keyway stress concentration factors

$$\left.\begin{aligned} K_t &= 2.2 \\ K_{ts} &= 3.0 \end{aligned}\right\} \text{Profile keyway}$$

For an end-mill profile keyway cutter of 0.010 in radius,

From Fig. 6-20: $q = 0.50$

From Fig. 6-21: $q_s = 0.65$

Eq. (6-32):

$$\begin{aligned} K_{fs} &= 1 + q_s(K_{ts} - 1) \\ &= 1 + 0.65(3.0 - 1) = 2.3 \\ K_f &= 1 + 0.50(2.2 - 1) = 1.6 \end{aligned}$$

Eq. (6-20): $k_b = \left(\dfrac{1.875}{0.30}\right)^{-0.107} = 0.822$

Eq. (6-18): $S_e = 0.883(0.822)(34.0) = 24.7\,\text{kpsi}$

Eq. (7-11):

$$\frac{1}{n} = \frac{16}{\pi(1.875^3)}\left\{4\left[\frac{1.6(2178)}{24\,700}\right]^2 + 3\left[\frac{2.3(2500)}{37\,500}\right]^2\right\}^{1/2}$$

$$= 0.353, \quad \text{from which } n = 2.83$$

Right-hand bearing shoulder

The text does not give minimum and maximum shoulder diameters for 03-series bearings (roller). Use $D = 1.75$ in.

$$\frac{r}{d} = \frac{0.030}{1.574} = 0.019, \qquad \frac{D}{d} = \frac{1.75}{1.574} = 1.11$$

From Fig. A-15-9,

$$K_t = 2.4$$

From Fig. A-15-8,

$$K_{ts} = 1.6$$

From Fig. 6-20, $\quad q = 0.65$

From Fig. 6-21, $\quad q_s = 0.83$

$$K_f = 1 + 0.65(2.4 - 1) = 1.91$$

$$K_{fs} = 1 + 0.83(1.6 - 1) = 1.50$$

$$M = 2178\left(\frac{0.453}{2}\right) = 493 \text{ lbf} \cdot \text{in}$$

Eq. (7-11):

$$\frac{1}{n} = \frac{16}{\pi(1.574^3)}\left[4\left(\frac{1.91(493)}{24\,700}\right)^2 + 3\left(\frac{1.50(2500)}{37\,500}\right)^2\right]^{1/2}$$

$$= 0.247, \quad \text{from which } n = 4.05$$

Overhanging coupling keyway

There is no bending moment, thus Eq. (7-11) reduces to:

$$\frac{1}{n} = \frac{16\sqrt{3}K_{fs}T_m}{\pi d^3 S_y} = \frac{16\sqrt{3}(1.50)(2500)}{\pi(1.5^3)(37\,500)}$$

$$= 0.261 \quad \text{from which } n = 3.83$$

(b) One could take pains to model this shaft exactly, using say finite element software. However, for the bearings and the gear, the shaft is basically of uniform diameter, 1.875 in. The reductions in diameter at the bearings will change the results insignificantly. Use $E = 30(10^6)$ psi.

To the left of the load:

$$\theta_{AB} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2)$$

$$= \frac{1449(2)(3x^2 + 2^2 - 11^2)}{6(30)(10^6)(\pi/64)(1.825^4)(11)}$$

$$= 2.4124(10^{-6})(3x^2 - 117)$$

At $x = 0$: $\quad \theta = -2.823(10^{-4})$ rad

At $x = 9$ in: $\quad \theta = 3.040(10^{-4})$ rad

At $x = 11$ in:
$$\theta = \frac{1449(9)(11^2 - 9^2)}{6(30)(10^6)(\pi/64)(1.875^4)(11)}$$
$$= 4.342(10^{-4})\text{ rad}$$

Obtain allowable slopes from Table 7-2.

Left bearing:

$$n_{fs} = \frac{\text{Allowable slope}}{\text{Actual slope}}$$
$$= \frac{0.001}{0.000\,282\,3} = 3.54$$

Right bearing:

$$n_{fs} = \frac{0.0008}{0.000\,434\,2} = 1.84$$

Gear mesh slope:

Table 7-2 recommends a minimum relative slope of 0.0005 rad. While we don't know the slope on the next shaft, we know that it will need to have a larger diameter and be stiffer. At the moment we can say

$$n_{fs} < \frac{0.0005}{0.000\,304} = 1.64$$

7-9 The solution to Problem 7-8 may be used as an example of the analysis process for a similar situation.

7-10 If you have a finite element program available, it is highly recommended. Beam deflection programs can be implemented but this is time consuming and the programs have narrow applications. Here we will demonstrate how the problem can be simplified and solved using singularity functions.

Deflection: First we will ignore the steps near the bearings where the bending moments are low. Thus let the 30 mm dia. be 35 mm. Secondly, the 55 mm dia. is very thin, 10 mm. The full bending stresses will not develop at the outer fibers so full stiffness will not develop either. Thus, ignore this step and let the diameter be 45 mm.

Statics: Left support: $R_1 = 7(315 - 140)/315 = 3.889$ kN

Right support: $R_2 = 7(140)/315 = 3.111$ kN

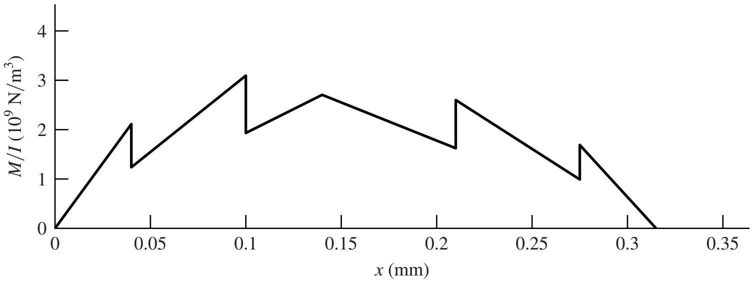
Determine the bending moment at each step.

x(mm)	0	40	100	140	210	275	315
M(N · m)	0	155.56	388.89	544.44	326.67	124.44	0

$I_{35} = (\pi/64)(0.035^4) = 7.366(10^{-8})\text{ m}^4,\ I_{40} = 1.257(10^{-7})\text{ m}^4,\ I_{45} = 2.013(10^{-7})\text{ m}^4$

Plot M/I as a function of x.

x (m)	$M/I\,(10^9\ \text{N/m}^3)$	Step	Slope	Δ Slope
0	0		52.8	
0.04	2.112			
0.04	1.2375	−0.8745	30.942	−21.86
0.1	3.094			
0.1	1.932	−1.162	19.325	−11.617
0.14	2.705			
0.14	2.705	0	−15.457	−34.78
0.21	1.623			
0.21	2.6	0.977	−24.769	−9.312
0.275	0.99			
0.275	1.6894	0.6994	−42.235	−17.47
0.315	0			



The steps and the change of slopes are evaluated in the table. From these, the function M/I can be generated:

$$M/I = \big[52.8x - 0.8745\langle x - 0.04\rangle^0 - 21.86\langle x - 0.04\rangle^1 - 1.162\langle x - 0.1\rangle^0 - 11.617\langle x - 0.1\rangle^1 - 34.78\langle x - 0.14\rangle^1 + 0.977\langle x - 0.21\rangle^0 - 9.312\langle x - 0.21\rangle^1 + 0.6994\langle x - 0.275\rangle^0 - 17.47\langle x - 0.275\rangle^1\big]\,10^9$$

Integrate twice:

$$E\frac{dy}{dx} = \big[26.4x^2 - 0.8745\langle x - 0.04\rangle^1 - 10.93\langle x - 0.04\rangle^2 - 1.162\langle x - 0.1\rangle^1 - 5.81\langle x - 0.1\rangle^2 - 17.39\langle x - 0.14\rangle^2 + 0.977\langle x - 0.21\rangle^1 - 4.655\langle x - 0.21\rangle^2 + 0.6994\langle x - 0.275\rangle^1 - 8.735\langle x - 0.275\rangle^2 + C_1\big]\,10^9 \qquad (1)$$

$$Ey = \big[8.8x^3 - 0.4373\langle x - 0.04\rangle^2 - 3.643\langle x - 0.04\rangle^3 - 0.581\langle x - 0.1\rangle^2 - 1.937\langle x - 0.1\rangle^3 - 5.797\langle x - 0.14\rangle^3 + 0.4885\langle x - 0.21\rangle^2 - 1.552\langle x - 0.21\rangle^3 + 0.3497\langle x - 0.275\rangle^2 - 2.912\langle x - 0.275\rangle^3 + C_1x + C_2\big]\,10^9$$

Boundary conditions: $y = 0$ at $x = 0$ yields $C_2 = 0$;

$y = 0$ at $x = 0.315$ m yields $C_1 = -0.295\,25$ N/m^2.

Equation (1) with $C_1 = -0.295\,25$ provides the slopes at the bearings and gear. The following table gives the results in the second column. The third column gives the results from a similar finite element model. The fourth column gives the result of a full model which models the 35 and 55 mm diameter steps.

x (mm)	θ (rad)	F.E. Model	Full F.E. Model
0	−0.001 4260	−0.001 4270	−0.001 4160
140	−0.000 1466	−0.000 1467	−0.000 1646
315	0.001 3120	0.001 3280	0.001 3150

The main discrepancy between the results is at the gear location $(x = 140 \text{ mm})$. The larger value in the full model is caused by the stiffer 55 mm diameter step. As was stated earlier, this step is not as stiff as modeling implicates, so the exact answer is somewhere between the full model and the simplified model which in any event is a small value. As expected, modeling the 30 mm dia. as 35 mm does not affect the results much.

It can be seen that the allowable slopes at the bearings are exceeded. Thus, either the load has to be reduced or the shaft "beefed" up. If the allowable slope is 0.001 rad, then the maximum load should be $F_{\max} = (0.001/0.001\,46)7 = 4.79$ kN. With a design factor this would be reduced further.

To increase the stiffness of the shaft, increase the diameters by $(0.001\,46/0.001)^{1/4} = 1.097$, from Eq. (7-18). Form a table:

Old d, mm	20.00	30.00	35.00	40.00	45.00	55.00
New ideal d, mm	21.95	32.92	38.41	43.89	49.38	60.35
Rounded up d, mm	22.00	34.00	40.00	44.00	50.00	62.00

Repeating the full finite element model results in

$$x = 0: \qquad \theta = -9.30 \times 10^{-4} \text{ rad}$$

$$x = 140 \text{ mm}: \quad \theta = -1.09 \times 10^{-4} \text{ rad}$$

$$x = 315 \text{ mm}: \quad \theta = 8.65 \times 10^{-4} \text{ rad}$$

Well within our goal. Have the students try a goal of 0.0005 rad at the bearings.

Strength: Due to stress concentrations and reduced shaft diameters, there are a number of locations to look at. A table of nominal stresses is given below. Note that torsion is only to the right of the 7 kN load. Using $\sigma = 32M/(\pi d^3)$ and $\tau = 16T/(\pi d^3)$,

x (mm)	0	15	40	100	110	140	210	275	300	330
σ (MPa)	0	22.0	37.0	61.9	47.8	60.9	52.0	39.6	17.6	0
τ (MPa)	0	0	0	0	0	6	8.5	12.7	20.2	68.1
σ' (MPa)	0	22.0	37.0	61.9	47.8	61.8	53.1	45.3	39.2	118.0

Table A-20 for AISI 1020 CD steel: $S_{ut} = 470$ MPa, $S_y = 390$ MPa

At $x = 210$ mm:

$$k_a = 4.51(470)^{-0.265} = 0.883, \quad k_b = (40/7.62)^{-0.107} = 0.837$$

$$S_e = 0.883(0.837)(0.5)(470) = 174 \text{ MPa}$$

$$D/d = 45/40 = 1.125, \ r/d = 2/40 = 0.05.$$

From Figs. A-15-8 and A-15-9, $K_t = 1.9$ and $K_{ts} = 1.32$.

From Figs. 6-20 and 6-21, $q = 0.75$ and $q_s = 0.92$,

$$K_f = 1 + 0.75(1.9 - 1) = 1.68, \text{ and } K_{fs} = 1 + 0.92(1.32 - 1) = 1.29.$$

From Eq. (7-11), with $M_m = T_a = 0$,

$$\frac{1}{n} = \frac{16}{\pi(0.04)^3}\left\{4\left[\frac{1.68(326.67)}{174(10^6)}\right]^2 + 3\left[\frac{1.29(107)}{390(10^6)}\right]^2\right\}^{1/2}$$

$$n = 1.98$$

At $x = 330$ mm: The von Mises stress is the highest but it comes from the steady torque only.

$$D/d = 30/20 = 1.5, \quad r/d = 2/20 = 0.1 \quad \Rightarrow \quad K_{ts} = 1.42,$$

$$q_s = 0.92 \quad \Rightarrow \quad K_{fs} = 1.39$$

$$\frac{1}{n} = \frac{16}{\pi(0.02)^3}\left(\sqrt{3}\right)\left[\frac{1.39(107)}{390(10^6)}\right]$$

$$n = 2.38$$

Check the other locations.

If worse-case is at $x = 210$ mm, the changes discussed for the slope criterion will improve the strength issue.

7-11 and 7-12 With these design tasks each student will travel different paths and almost all details will differ. The important points are

- The student gets a blank piece of paper, a statement of function, and some constraints–explicit and implied. At this point in the course, this is a good experience.
- It is a good preparation for the capstone design course.
- The adequacy of their design must be demonstrated and possibly include a designer's notebook.
- Many of the fundaments of the course, based on this text and this course, are useful. The student will find them useful and notice that he/she is doing it.
- Don't let the students create a time sink for themselves. Tell them how far you want them to go.

7-13 I used this task as a final exam when all of the students in the course had consistent test scores going into the final examination; it was my expectation that they would not change things much by taking the examination.

This problem is a learning experience. Following the task statement, the following guidance was added.

- Take the first half hour, resisting the temptation of putting pencil to paper, and decide what the problem really is.
- Take another twenty minutes to list several possible remedies.
- Pick one, and show your instructor how you would implement it.

The students' initial reaction is that he/she does not know much from the problem statement. Then, slowly the realization sets in that they do know some important things that the designer did not. They knew how it failed, where it failed, and that the design wasn't good enough; it was close, though.

Also, a fix at the bearing seat lead-in could transfer the problem to the shoulder fillet, and the problem may not be solved.

To many students' credit, they chose to keep the shaft geometry, and selected a new material to realize about twice the Brinell hardness.

7-14 In Eq. (7-24) set

$$I = \frac{\pi d^4}{64}, \quad A = \frac{\pi d^2}{4}$$

to obtain

$$\omega = \left(\frac{\pi}{l}\right)^2 \left(\frac{d}{4}\right)\sqrt{\frac{gE}{\gamma}} \tag{1}$$

or

$$d = \frac{4l^2\omega}{\pi^2}\sqrt{\frac{\gamma}{gE}} \tag{2}$$

(a) From Eq. (1) and Table A-5,

$$\omega = \left(\frac{\pi}{24}\right)^2 \left(\frac{1}{4}\right)\sqrt{\frac{386(30)(10^6)}{0.282}} = 868 \text{ rad/s} \quad \textit{Ans.}$$

(b) From Eq. (2),

$$d = \frac{4(24)^2(2)(868)}{\pi^2}\sqrt{\frac{0.282}{386(30)(10^6)}} = 2 \text{ in} \quad \textit{Ans.}$$

(c) From Eq. (2),

$$l\omega = \frac{\pi^2}{4}\frac{d}{l}\sqrt{\frac{gE}{\gamma}}$$

Since d/l is the same regardless of the scale.

$$l\omega = \text{constant} = 24(868) = 20\,832$$

$$\omega = \frac{20\,832}{12} = 1736 \text{ rad/s} \quad \textit{Ans.}$$

Thus the first critical speed doubles.

7-15 From Prob. 7-14, $\omega = 868$ rad/s

$$A = 0.7854 \text{ in}^2, \quad I = 0.04909 \text{ in}^4, \quad \gamma = 0.282 \text{ lbf/in}^3,$$

$$E = 30(10^6) \text{ psi}, \quad w = A\gamma l = 0.7854(0.282)(24) = 5.316 \text{ lbf}$$

One element:

$$\text{Eq. (7-24)}\quad \delta_{11} = \frac{12(12)(24^2 - 12^2 - 12^2)}{6(30)(10^6)(0.049\,09)(24)} = 1.956(10^{-4}) \text{ in/lbf}$$

$$y_1 = w_1\delta_{11} = 5.316(1.956)(10^{-4}) = 1.0398(10^{-3}) \text{ in}$$

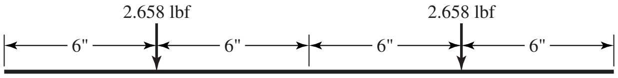
$$y_1^2 = 1.0812(10^{-6})$$

$$\sum wy = 5.316(1.0398)(10^{-3}) = 5.528(10^{-3})$$

$$\sum wy^2 = 5.316(1.0812)(10^{-6}) = 5.748(10^{-6})$$

$$\omega_1 = \sqrt{g\frac{\sum wy}{\sum wy^2}} = \sqrt{386\left[\frac{5.528(10^{-3})}{5.748(10^{-6})}\right]} = 609 \text{ rad/s} \quad (30\% \text{ low})$$

Two elements:



$$\delta_{11} = \delta_{22} = \frac{18(6)(24^2 - 18^2 - 6^2)}{6(30)(10^6)(0.049\,09)(24)} = 1.100(10^{-4}) \text{ in/lbf}$$

$$\delta_{12} = \delta_{21} = \frac{6(6)(24^2 - 6^2 - 6^2)}{6(30)(10^6)(0.049\,09)(24)} = 8.556(10^{-5}) \text{ in/lbf}$$

$$y_1 = w_1\delta_{11} + w_2\delta_{12} = 2.658(1.100)(10^{-4}) + 2.658(8.556)(10^{-5})$$
$$= 5.198(10^{-4}) \text{ in} = y_2,$$

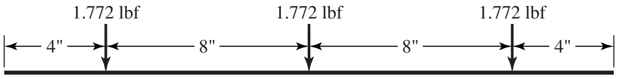
$$y_1^2 = y_2^2 = 2.702(10^{-7}) \text{ in}^2$$

$$\sum wy = 2(2.658)(5.198)(10^{-4}) = 2.763(10^{-3})$$

$$\sum wy^2 = 2(2.658)(2.702)(10^{-7}) = 1.436(10^{-6})$$

$$\omega_1 = \sqrt{386\left[\frac{2.763(10^{-3})}{1.436(10^{-6})}\right]} = 862 \;\text{ rad/s} \quad (0.7\% \text{ low})$$

Three elements:



$$\delta_{11} = \delta_{33} = \frac{20(4)(24^2 - 20^2 - 4^2)}{6(30)(10^6)(0.049\,09)(24)} = 6.036(10^{-5}) \text{ in/lbf}$$

$$\delta_{22} = \frac{12(12)(24^2 - 12^2 - 12^2)}{6(30)(10^6)(0.049\,09)(24)} = 1.956(10^{-4}) \text{ in/lbf}$$

$$\delta_{12} = \delta_{32} = \frac{12(4)(24^2 - 12^2 - 4^2)}{6(30)(10^6)(0.049\,09)(24)} = 9.416(10^{-5}) \text{ in/lbf}$$

$$\delta_{13} = \frac{4(4)(24^2 - 4^2 - 4^2)}{6(30)(10^6)(0.049\,09)(24)} = 4.104(10^{-5}) \text{ in/lbf}$$

$$y_1 = 1.772[6.036(10^{-5}) + 9.416(10^{-5}) + 4.104(10^{-5})] = 3.465(10^{-4})\text{ in}$$
$$y_2 = 1.772[9.416(10^{-5}) + 1.956(10^{-4}) + 9.416(10^{-5})] = 6.803(10^{-4})\text{ in}$$
$$y_3 = 1.772[4.104(10^{-5}) + 9.416(10^{-5}) + 6.036(10^{-5})] = 3.465(10^{-4})\text{ in}$$
$$\sum wy = 2.433(10^{-3}), \quad \sum wy^2 = 1.246(10^{-6})$$
$$\omega_1 = \sqrt{386\left[\frac{2.433(10^{-3})}{1.246(10^{-6})}\right]} = 868\text{ rad/s} \qquad \text{(same as in Prob. 7-14)}$$

The point was to show that convergence is rapid using a static deflection beam equation. The method works because:

- If a deflection curve is chosen which meets the boundary conditions of moment-free and deflection-free ends, and in this problem, of symmetry, the strain energy is not very sensitive to the equation used.
- Since the static bending equation is available, and meets the moment-free and deflection-free ends, it works.

7-16 **(a)** For two bodies, Eq. (7-26) is

$$\begin{vmatrix} (m_1\delta_{11} - 1/\omega^2) & m_2\delta_{12} \\ m_1\delta_{21} & (m_2\delta_{22} - 1/\omega^2) \end{vmatrix} = 0$$

Expanding the determinant yields,

$$\left(\frac{1}{\omega^2}\right)^2 - (m_1\delta_{11} + m_2\delta_{22})\left(\frac{1}{\omega_1^2}\right) + m_1m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21}) = 0 \qquad (1)$$

Eq. (1) has two roots $1/\omega_1^2$ and $1/\omega_2^2$. Thus

$$\left(\frac{1}{\omega^2} - \frac{1}{\omega_1^2}\right)\left(\frac{1}{\omega^2} - \frac{1}{\omega_2^2}\right) = 0$$

or,

$$\left(\frac{1}{\omega^2}\right)^2 + \left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}\right)\left(\frac{1}{\omega}\right)^2 + \left(\frac{1}{\omega_1^2}\right)\left(\frac{1}{\omega_2^2}\right) = 0 \qquad (2)$$

Equate the third terms of Eqs. (1) and (2), which must be identical.

$$\frac{1}{\omega_1^2}\frac{1}{\omega_2^2} = m_1m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21}) \quad \Rightarrow \quad \frac{1}{\omega_2^2} = \omega_1^2 m_1m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21})$$

and it follows that

$$\omega_2 = \frac{1}{\omega_1}\sqrt{\frac{g^2}{w_1w_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21})}} \quad Ans.$$

(b) In Ex. 7-5, Part (b) the first critical speed of the two-disk shaft ($w_1 = 35$ lbf, $w_2 = 55$ lbf) is $\omega_1 = 124.7$ rad/s. From part (a), using influence coefficients

$$\omega_2 = \frac{1}{124.7}\sqrt{\frac{386^2}{35(55)[2.061(3.534) - 2.234^2](10^{-8})}} = 466\text{ rad/s} \quad Ans.$$

7-17 In Eq. (7-22) the term $\sqrt{I/A}$ appears. For a hollow unform diameter shaft,

$$\sqrt{\frac{I}{A}} = \sqrt{\frac{\pi\left(d_o^4 - d_i^4\right)/64}{\pi\left(d_o^2 - d_i^2\right)/4}} = \sqrt{\frac{1}{16}\frac{\left(d_o^2 + d_i^2\right)\left(d_o^2 - d_i^2\right)}{d_o^2 - d_i^2}} = \frac{1}{4}\sqrt{d_o^2 + d_i^2}$$

This means that when a solid shaft is hollowed out, the critical speed increases beyond that of the solid shaft. By how much?

$$\frac{\frac{1}{4}\sqrt{d_o^2 + d_i^2}}{\frac{1}{4}\sqrt{d_o^2}} = \sqrt{1 + \left(\frac{d_i}{d_o}\right)^2}$$

The possible values of d_i are $0 \le d_i \le d_o$, so the range of critical speeds is

$$\omega_s\sqrt{1+0} \text{ to about } \omega_s\sqrt{1+1}$$

or from ω_s to $\sqrt{2}\omega_s$. *Ans.*

7-18 All steps will be modeled using singularity functions with a spreadsheet. Programming both loads will enable the user to first set the left load to 1, the right load to 0 and calculate δ_{11} and δ_{21}. Then setting left load to 0 and the right to 1 to get δ_{12} and δ_{22}. The spreadsheet shown on the next page shows the δ_{11} and δ_{21} calculation. Table for M/I vs x is easy to make. The equation for M/I is:

$$\begin{aligned} M/I = {} & D13x + C15\langle x-1\rangle^0 + E15\langle x-1\rangle^1 + E17\langle x-2\rangle^1 \\ & + C19\langle x-9\rangle^0 + E19\langle x-9\rangle^1 + E21\langle x-14\rangle^1 \\ & + C23\langle x-15\rangle^0 + E23\langle x-15\rangle^1 \end{aligned}$$

Integrating twice gives the equation for Ey. Boundary conditions $y = 0$ at $x = 0$ and at $x = 16$ inches provide integration constants ($C_2 = 0$). Substitution back into the deflection equation at $x = 2$, 14 inches provides the δ's. The results are: $\delta_{11} - 2.917(10^{-7})$, $\delta_{12} = \delta_{21} = 1.627(10^{-7})$, $\delta_{22} = 2.231(10^{-7})$. This can be verified by finite element analysis.

$$y_1 = 20(2.917)(10^{-7}) + 35(1.627)(10^{-7}) = 1.153(10^{-5})$$

$$y_2 = 20(1.627)(10^{-7}) + 35(2.231)(10^{-7}) = 1.106(10^{-5})$$

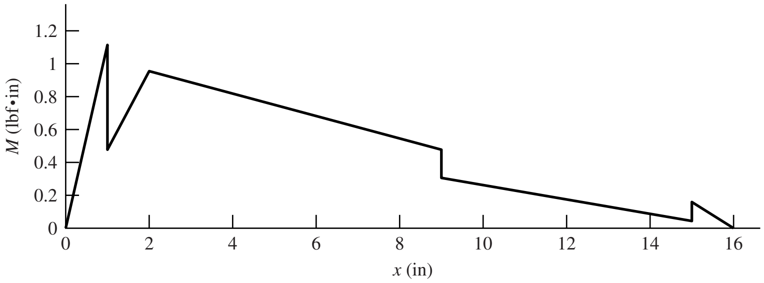
$$y_1^2 = 1.329(10^{-10}), \quad y_2^2 = 1.224(10^{-10})$$

$$\sum wy = 6.177(10^{-4}), \quad \sum wy^2 = 6.942(10^{-9})$$

Neglecting the shaft, Eq. (7-23) gives

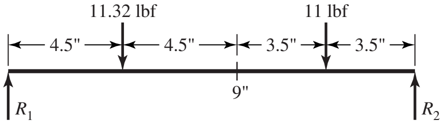
$$\omega_1 = \sqrt{386\left[\frac{6.177(10^{-4})}{6.942(10^{-9})}\right]} = 5860 \text{ rad/s} \quad \text{or} \quad 55\,970 \text{ rev/min} \quad \textit{Ans.}$$

	A	B	C	D	E	F	G	H	I
1		$F_1 = 1$		$F_2 = 0$	$R_1 = 0.875$ (left reaction)				
2									
3	x	M		$I_1 = I_4 = 0.7854$					
4	0	0		$I_2 = 1.833$					
5	1	0.875		$I_3 = 2.861$					
6	2	1.75							
7	9	0.875							
8	14	0.25							
9	15	0.125							
10	16	0							
11									
12	x	M/I	step	slope	Δ slope				
13	0	0		1.114 082					
14	1	1.114 082							
15	1	0.477 36	−0.636 722 477	0.477 36	−0.636 72				
16	2	0.954 719							
17	2	0.954 719	0	−0.068 19	−0.545 55				
18	9	0.477 36							
19	9	0.305 837	−0.171 522 4	−0.043 69	0.024 503				
20	14	0.087 382							
21	14	0.087 382	0	−0.043 69	0				
22	15	0.043 691							
23	15	0.159 155	0.115 463 554	−0.159 15	−0.115 46				
24	16	0							
25									
26		$C_1 = -4.906\,001\,093$							
27									
28									
29		$\delta_{11} =$ 2.91701E-07							
30		$\delta_{21} =$ 1.6266E-07							



Repeat for $F_1 = 0$ and $F_2 = 1$.

Modeling the shaft separately using 2 elements gives approximately



The spreadsheet can be easily modified to give

$$\delta_{11} = 9.605(10^{-7}), \quad \delta_{12} = \delta_{21} = 5.718(10^{-7}), \quad \delta_{22} = 5.472(10^{-7})$$

$$y_1 = 1.716(10^{-5}), \quad y_2 = 1.249(10^{-5}), \quad y_1^2 = 2.946(10^{-10}),$$

$$y_2^2 = 1.561(10^{-10}), \quad \sum wy = 3.316(10^{-4}), \quad \sum wy^2 = 5.052(10^{-9})$$

$$\omega_1 = \sqrt{386\left[\frac{3.316(10^{-4})}{5.052(10^{-9})}\right]} = 5034 \text{ rad/s} \quad \textit{Ans.}$$

A finite element model of the exact shaft gives $\omega_1 = 5340$ rad/s. The simple model is 5.7% low.

Combination Using Dunkerley's equation, Eq. (7-32):

$$\frac{1}{\omega_1^2} = \frac{1}{5860^2} + \frac{1}{5034^2} \quad \Rightarrow \quad 3819 \text{ rad/s} \quad \textit{Ans.}$$

7-19 We must not let the basis of the stress concentration factor, as presented, impose a viewpoint on the designer. Table A-16 shows K_{ts} as a decreasing monotonic as a function of a/D. All is not what it seems.

Let us change the basis for data presentation to the full section rather than the net section.

$$\tau = K_{ts}\tau_0 = K'_{ts}\tau'_0$$

$$K_{ts} = \frac{32T}{\pi AD^3} = K'_{ts}\left(\frac{32T}{\pi D^3}\right)$$

Therefore

$$K'_{ts} = \frac{K_{ts}}{A}$$

Form a table:

(a/D)	A	K_{ts}	K'_{ts}
0.050	0.95	1.77	1.86
0.075	0.93	1.71	1.84
0.100	0.92	1.68	1.83 ← minimum
0.125	0.89	1.64	1.84
0.150	0.87	1.62	1.86
0.175	0.85	1.60	1.88
0.200	0.83	1.58	1.90

K'_{ts} has the following attributes:

- It exhibits a minimum;
- It changes little over a wide range;
- Its minimum is a stationary point minimum at $a/D \doteq 0.100$;
- Our knowledge of the minima location is

$$0.075 \le (a/D) \le 0.125$$

We can form a design rule: in torsion, the pin diameter should be about 1/10 of the shaft diameter, for greatest shaft capacity. However, it is not catastrophic if one forgets the rule.

7-20 Choose 15 mm as basic size, D, d. Table 7-9: fit is designated as 15H7/h6. From Table A-11, the tolerance grades are $\Delta D = 0.018$ mm and $\Delta d = 0.011$ mm.

Hole: Eq. (7-36)

$$D_{\text{max}} = D + \Delta D = 15 + 0.018 = 15.018 \text{ mm} \quad \textit{Ans.}$$
$$D_{\text{min}} = D = 15.000 \text{ mm} \quad \textit{Ans.}$$

Shaft: From Table A-12, fundamental deviation $\delta_F = 0$. From Eq. (2-39)

$$d_{\text{max}} = d + \delta_F = 15.000 + 0 = 15.000 \text{ mm} \quad \textit{Ans.}$$
$$d_{\text{min}} = d + \delta_R - \Delta d = 15.000 + 0 - 0.011 = 14.989 \text{ mm} \quad \textit{Ans.}$$

7-21 Choose 45 mm as basic size. Table 7-9 designates fit as 45H7/s6. From Table A-11, the tolerance grades are $\Delta D = 0.025$ mm and $\Delta d = 0.016$ mm

Hole: Eq. (7-36)

$$D_{\text{max}} = D + \Delta D = 45.000 + 0.025 = 45.025 \text{ mm} \quad \textit{Ans.}$$
$$D_{\text{min}} = D = 45.000 \text{ mm} \quad \textit{Ans.}$$

Shaft: From Table A-12, fundamental deviation $\delta_F = +0.043$ mm. From Eq. (7-38)

$$d_{\text{min}} = d + \delta_F = 45.000 + 0.043 = 45.043 \text{ mm} \quad \textit{Ans.}$$
$$d_{\text{max}} = d + \delta_F + \Delta d = 45.000 + 0.043 + 0.016 = 45.059 \text{ mm} \quad \textit{Ans.}$$

7-22 Choose 50 mm as basic size. From Table 7-9 fit is 50H7/g6. From Table A-11, the tolerance grades are $\Delta D = 0.025$ mm and $\Delta d = 0.016$ mm.

Hole:

$$D_{\text{max}} = D + \Delta D = 50 + 0.025 = 50.025 \text{ mm} \quad \textit{Ans.}$$
$$D_{\text{min}} = D = 50.000 \text{ mm} \quad \textit{Ans.}$$

Shaft: From Table A-12 fundamental deviation = −0.009 mm

$$d_{\text{max}} = d + \delta_F = 50.000 + (-0.009) = 49.991 \text{ mm} \quad \textit{Ans.}$$
$$\begin{aligned} d_{\text{min}} &= d + \delta_F - \Delta d \\ &= 50.000 + (-0.009) - 0.016 \\ &= 49.975 \text{ mm} \end{aligned}$$

7-23 Choose the basic size as 1.000 in. From Table 7-9, for 1.0 in, the fit is H8/f7. From Table A-13, the tolerance grades are $\Delta D = 0.0013$ in and $\Delta d = 0.0008$ in.

Hole:

$$D_{\max} = D + (\Delta D)_{\text{hole}} = 1.000 + 0.0013 = 1.0013 \text{ in} \quad \textit{Ans.}$$

$$D_{\min} = D = 1.0000 \text{ in} \quad \textit{Ans.}$$

Shaft: From Table A-14: Fundamental deviation $= -0.0008$ in

$$d_{\max} = d + \delta_F = 1.0000 + (-0.0008) = 0.9992 \text{ in} \quad \textit{Ans.}$$

$$d_{\min} = d + \delta_F - \Delta d = 1.0000 + (-0.0008) - 0.0008 = 0.9984 \text{ in} \quad \textit{Ans.}$$

Alternatively,

$$d_{\min} = d_{\max} - \Delta d = 0.9992 - 0.0008 = 0.9984 \text{ in.} \quad \textit{Ans.}$$

7-24 **(a)** Basic size is $D = d = 1.5$ in.

Table 7-9: H7/s6 is specified for medium drive fit.

Table A-13: Tolerance grades are $\Delta D = 0.001$ in and $\Delta d = 0.0006$ in.

Table A-14: Fundamental deviation is $\delta_F = 0.0017$ in.

Eq. (7-36): $D_{\max} = D + \Delta D = 1.501$ in *Ans.*

$D_{\min} = D = 1.500$ in *Ans.*

Eq. (7-37): $d_{\max} = d + \delta_F + \Delta d = 1.5 + 0.0017 + 0.0006 = 1.5023$ in *Ans.*

Eq. (7-38): $d_{\min} = d + \delta_F = 1.5 + 0.0017 + 1.5017$ in *Ans.*

(b) Eq. (7-42): $\delta_{\min} = d_{\min} - D_{\max} = 1.5017 - 1.501 = 0.0007$ in

Eq. (7-43): $\delta_{\max} = d_{\max} - D_{\min} = 1.5023 - 1.500 = 0.0023$ in

Eq. (7-40):

$$p_{\max} = \frac{E\delta_{\max}}{2d^3}\left[\frac{(d_o^2 - d^2)(d^2 - d_i^2)}{d_o^2 - d_i^2}\right]$$

$$= \frac{(30)(10^6)(0.0023)}{2(1.5)^3}\left[\frac{(2.5^2 - 1.5^2)(1.5^2 - 0)}{2.5^2 - 0}\right] = 14\,720 \text{ psi} \quad \textit{Ans.}$$

$$p_{\min} = \frac{E\delta_{\min}}{2d^3}\left[\frac{\left(d_o^2 - d^2\right)\left(d^2 - d_i^2\right)}{d_o^2 - d_i^2}\right]$$

$$= \frac{(30)(10^6)(0.0007)}{2(1.5)^3}\left[\frac{(2.5^2 - 1.5^2)(1.5^2 - 0)}{2.5^2 - 0}\right] = 4480 \text{ psi} \quad \textit{Ans.}$$

(c) For the shaft:

Eq. (7-44): $\sigma_{t,\text{shaft}} = -p = -14\,720$ psi

Eq. (7-46): $\sigma_{r,\text{shaft}} = -p = -14\,720$ psi

Eq. (5-13):

$$\sigma' = (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)^{1/2}$$

$$= [(-14\,720)^2 - (-14\,720)(-14\,720) + (-14\,720)^2]^{1/2}$$

$$= 14\,720 \text{ psi}$$

$$n = S_y/\sigma' = 57\,000/14\,720 = 3.9 \quad \textit{Ans.}$$

For the hub:

Eq. (7-45): $$\sigma_{t,\text{hub}} = p\frac{d_o^2 + d^2}{d_o^2 - d^2} = (14\,720)\left(\frac{2.5^2 + 1.5^2}{2.5^2 - 1.5^2}\right) = 31\,280 \text{ psi}$$

Eq. (7-46): $$\sigma_{r,\text{hub}} = -p = -14\,720 \text{ psi}$$

Eq. (5-13): $$\sigma' = (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)^{1/2}$$

$$= [(31\,280)^2 - (31\,280)(-14\,720) + (-14\,720)^2]^{1/2} = 40\,689 \text{ psi}$$

$$n = S_y/\sigma' = 85\,000/40\,689 = 2.1 \quad \textit{Ans.}$$

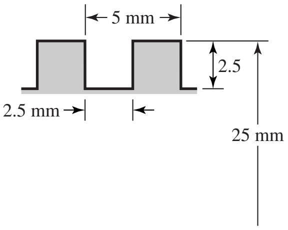
(d) Eq. (7-49) $$T = (\pi/2) f p_{\min} l d^2$$

$$= (\pi/2)(0.3)(4480)(2)(1.5)^2 = 9500 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

Chapter 8

8-1

(a)



Thread depth = 2.5 mm *Ans.*

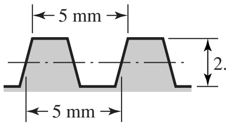
Width = 2.5 mm *Ans.*

$$d_m = 25 - 1.25 - 1.25 = 22.5 \text{ mm}$$

$$d_r = 25 - 5 = 20 \text{ mm}$$

$$l = p = 5 \text{ mm} \quad \textit{Ans.}$$

(b)



Thread depth = 2.5 mm *Ans.*

Width at pitch line = 2.5 mm *Ans.*

$$d_m = 22.5 \text{ mm}$$

$$d_r = 20 \text{ mm}$$

$$l = p = 5 \text{ mm} \quad \textit{Ans.}$$

8-2 From Table 8-1,

$$d_r = d - 1.226\,869p$$

$$d_m = d - 0.649\,519p$$

$$\bar{d} = \frac{d - 1.226\,869p + d - 0.649\,519p}{2} = d - 0.938\,194p$$

$$A_t = \frac{\pi \bar{d}^2}{4} = \frac{\pi}{4}(d - 0.938\,194p)^2 \quad \textit{Ans.}$$

8-3 From Eq. (*c*) of Sec. 8-2,

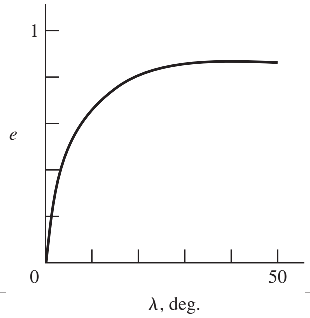
$$P = F\frac{\tan\lambda + f}{1 - f\tan\lambda}$$

$$T = \frac{Pd_m}{2} = \frac{Fd_m}{2}\,\frac{\tan\lambda + f}{1 - f\tan\lambda}$$

$$e = \frac{T_0}{T} = \frac{Fl/(2\pi)}{Fd_m/2}\,\frac{1 - f\tan\lambda}{\tan\lambda + f} = \tan\lambda\frac{1 - f\tan\lambda}{\tan\lambda + f} \quad \textit{Ans.}$$

Using $f = 0.08$, form a table and plot the efficiency curve.

λ, deg.	e
0	0
10	0.678
20	0.796
30	0.838
40	0.8517
45	0.8519



8-4 Given $F = 6$ kN, $l = 5$ mm, and $d_m = 22.5$ mm, the torque required to raise the load is found using Eqs. (8-1) and (8-6)

$$T_R = \frac{6(22.5)}{2}\left[\frac{5 + \pi(0.08)(22.5)}{\pi(22.5) - 0.08(5)}\right] + \frac{6(0.05)(40)}{2}$$

$$= 10.23 + 6 = 16.23 \text{ N} \cdot \text{m} \quad Ans.$$

The torque required to lower the load, from Eqs. (8-2) and (8-6) is

$$T_L = \frac{6(22.5)}{2}\left[\frac{\pi(0.08)22.5 - 5}{\pi(22.5) + 0.08(5)}\right] + \frac{6(0.05)(40)}{2}$$

$$= 0.622 + 6 = 6.622 \text{ N} \cdot \text{m} \quad Ans.$$

Since T_L is positive, the thread is self-locking. The efficiency is

Eq. (8-4): $$e = \frac{6(5)}{2\pi(16.23)} = 0.294 \quad Ans.$$

8-5 Collar (thrust) bearings, at the bottom of the screws, must bear on the collars. The bottom segment of the screws must be in compression. Where as tension specimens and their grips must be in tension. Both screws must be of the same-hand threads.

8-6 Screws rotate at an angular rate of

$$n = \frac{1720}{75} = 22.9 \text{ rev/min}$$

(a) The lead is 0.5 in, so the linear speed of the press head is

$$V = 22.9(0.5) = 11.5 \text{ in/min} \quad Ans.$$

(b) $F = 2500$ lbf/screw

$$d_m = 3 - 0.25 = 2.75 \text{ in}$$

$$\sec\alpha = 1/\cos(29/2) = 1.033$$

Eq. (8-5):

$$T_R = \frac{2500(2.75)}{2}\left(\frac{0.5 + \pi(0.05)(2.75)(1.033)}{\pi(2.75) - 0.5(0.05)(1.033)}\right) = 377.6 \text{ lbf} \cdot \text{in}$$

Eq. (8-6):

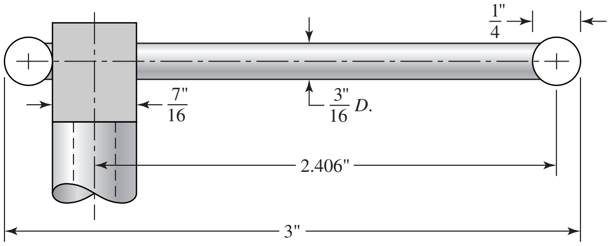
$$T_c = 2500(0.06)(5/2) = 375 \text{ lbf} \cdot \text{in}$$

$$T_{\text{total}} = 377.6 + 375 = 753 \text{ lbf} \cdot \text{in/screw}$$

$$T_{\text{motor}} = \frac{753(2)}{75(0.95)} = 21.1 \text{ lbf} \cdot \text{in}$$

$$H = \frac{Tn}{63\,025} = \frac{21.1(1720)}{63\,025} = 0.58 \text{ hp} \quad Ans.$$

8-7 The force F is perpendicular to the paper.



$$L = 3 - \frac{1}{8} - \frac{1}{4} - \frac{7}{32} = 2.406 \text{ in}$$

$$T = 2.406F$$

$$M = \left(L - \frac{7}{32}\right)F = \left(2.406 - \frac{7}{32}\right)F = 2.188F$$

$$S_y = 41 \text{ kpsi}$$

$$\sigma = S_y = \frac{32M}{\pi d^3} = \frac{32(2.188)F}{\pi(0.1875)^3} = 41\,000$$

$$F = 12.13 \text{ lbf}$$

$$T = 2.406(12.13) = 29.2 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

(b) Eq. (8-5), $2\alpha = 60^\circ$, $l = 1/14 = 0.0714$ in, $f = 0.075$, $\sec\alpha = 1.155$, $p = 1/14$ in

$$d_m = \frac{7}{16} - 0.649\,519\left(\frac{1}{14}\right) = 0.3911 \text{ in}$$

$$T_R = \frac{F_{\text{clamp}}(0.3911)}{2}\left(\frac{\text{Num}}{\text{Den}}\right)$$

$$\text{Num} = 0.0714 + \pi(0.075)(0.3911)(1.155)$$

$$\text{Den} = \pi(0.3911) - 0.075(0.0714)(1.155)$$

$$T = 0.028\,45F_{\text{clamp}}$$

$$F_{\text{clamp}} = \frac{T}{0.028\,45} = \frac{29.2}{0.028\,45} = 1030 \text{ lbf} \quad \textit{Ans.}$$

(c) The column has one end fixed and the other end pivoted. Base decision on the mean diameter column. Input: $C = 1.2$, $D = 0.391$ in, $S_y = 41$ kpsi, $E = 30(10^6)$ psi, $L = 4.1875$ in, $k = D/4 = 0.097\,75$ in, $L/k = 42.8$.

For this J. B. Johnson column, the critical load represents the limiting clamping force for bucking. Thus, $F_{\text{clamp}} = P_{\text{cr}} = 4663$ lbf.

(d) This is a subject for class discussion.

8-8

$$T = 6(2.75) = 16.5 \text{ lbf} \cdot \text{in}$$

$$d_m = \frac{5}{8} - \frac{1}{12} = 0.5417 \text{ in}$$

$$l = \frac{1}{6} = 0.1667 \text{ in}, \quad \alpha = \frac{29^\circ}{2} = 14.5^\circ, \quad \sec 14.5^\circ = 1.033$$

Eq. (8-5): $$T = 0.5417(F/2)\left[\frac{0.1667 + \pi(0.15)(0.5417)(1.033)}{\pi(0.5417) - 0.15(0.1667)(1.033)}\right] = 0.0696F$$

Eq. (8-6): $$T_c = 0.15(7/16)(F/2) = 0.032\,81F$$

$$T_{\text{total}} = (0.0696 + 0.0328)F = 0.1024F$$

$$F = \frac{16.5}{0.1024} = 161 \text{ lbf} \quad \textit{Ans.}$$

8-9 $d_m = 40 - 3 = 37$ mm, $l = 2(6) = 12$ mm

From Eq. (8-1) and Eq. (8-6)

$$T_R = \frac{10(37)}{2}\left[\frac{12 + \pi(0.10)(37)}{\pi(37) - 0.10(12)}\right] + \frac{10(0.15)(60)}{2}$$

$$= 38.0 + 45 = 83.0 \text{ N} \cdot \text{m}$$

Since $n = V/l = 48/12 = 4$ rev/s

$$\omega = 2\pi n = 2\pi(4) = 8\pi \text{ rad/s}$$

so the power is

$$H = T\omega = 83.0(8\pi) = 2086\ W \quad \textit{Ans.}$$

8-10

(a) $d_m = 36 - 3 = 33$ mm, $l = p = 6$ mm

From Eqs. (8-1) and (8-6)

$$T = \frac{33F}{2}\left[\frac{6 + \pi(0.14)(33)}{\pi(33) - 0.14(6)}\right] + \frac{0.09(90)F}{2}$$

$$= (3.292 + 4.050)F = 7.34F \text{ N} \cdot \text{m}$$

$$\omega = 2\pi n = 2\pi(1) = 2\pi \text{ rad/s}$$

$$H = T\omega$$

$$T = \frac{H}{\omega} = \frac{3000}{2\pi} = 477 \text{ N} \cdot \text{m}$$

$$F = \frac{477}{7.34} = 65.0 \text{ kN} \quad \textit{Ans.}$$

(b) $e = \dfrac{Fl}{2\pi T} = \dfrac{65.0(6)}{2\pi(477)} = 0.130 \quad \textit{Ans.}$

8-11

(a) $L_T = 2D + \dfrac{1}{4} = 2(0.5) + 0.25 = 1.25$ in *Ans.*

(b) From Table A-32 the washer thickness is 0.109 in. Thus,

$$l = 0.5 + 0.5 + 0.109 = 1.109 \text{ in} \quad \textit{Ans.}$$

(c) From Table A-31, $H = \dfrac{7}{16} = 0.4375$ in *Ans.*

(d) $l + H = 1.109 + 0.4375 = 1.5465$ in
This would be rounded to 1.75 in per Table A-17. The bolt is long enough. *Ans.*

(e) $l_d = L - L_T = 1.75 - 1.25 = 0.500$ in *Ans.*

$l_t = l - l_d = 1.109 - 0.500 = 0.609$ in *Ans.*

These lengths are needed to estimate bolt spring rate k_b.

Note: In an analysis problem, you need not know the fastener's length at the outset, although you can certainly check, if appropriate.

8-12

(a) $L_T = 2D + 6 = 2(14) + 6 = 34$ mm *Ans.*

(b) From Table A-33, the maximum washer thickness is 3.5 mm. Thus, the grip is, $l = 14 + 14 + 3.5 = 31.5$ mm *Ans.*

(c) From Table A-31, $H = 12.8$ mm

(d) $l + H = 31.5 + 12.8 = 44.3$ mm
Adding one or two threads and rounding up to $L = 50$ mm. The bolt is long enough. *Ans.*

(e) $l_d = L - L_T = 50 - 34 = 16$ mm *Ans.*

$l_t = l - l_d = 31.5 - 16 = 15.5$ mm *Ans.*
These lengths are needed to estimate the bolt spring rate k_b.

8-13

(a) $L_T = 2D + \dfrac{1}{4} = 2(0.5) + 0.25 = 1.25$ in *Ans.*

(b) $l' > h + \dfrac{d}{2} = t_1 + \dfrac{d}{2} = 0.875 + \dfrac{0.5}{2} = 1.125$ in *Ans.*

(c) $L > h + 1.5d = t_1 + 1.5d = 0.875 + 1.5(0.5) = 1.625$ in
From Table A-17, this rounds to 1.75 in. The cap screw is long enough. *Ans.*

(d) $l_d = L - L_T = 1.75 - 1.25 = 0.500$ in *Ans.*

$l_t = l' - l_d = 1.125 - 0.5 = 0.625$ in *Ans.*

8-14

(a) $L_T = 2(12) + 6 = 30$ mm *Ans.*

(b) $l' = h + \dfrac{d}{2} = t_1 + \dfrac{d}{2} = 20 + \dfrac{12}{2} = 26$ mm *Ans.*

(c) $L > h + 1.5d = t_1 + 1.5d = 20 + 1.5(12) = 38$ mm
This rounds to 40 mm (Table A-17). The fastener is long enough. *Ans.*

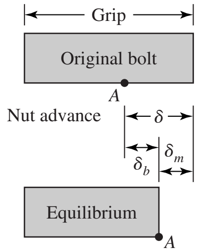
(d) $l_d = L - L_T = 40 - 30 = 10$ mm *Ans.*

$l_T = l' - l_d = 26 - 10 = 16$ mm *Ans.*

8-15

(a)
$$A_d = 0.7854(0.75)^2 = 0.442 \text{ in}^2$$
$$A_{\text{tube}} = 0.7854(1.125^2 - 0.75^2) = 0.552 \text{ in}^2$$
$$k_b = \frac{A_d E}{\text{grip}} = \frac{0.442(30)(10^6)}{13} = 1.02(10^6) \text{ lbf/in} \quad \textit{Ans.}$$
$$k_m = \frac{A_{\text{tube}} E}{13} = \frac{0.552(30)(10^6)}{13} = 1.27(10^6) \text{ lbf/in} \quad \textit{Ans.}$$
$$C = \frac{1.02}{1.02 + 1.27} = 0.445 \quad \textit{Ans.}$$

(b)



$$\delta = \frac{1}{16} \cdot \frac{1}{3} = \frac{1}{48} = 0.020\,83 \text{ in}$$
$$|\delta_b| = \left(\frac{|P|l}{AE}\right)_b = \frac{(13 - 0.020\,83)}{0.442(30)(10^6)}|P| = 9.79(10^{-7})|P| \text{ in}$$
$$|\delta_m| = \left(\frac{|P|l}{AE}\right)_m = \frac{|P|(13)}{0.552(30)(10^6)} = 7.85(10^{-7})|P| \text{ in}$$
$$|\delta_b| + |\delta_m| = \delta = 0.020\,83$$
$$9.79(10^{-7})|P| + 7.85(10^{-7})|P| = 0.020\,83$$
$$F_i = |P| = \frac{0.020\,83}{9.79(10^{-7}) + 7.85(10^{-7})} = 11\,810 \text{ lbf} \quad \textit{Ans.}$$

(c) At opening load P_0
$$9.79(10^{-7})P_0 = 0.020\,83$$
$$P_0 = \frac{0.020\,83}{9.79(10^{-7})} = 21\,280 \text{ lbf} \quad \textit{Ans.}$$

As a check use $F_i = (1 - C)P_0$
$$P_0 = \frac{F_i}{1 - C} = \frac{11\,810}{1 - 0.445} = 21\,280 \text{ lbf}$$

8-16 The movement is known at one location when the nut is free to turn
$$\delta = pt = t/N$$
Letting N_t represent the turn of the nut from snug tight, $N_t = \theta/360^\circ$ and $\delta = N_t/N$. The elongation of the bolt δ_b is
$$\delta_b = \frac{F_i}{k_b}$$
The advance of the nut along the bolt is the algebraic sum of $|\delta_b|$ and $|\delta_m|$

$$|\delta_b| + |\delta_m| = \frac{N_t}{N}$$

$$\frac{F_i}{k_b} + \frac{F_i}{k_m} = \frac{N_t}{N}$$

$$N_t = NF_i\left[\frac{1}{k_b} + \frac{1}{k_m}\right] = \left(\frac{k_b + k_m}{k_b k_m}\right)F_i N = \frac{\theta}{360^\circ} \quad \textit{Ans.}$$

As a check invert Prob. 8-15. What Turn-of-Nut will induce $F_i = 11\,808$ lbf?

$$N_t = 16(11\,808)\left(\frac{1}{1.02(10^6)} + \frac{1}{1.27(10^6)}\right)$$

$$= 0.334 \text{ turns} \doteq 1/3 \text{ turn} \quad \text{(checks)}$$

The relationship between the Turn-of-Nut method and the Torque Wrench method is as follows.

$$N_t = \left(\frac{k_b + k_m}{k_b k_m}\right)F_i N \qquad \text{(Turn-of-Nut)}$$

$$T = KF_i d \qquad \text{(Torque Wrench)}$$

Eliminate F_i

$$N_t = \left(\frac{k_b + k_m}{k_b k_m}\right)\frac{NT}{Kd} = \frac{\theta}{360^\circ} \quad \textit{Ans.}$$

8-17

(a) From Ex. 8-4, $F_i = 14.4$ kip, $k_b = 5.21(10^6)$ lbf/in, $k_m = 8.95(10^6)$ lbf/in

Eq. (8-27): $\quad T = kF_i d = 0.2(14.4)(10^3)(5/8) = 1800 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$

From Prob. 8-16,

$$t = NF_i\left(\frac{1}{k_b} + \frac{1}{k_m}\right) = 16(14.4)(10^3)\left[\frac{1}{5.21(10^6)} + \frac{1}{8.95(10^6)}\right]$$

$$= 0.132 \text{ turns} = 47.5^\circ \quad \textit{Ans.}$$

Bolt group is $(1.5)/(5/8) = 2.4$ diameters. Answer is lower than RB&W recommendations.

(b) From Ex. 8-5, $F_i = 14.4$ kip, $k_b = 6.78$ Mlbf/in, and $k_m = 17.4$ Mlbf/in

$$T = 0.2(14.4)(10^3)(5/8) = 1800 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$t = 11(14.4)(10^3)\left[\frac{1}{6.78(10^6)} + \frac{1}{17.4(10^6)}\right]$$

$$= 0.0325 = 11.7^\circ \quad \textit{Ans.} \quad \text{Again lower than RB\&W.}$$

8-18 From Eq. (8-22) for the conical frusta, with $d/l = 0.5$

$$\left.\frac{k_m}{Ed}\right|_{(d/l)=0.5} = \frac{0.5774\pi}{2\ln\{5[0.5774 + 0.5(0.5)]/[0.5774 + 2.5(0.5)]\}} = 1.11$$

Eq. (8-23), from the Wileman *et al.* finite element study, using the general expression,

$$\left.\frac{k_m}{Ed}\right|_{(d/l)=0.5} = 0.789\,52\exp[0.629\,14(0.5)] = 1.08$$

8-19 For cast iron, from Table 8-8: $A = 0.778\,71$, $B = 0.616\,16$, $E = 14.5$ Mpsi

$$k_m = 14.5(10^6)(0.625)(0.778\,71)\exp\left(0.616\,16\frac{0.625}{1.5}\right) = 9.12(10^6)\text{ lbf/in}$$

This member's spring rate applies to both members. We need k_m for the upper member which represents half of the joint.

$$k_{ci} = 2k_m = 2[9.12(10^6)] = 18.24(10^6)\text{ lbf/in}$$

For steel from Table 8-8: $A = 0.787\,15$, $B = 0.628\,73$, $E = 30$ Mpsi

$$k_m = 30(10^6)(0.625)(0.787\,15)\exp\left(0.628\,73\frac{0.625}{1.5}\right) = 19.18(10^6)\text{ lbf/in}$$

$$k_{\text{steel}} = 2k_m = 2(19.18)(10^6) = 38.36(10^6)\text{ lbf/in}$$

For springs in series

$$\frac{1}{k_m} = \frac{1}{k_{ci}} + \frac{1}{k_{\text{steel}}} = \frac{1}{18.24(10^6)} + \frac{1}{38.36(10^6)}$$

$$k_m = 12.4(10^6)\text{ lbf/in} \quad \textit{Ans.}$$

8-20 The external tensile load per bolt is

$$P = \frac{1}{10}\left(\frac{\pi}{4}\right)(150)^2(6)(10^{-3}) = 10.6\text{ kN}$$

Also, $l = 40$ mm and from Table A-31, for $d = 12$ mm, $H = 10.8$ mm. No washer is specified.

$$L_T = 2D + 6 = 2(12) + 6 = 30\text{ mm}$$

$$l + H = 40 + 10.8 = 50.8\text{ mm}$$

Table A-17: $L = 60$ mm

$$l_d = 60 - 30 = 30\text{ mm}$$

$$l_t = 45 - 30 = 15\text{ mm}$$

$$A_d = \frac{\pi(12)^2}{4} = 113\text{ mm}^2$$

Table 8-1: $A_t = 84.3\text{ mm}^2$

Eq. (8-17):

$$k_b = \frac{113(84.3)(207)}{113(15) + 84.3(30)} = 466.8\text{ MN/m}$$

Steel: Using Eq. (8-23) for $A = 0.787\,15$, $B = 0.628\,73$ and $E = 207$ GPa

Eq. (8-23): $k_m = 207(12)(0.787\,15)\exp[(0.628\,73)(12/40)] = 2361$ MN/m

$$k_s = 2k_m = 4722 \text{ MN/m}$$

Cast iron: $A = 0.778\,71$, $B = 0.616\,16$, $E = 100$ GPa

$$k_m = 100(12)(0.778\,71)\exp[(0.616\,16)(12/40)] = 1124 \text{ MN/m}$$

$$k_{ci} = 2k_m = 2248 \text{ MN/m}$$

$$\frac{1}{k_m} = \frac{1}{k_s} + \frac{1}{k_{ci}} \quad \Rightarrow \quad k_m = 1523 \text{ MN/m}$$

$$C = \frac{466.8}{466.8 + 1523} = 0.2346$$

Table 8-1: $A_t = 84.3$ mm^2, Table 8-11, $S_p = 600$ MPa

Eqs. (8-30) and (8-31): $F_i = 0.75(84.3)(600)(10^{-3}) = 37.9$ kN

Eq. (8-28):

$$n = \frac{S_p A_t - F_i}{CP} = \frac{600(10^{-3})(84.3) - 37.9}{0.2346(10.6)} = 5.1 \quad \textit{Ans.}$$

8-21 Computer programs will vary.

8-22 $D_3 = 150$ mm, $A = 100$ mm, $B = 200$ mm, $C = 300$ mm, $D = 20$ mm, $E = 25$ mm. ISO 8.8 bolts: $d = 12$ mm, $p = 1.75$ mm, coarse pitch of $p = 6$ MPa.

$$P = \frac{1}{10}\left(\frac{\pi}{4}\right)(150^2)(6)(10^{-3}) = 10.6 \text{ kN/bolt}$$

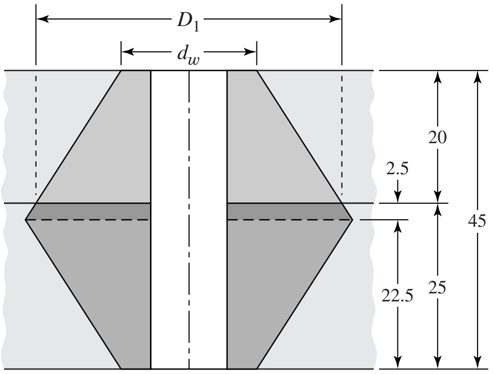
$$l = D + E = 20 + 25 = 45 \text{ mm}$$

$$L_T = 2D + 6 = 2(12) + 6 = 30 \text{ mm}$$

Table A-31: $H = 10.8$ mm

$$l + H = 45 + 10.8 = 55.8 \text{ mm}$$

Table A-17: $L = 60$ mm



$$l_d = 60 - 30 = 30 \text{ mm}, \quad l_t = 45 - 30 = 15 \text{ mm}, \quad A_d = \pi(12^2/4) = 113 \text{ mm}^2$$

Table 8-1: $A_t = 84.3$ mm^2

Eq. (8-17):

$$k_b = \frac{113(84.3)(207)}{113(15) + 84.3(30)} = 466.8 \text{ MN/m}$$

There are three frusta: $d_m = 1.5(12) = 18$ mm

$$D_1 = (20 \tan 30^\circ)2 + d_w = (20 \tan 30^\circ)2 + 18 = 41.09 \text{ mm}$$

Upper Frustum: $t = 20$ mm, $E = 207$ GPa, $D = 1.5(12) = 18$ mm

Eq. (8-20): $$k_1 = 4470 \text{ MN/m}$$

Central Frustum: $t = 2.5$ mm, $D = 41.09$ mm, $E = 100$ GPa (Table A-5) $\Rightarrow$ $k_2 =$ 52 230 MN/m

Lower Frustum: $t = 22.5$ mm, $E = 100$ GPa, $D = 18$ mm $\Rightarrow$ $k_3 = 2074$ MN/m

From Eq. (8-18): $k_m = [(1/4470) + (1/52\,230) + (1/2074)]^{-1} = 1379$ MN/m

Eq. (*e*), p. 421: $$C = \frac{466.8}{466.8 + 1379} = 0.253$$

Eqs. (8-30) and (8-31):

$$F_i = K F_p = K A_t S_p = 0.75(84.3)(600)(10^{-3}) = 37.9 \text{ kN}$$

Eq. (8-28): $$n = \frac{S_p A_t - F_i}{C P} = \frac{600(10^{-3})(84.3) - 37.9}{0.253(10.6)} = 4.73 \quad \textit{Ans.}$$

8-23 $P = \dfrac{1}{8}\left(\dfrac{\pi}{4}\right)(120^2)(6)(10^{-3}) = 8.48$ kN

From Fig. 8-21, $t_1 = h = 20$ mm and $t_2 = 25$ mm

$$l = 20 + 12/2 = 26 \text{ mm}$$

$$t = 0 \quad \text{(no washer)}, \qquad L_T = 2(12) + 6 = 30 \text{ mm}$$

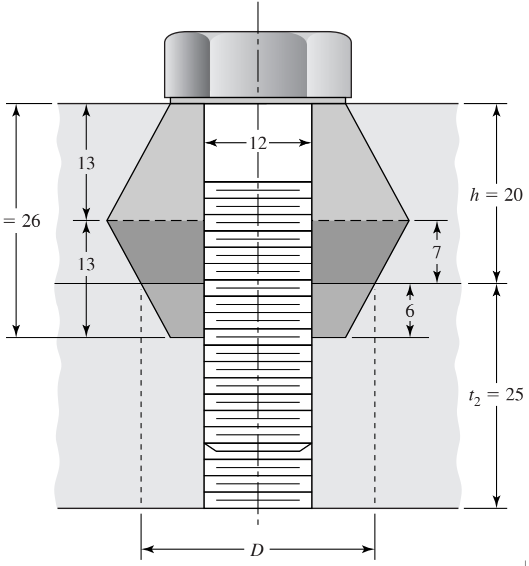
$$L > h + 1.5d = 20 + 1.5(12) = 38 \text{ mm}$$

Use 40 mm cap screws.

$$l_d = 40 - 30 = 10 \text{ mm}$$

$$l_t = l - l_d = 26 - 10 = 16 \text{ mm}$$

$$A_d = 113 \text{ mm}^2, \quad A_t = 84.3 \text{ mm}^2$$



Eq. (8-17):

$$k_b = \frac{113(84.3)(207)}{113(16) + 84.3(10)}$$

$$= 744 \text{ MN/m} \quad \textit{Ans.}$$

$$d_w = 1.5(12) = 18 \text{ mm}$$

$$D = 18 + 2(6)(\tan 30) = 24.9 \text{ mm}$$

From Eq. (8-20):

Top frustum: $D = 18, t = 13, E = 207$ GPa $\Rightarrow$ $k_1 = 5316$ MN/m

Mid-frustum: $t = 7, E = 207$ GPa, $D = 24.9$ mm $\Rightarrow$ $k_2 = 15\,620$ MN/m

Bottom frustum: $D = 18, t = 6, E = 100$ GPa $\Rightarrow$ $k_3 = 3887$ MN/m

$$k_m = \frac{1}{(1/5316) + (1/55\,620) + (1/3887)} = 2158 \text{ MN/m} \quad Ans.$$

$$C = \frac{744}{744 + 2158} = 0.256 \quad Ans.$$

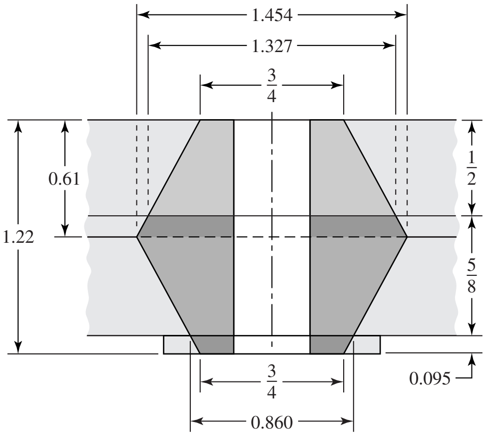
From Prob. 8-22, $F_i = 37.9$ kN

$$n = \frac{S_p A_t - F_i}{CP} = \frac{600(0.0843) - 37.9}{0.256(8.48)} = 5.84 \quad Ans.$$

8-24 Calculation of bolt stiffness:

$$H = 7/16 \text{ in}$$
$$L_T = 2(1/2) + 1/4 = 1\,1/4 \text{ in}$$
$$l = 1/2 + 5/8 + 0.095 = 1.22 \text{ in}$$
$$L > 1.125 + 7/16 + 0.095 = 1.66 \text{ in}$$

Use $L = 1.75$ in



$$l_d = L - L_T = 1.75 - 1.25 = 0.500 \text{ in}$$
$$l_t = 1.125 + 0.095 - 0.500 = 0.72 \text{ in}$$
$$A_d = \pi(0.50^2)/4 = 0.1963 \text{ in}^2$$
$$A_t = 0.1419 \text{ in}^2 \text{ (UNC)}$$
$$k_t = \frac{A_t E}{l_t} = \frac{0.1419(30)}{0.72} = 5.9125 \text{ Mlbf/in}$$
$$k_d = \frac{A_d E}{l_d} = \frac{0.1963(30)}{0.500} = 11.778 \text{ Mlbf/in}$$
$$k_b = \frac{1}{(1/5.9125) + (1/11.778)} = 3.936 \text{ Mlbf/in} \quad Ans.$$

Member stiffness for four frusta and joint constant C using Eqs. (8-20) and (e).

Top frustum: $D = 0.75, t = 0.5, d = 0.5, E = 30 \quad \Rightarrow \quad k_1 = 33.30$ Mlbf/in

2nd frustum: $D = 1.327, t = 0.11, d = 0.5, E = 14.5 \quad \Rightarrow \quad k_2 = 173.8$ Mlbf/in

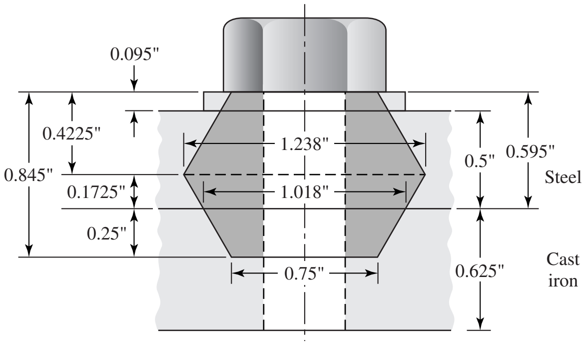
3rd frustum: $D = 0.860, t = 0.515, E = 14.5 \quad \Rightarrow \quad k_3 = 21.47$ Mlbf/in

Fourth frustum: $D = 0.75, t = 0.095, d = 0.5, E = 30 \quad \Rightarrow \quad k_4 = 97.27$ Mlbf/in

$$k_m = \left(\sum_{i=1}^{4} 1/k_i \right)^{-1} = 10.79 \text{ Mlbf/in} \quad \textit{Ans.}$$

$$C = 3.94/(3.94 + 10.79) = 0.267 \quad \textit{Ans.}$$

8-25



$$k_b = \frac{A_t E}{l} = \frac{0.1419(30)}{0.845} = 5.04 \text{ Mlbf/in} \quad \textit{Ans.}$$

From Fig. 8-21,

$$h = \frac{1}{2} + 0.095 = 0.595 \text{ in}$$

$$l = h + \frac{d}{2} = 0.595 + \frac{0.5}{2} = 0.845$$

$$D_1 = 0.75 + 0.845 \tan 30^\circ = 1.238 \text{ in}$$

$$l/2 = 0.845/2 = 0.4225 \text{ in}$$

From Eq. (8-20):

Frustum 1: $D = 0.75, t = 0.4225$ in, $d = 0.5$ in, $E = 30$ Mpsi $\Rightarrow k_1 = 36.14$ Mlbf/in

Frustum 2: $D = 1.018$ in, $t = 0.1725$ in, $E = 70$ Mpsi, $d = 0.5$ in $\Rightarrow k_2 = 134.6$ Mlbf/in

Frustum 3: $D = 0.75, t = 0.25$ in, $d = 0.5$ in, $E = 14.5$ Mpsi $\Rightarrow k_3 = 23.49$ Mlbf/in

$$k_m = \frac{1}{(1/36.14) + (1/134.6) + (1/23.49)} = 12.87 \text{ Mlbf/in} \quad \textit{Ans.}$$

$$C = \frac{5.04}{5.04 + 12.87} = 0.281 \quad \textit{Ans.}$$

8-26 Refer to Prob. 8-24 and its solution. Additional information: $A = 3.5$ in, $D_s = 4.25$ in, static pressure 1500 psi, $D_b = 6$ in, C (joint constant) $= 0.267$, ten SAE grade 5 bolts.

$$P = \frac{1}{10}\,\frac{\pi(4.25^2)}{4}(1500) = 2128 \text{ lbf}$$

From Tables 8-2 and 8-9,

$$A_t = 0.1419 \text{ in}^2$$

$$S_p = 85\,000 \text{ psi}$$

$$F_i = 0.75(0.1419)(85) = 9.046 \text{ kip}$$

From Eq. (8-28),

$$n = \frac{S_p A_t - F_i}{CP} = \frac{85(0.1419) - 9.046}{0.267(2.128)} = 5.31 \quad \textit{Ans.}$$

8-27 From Fig. 8-21, $t_1 = 0.25$ in

$$h = 0.25 + 0.065 = 0.315 \text{ in}$$

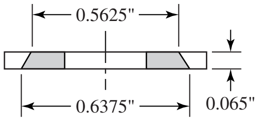
$$l = h + (d/2) = 0.315 + (3/16) = 0.5025 \text{ in}$$

$$D_1 = 1.5(0.375) + 0.577(0.5025) = 0.8524 \text{ in}$$

$$D_2 = 1.5(0.375) = 0.5625 \text{ in}$$

$$l/2 = 0.5025/2 = 0.251\,25 \text{ in}$$

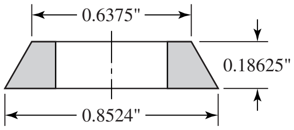
Frustum 1: Washer



$$E = 30 \text{ Mpsi}, \quad t = 0.065 \text{ in}, \quad D = 0.5625 \text{ in}$$

$$k = 78.57 \text{ Mlbf/in} \qquad \text{(by computer)}$$

Frustum 2: Cap portion

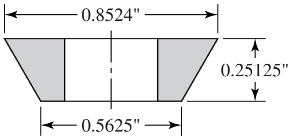


$$E = 14 \text{ Mpsi}, \quad t = 0.186\,25 \text{ in}$$

$$D = 0.5625 + 2(0.065)(0.577) = 0.6375 \text{ in}$$

$$k = 23.46 \text{ Mlbf/in} \qquad \text{(by computer)}$$

Frustum 3: Frame and Cap



$$E = 14 \text{ Mpsi}, \quad t = 0.251\,25 \text{ in}, \quad D = 0.5625 \text{ in}$$

$$k = 14.31 \text{ Mlbf/in} \qquad \text{(by computer)}$$

$$k_m = \frac{1}{(1/78.57) + (1/23.46) + (1/14.31)} = 7.99 \text{ Mlbf/in} \quad \textit{Ans.}$$

For the bolt, $L_T = 2(3/8) + (1/4) = 1$ in. So the bolt is threaded all the way. Since $A_t = 0.0775 \text{ in}^2$

$$k_b = \frac{0.0775(30)}{0.5025} = 4.63 \text{ Mlbf/in} \quad \textit{Ans.}$$

8-28

(a) $F_b' = RF_{b,\max}' \sin\theta$

Half of the external moment is contributed by the line load in the interval $0 \le \theta \le \pi$.

$$\frac{M}{2} = \int_0^\pi F_b' R^2 \sin\theta \, d\theta = \int_0^\pi F_{b,\max}' R^2 \sin^2\theta \, d\theta$$

$$\frac{M}{2} = \frac{\pi}{2} F_{b,\max}' R^2$$

from which $F_{b,\max}' = \dfrac{M}{\pi R^2}$

$$F_{\max} = \int_{\phi_1}^{\phi_2} F_b' R \sin\theta \, d\theta = \frac{M}{\pi R^2} \int_{\phi_1}^{\phi_2} R \sin\theta \, d\theta = \frac{M}{\pi R}(\cos\phi_1 - \cos\phi_2)$$

Noting $\phi_1 = 75^\circ, \phi_2 = 105^\circ$

$$F_{\max} = \frac{12\,000}{\pi(8/2)}(\cos 75^\circ - \cos 105^\circ) = 494 \text{ lbf} \quad \textit{Ans.}$$

(b)
$$F_{\max} = F_{b,\max}' R\Delta\phi = \frac{M}{\pi R^2}(R)\left(\frac{2\pi}{N}\right) = \frac{2M}{RN}$$

$$F_{\max} = \frac{2(12\,000)}{(8/2)(12)} = 500 \text{ lbf} \quad \textit{Ans.}$$

(c) $F = F_{\max}\sin\theta$

$$M = 2F_{\max}R[(1)\sin^2 90^\circ + 2\sin^2 60^\circ + 2\sin^2 30^\circ + (1)\sin^2(0)] = 6F_{\max}R$$

from which

$$F_{\max} = \frac{M}{6R} = \frac{12\,000}{6(8/2)} = 500 \text{ lbf} \quad \textit{Ans.}$$

The simple general equation resulted from part (*b*)

$$F_{\max} = \frac{2M}{RN}$$

8-29 **(a)** Table 8-11: $S_p = 600$ MPa

Eq. (8-30): $F_i = 0.9A_tS_p = 0.9(245)(600)(10^{-3}) = 132.3$ kN

Table (8-15): $K = 0.18$

Eq. (8-27) $T = 0.18(132.3)(20) = 476 \text{ N}\cdot\text{m}$ *Ans.*

(b) Washers: $t = 3.4$ mm, $d = 20$ mm, $D = 30$ mm, $E = 207$ GPa $\Rightarrow$ $k_1 = 42\,175$ MN/m

Cast iron: $t = 20$ mm, $d = 20$ mm, $D = 30 + 2(3.4)\tan 30^\circ = 33.93$ mm,

$E = 135$ GPa $\quad\Rightarrow\quad k_2 = 7885$ MN/m

Steel: $t = 20$ mm, $d = 20$ mm, $D = 33.93$ mm, $E = 207$ GPa $\Rightarrow k_3 = 12\,090$ MN/m

$$k_m = (2/42\,175 + 1/7885 + 1/12\,090)^{-1} = 3892 \text{ MN/m}$$

Bolt: $l = 46.8$ mm. Nut: $H = 18$ mm. $L > 46.8 + 18 = 64.8$ mm. Use $L = 80$ mm.

$$L_T = 2(20) + 6 = 46 \text{ mm}, \; l_d = 80 - 46 = 34 \text{ mm}, \; l_t = 46.8 - 34 = 12.8 \text{ mm},$$

$$A_t = 245 \text{ mm}^2, \quad A_d = \pi 20^2/4 = 314.2 \text{ mm}^2$$

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{314.2(245)(207)}{314.2(12.8) + 245(34)} = 1290 \text{ MN/m}$$

$$C = 1290/(1290 + 3892) = 0.2489, \quad S_p = 600 \text{ MPa}, \quad F_i = 132.3 \text{ kN}$$

$$n = \frac{S_p A_t - F_i}{C(P/N)} = \frac{600(0.245) - 132.3}{0.2489(15/4)} = 15.7 \quad \textit{Ans.}$$

Bolts are a bit oversized for the load.

8-30 **(a)** ISO M 20 × 2.5 grade 8.8 coarse pitch bolts, lubricated.

Table 8-2 $\quad A_t = 245 \text{ mm}^2$

Table 8-11 $\quad S_p = 600$ MPa

$$A_d = \pi(20)^2/4 = 314.2 \text{ mm}^2$$

$$F_p = 245(0.600) = 147 \text{ kN}$$

$$F_i = 0.90F_p = 0.90(147) = 132.3 \text{ kN}$$

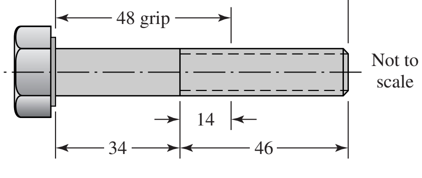
$$T = 0.18(132.3)(20) = 476 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

(b) $L \geq l + H = 48 + 18 = 66$ mm. Therefore, set $L = 80$ mm per Table A-17.

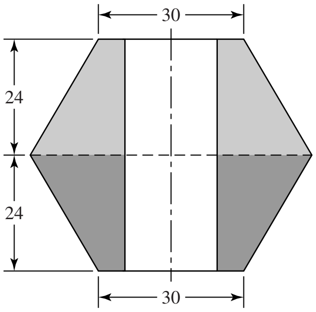
$$L_T = 2D + 6 = 2(20) + 6 = 46 \text{ mm}$$

$$l_d = L - L_T = 80 - 46 = 34 \text{ mm}$$

$$l_t = l - l_d = 48 - 34 = 14 \text{ mm}$$



$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{314.2(245)(207)}{314.2(14) + 245(34)} = 1251.9 \text{ MN/m}$$



Use Wileman *et al.*

Eq. (8-23)

$$A = 0.787\,15, \quad B = 0.628\,73$$

$$\frac{k_m}{Ed} = A \exp\left(\frac{Bd}{L_G}\right) = 0.787\,15 \exp\left[0.628\,73\left(\frac{20}{48}\right)\right] = 1.0229$$

$$k_m = 1.0229(207)(20) = 4235 \text{ MN/m}$$

$$C = \frac{1251.9}{1251.9 + 4235} = 0.228$$

Bolts carry 0.228 of the external load; members carry 0.772 of the external load. *Ans.* Thus, the actual loads are

$$F_b = CP + F_i = 0.228(20) + 132.3 = 136.9 \text{ kN}$$

$$F_m = (1 - C)P - F_i = (1 - 0.228)20 - 132.3 = -116.9 \text{ kN}$$

8-31 Given $p_{\max} = 6$ MPa, $p_{\min} = 0$ and from Prob. 8-20 solution, $C = 0.2346$, $F_i = 37.9$ kN, $A_t = 84.3$ mm^2.

For 6 MPa, $P = 10.6$ kN per bolt

$$\sigma_i = \frac{F_i}{A_t} = \frac{37.9(10^3)}{84.3} = 450 \text{ MPa}$$

Eq. (8-35):

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.2346(10.6)(10^3)}{2(84.3)} = 14.75 \text{ MPa}$$

$$\sigma_m = \sigma_a + \sigma_i = 14.75 + 450 = 464.8 \text{ MPa}$$

(a) Goodman Eq. (8-40) for 8.8 bolts with $S_e = 129$ MPa, $S_{ut} = 830$ MPa

$$S_a = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut} + S_e} = \frac{129(830 - 450)}{830 + 129} = 51.12 \text{ MPa}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{51.12}{14.75} = 3.47 \quad \textit{Ans.}$$

(b) Gerber Eq. (8-42)

$$S_a = \frac{1}{2S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(129)}\left[830\sqrt{830^2 + 4(129)(129 + 450)} - 830^2 - 2(450)(129)\right]$$

$$= 76.99 \text{ MPa}$$

$$n_f = \frac{76.99}{14.75} = 5.22 \quad \textit{Ans.}$$

(c) ASME-elliptic Eq. (8-43) with $S_p = 600$ MPa

$$S_a = \frac{S_e}{S_p^2 + S_e^2}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{129}{600^2 + 129^2}\left[600\sqrt{600^2 + 129^2 - 450^2} - 450(129)\right] = 65.87 \text{ MPa}$$

$$n_f = \frac{65.87}{14.75} = 4.47 \quad \textit{Ans.}$$

8-32

$$P = \frac{pA}{N} = \frac{\pi D^2 p}{4N} = \frac{\pi(0.9^2)(550)}{4(36)} = 9.72 \text{ kN/bolt}$$

Table 8-11: $S_p = 830$ MPa, $S_{ut} = 1040$ MPa, $S_y = 940$ MPa

Table 8-1: $A_t = 58 \text{ mm}^2$

$$A_d = \pi(10^2)/4 = 78.5 \text{ mm}^2$$

$$l = D + E = 20 + 25 = 45 \text{ mm}$$

$$L_T = 2(10) + 6 = 26 \text{ mm}$$

Table A-31: $H = 8.4$ mm

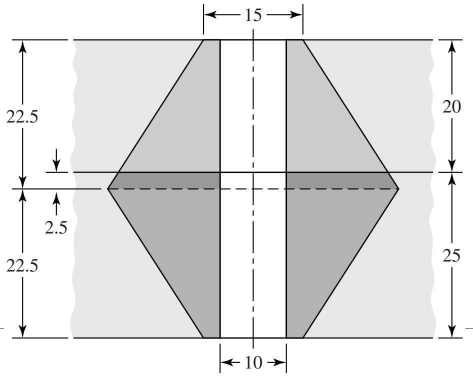
$$L \geq l + H = 45 + 8.4 = 53.4 \text{ mm}$$

Choose $L = 60$ mm from Table A-17

$$l_d = L - L_T = 60 - 26 = 34 \text{ mm}$$

$$l_t = l - l_d = 45 - 34 = 11 \text{ mm}$$

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{78.5(58)(207)}{78.5(11) + 58(34)} = 332.4 \text{ MN/m}$$



Frustum 1: Top, $E = 207, t = 20$ mm, $d = 10$ mm, $D = 15$ mm

$$k_1 = \frac{0.5774\pi(207)(10)}{\ln\left\{\left[\dfrac{1.155(20)+15-10}{1.155(20)+15+10}\right]\left(\dfrac{15+10}{15-10}\right)\right\}}$$

$$= 3503 \text{ MN/m}$$

Frustum 2: Middle, $E = 96$ GPa, $D = 38.09$ mm, $t = 2.5$ mm, $d = 10$ mm

$$k_2 = \frac{0.5774\pi(96)(10)}{\ln\left\{\left[\dfrac{1.155(2.5)+38.09-10}{1.155(2.5)+38.09+10}\right]\left(\dfrac{38.09+10}{38.09-10}\right)\right\}}$$

$$= 44\,044 \text{ MN/m}$$

could be neglected due to its small influence on k_m.

Frustum 3: Bottom, $E = 96$ GPa, $t = 22.5$ mm, $d = 10$ mm, $D = 15$ mm

$$k_3 = \frac{0.5774\pi(96)(10)}{\ln\left\{\left[\dfrac{1.155(22.5)+15-10}{1.155(22.5)+15+10}\right]\left(\dfrac{15+10}{15-10}\right)\right\}}$$

$$= 1567 \text{ MN/m}$$

$$k_m = \frac{1}{(1/3503)+(1/44\,044)+(1/1567)} = 1057 \text{ MN/m}$$

$$C = \frac{332.4}{332.4+1057} = 0.239$$

$$F_i = 0.75A_tS_p = 0.75(58)(830)(10^{-3}) = 36.1 \text{ kN}$$

Table 8-17: $S_e = 162$ MPa

$$\sigma_i = \frac{F_i}{A_t} = \frac{36.1(10^3)}{58} = 622 \text{ MPa}$$

(a) Goodman Eq. (8-40)

$$S_a = \frac{S_e(S_{ut}-\sigma_i)}{S_{ut}+S_e} = \frac{162(1040-622)}{1040+162} = 56.34 \text{ MPa}$$

$$n_f = \frac{56.34}{20} = 2.82 \quad \textit{Ans.}$$

(b) Gerber Eq. (8-42)

$$S_a = \frac{1}{2S_e}\left[S_{ut}\sqrt{S_{ut}^2+4S_e(S_e+\sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(162)}\left[1040\sqrt{1040^2+4(162)(162+622)} - 1040^2 - 2(622)(162)\right]$$

$$= 86.8 \text{ MPa}$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.239(9.72)(10^3)}{2(58)} = 20 \text{ MPa}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{86.8}{20} = 4.34 \quad Ans.$$

(c) ASME elliptic

$$S_a = \frac{S_e}{S_p^2 + S_e^2}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{162}{830^2 + 162^2}\left[830\sqrt{830^2 + 162^2 - 622^2} - 622(162)\right] = 84.90 \text{ MPa}$$

$$n_f = \frac{84.90}{20} = 4.24 \quad Ans.$$

8-33 Let the repeatedly-applied load be designated as P. From Table A-22, $S_{ut} =$ 93.7 kpsi. Referring to the Figure of Prob. 3-74, the following notation will be used for the radii of Section AA.

$$r_i = 1 \text{ in}, \quad r_o = 2 \text{ in}, \quad r_c = 1.5 \text{ in}$$

From Table 4-5, with $R = 0.5$ in

$$r_n = \frac{0.5^2}{2\left(1.5 - \sqrt{1.5^2 - 0.5^2}\right)} = 1.457\,107 \text{ in}$$

$$e = r_c - r_n = 1.5 - 1.457\,107 = 0.042\,893 \text{ in}$$

$$c_o = r_o - r_n = 2 - 1.457\,109 = 0.542\,893 \text{ in}$$

$$c_i = r_n - r_i = 1.457\,107 - 1 = 0.457\,107 \text{ in}$$

$$A = \pi(1^2)/4 = 0.7854 \text{ in}^2$$

If P is the maximum load

$$M = Pr_c = 1.5P$$

$$\sigma_i = \frac{P}{A}\left(1 + \frac{r_c c_i}{er_i}\right) = \frac{P}{0.7854}\left(1 + \frac{1.5(0.457)}{0.0429(1)}\right) = 21.62P$$

$$\sigma_a = \sigma_m = \frac{\sigma_i}{2} = \frac{21.62P}{2} = 10.81P$$

(a) *Eye:* Section AA

$$k_a = 14.4(93.7)^{-0.718} = 0.553$$

$$d_e = 0.37d = 0.37(1) = 0.37 \text{ in}$$

$$k_b = \left(\frac{0.37}{0.30}\right)^{-0.107} = 0.978$$

$$k_c = 0.85$$

$$S_e' = 0.5(93.7) = 46.85 \text{ kpsi}$$

$$S_e = 0.553(0.978)(0.85)(46.85) = 21.5 \text{ kpsi}$$

Since no stress concentration exists, use a load line slope of 1. From Table 7-10 for Gerber

$$S_a = \frac{93.7^2}{2(21.5)}\left[-1+\sqrt{1+\left(\frac{2(21.5)}{93.7}\right)^2}\,\right] = 20.47 \text{ kpsi}$$

Note the mere 5 percent degrading of S_e in S_a

$$n_f = \frac{S_a}{\sigma_a} = \frac{20.47(10^3)}{10.81P} = \frac{1894}{P}$$

Thread: Die cut. Table 8-17 gives 18.6 kpsi for rolled threads. Use Table 8-16 to find S_e for die cut threads

$$S_e = 18.6(3.0/3.8) = 14.7 \text{ kpsi}$$

Table 8-2:

$$\begin{aligned} A_t &= 0.663 \text{ in}^2 \\ \sigma &= P/A_t = P/0.663 = 1.51P \\ \sigma_a &= \sigma_m = \sigma/2 = 1.51P/2 = 0.755P \end{aligned}$$

From Table 7-10, Gerber

$$S_a = \frac{120^2}{2(14.7)}\left[-1+\sqrt{1+\left(\frac{2(14.7)}{120}\right)^2}\,\right] = 14.5 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{14\,500}{0.755P} = \frac{19\,200}{P}$$

Comparing $1894/P$ with $19\,200/P$, we conclude that the *eye* is weaker in fatigue. *Ans.*

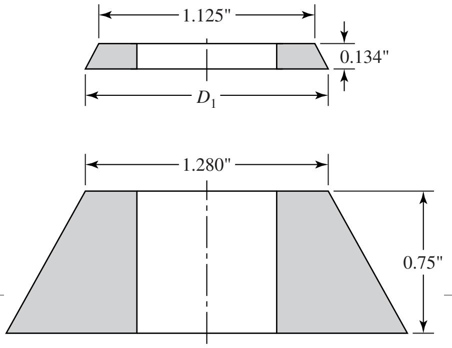
(b) Strengthening steps can include heat treatment, cold forming, cross section change (a round is a poor cross section for a curved bar in bending because the bulk of the material is located where the stress is small). *Ans.*

(c) For $n_f = 2$

$$P = \frac{1894}{2} = 947 \text{ lbf, max. load} \quad \textit{Ans.}$$

8-34 **(a)** $L \geq 1.5 + 2(0.134) + \dfrac{41}{64} = 2.41$ in. Use $L = 2\frac{1}{2}$ in *Ans.*

(b) Four frusta: Two washers and two members



Washer: $E = 30$ Mpsi, $t = 0.134$ in, $D = 1.125$ in, $d = 0.75$ in

Eq. (8-20): $k_1 = 153.3$ Mlbf/in

Member: $E = 16$ Mpsi, $t = 0.75$ in, $D = 1.280$ in, $d = 0.75$ in

Eq. (8-20): $k_2 = 35.5$ Mlbf/in

$$k_m = \frac{1}{(2/153.3) + (2/35.5)} = 14.41 \text{ Mlbf/in} \quad \textit{Ans.}$$

Bolt:

$$\begin{aligned}
L_T &= 2(3/4) + 1/4 = 1^3\!/_4 \text{ in}\\
l &= 2(0.134) + 2(0.75) = 1.768 \text{ in}\\
l_d &= L - L_T = 2.50 - 1.75 = 0.75 \text{ in}\\
l_t &= l - l_d = 1.768 - 0.75 = 1.018 \text{ in}\\
A_t &= 0.373 \text{ in}^2 \quad \text{(Table 8-2)}\\
A_d &= \pi(0.75)^2/4 = 0.442 \text{ in}^2\\
k_b &= \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.442(0.373)(30)}{0.442(1.018) + 0.373(0.75)} = 6.78 \text{ Mlbf/in} \quad \textit{Ans.}
\end{aligned}$$

$$C = \frac{6.78}{6.78 + 14.41} = 0.320 \quad \textit{Ans.}$$

(c) From Eq. (8-40), Goodman with $S_e = 18.6$ kpsi, $S_{ut} = 120$ kpsi

$$S_a = \frac{18.6[120 - (25/0.373)]}{120 + 18.6} = 7.11 \text{ kpsi}$$

The stress components are

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.320(6)}{2(0.373)} = 2.574 \text{ kpsi}$$

$$\sigma_m = \sigma_a + \frac{F_i}{A_t} = 2.574 + \frac{25}{0.373} = 69.6 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{7.11}{2.574} = 2.76 \quad \textit{Ans.}$$

(d) Eq. (8-42) for Gerber

$$S_a = \frac{1}{2(18.6)}\left[120\sqrt{120^2 + 4(18.6)\left(18.6 + \frac{25}{0.373}\right)} - 120^2 - 2\left(\frac{25}{0.373}\right)18.6\right]$$
$$= 10.78 \text{ kpsi}$$

$$n_f = \frac{10.78}{2.574} = 4.19 \quad \textit{Ans.}$$

(e) $n_{\text{proof}} = \dfrac{85}{2.654 + 69.8} = 1.17 \quad \textit{Ans.}$

8-35

(a) Table 8-2: $A_t = 0.1419\ \text{in}^2$

Table 8-9: $S_p = 85\ \text{kpsi},\ \ S_{ut} = 120\ \text{kpsi}$

Table 8-17: $S_e = 18.6\ \text{kpsi}$

$$F_i = 0.75A_tS_p = 0.75(0.1419)(85) = 9.046\ \text{kip}$$

$$C = \frac{4.94}{4.94 + 15.97} = 0.236$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.236P}{2(0.1419)} = 0.832P\ \text{kpsi}$$

Eq. (8-40) for Goodman criterion

$$S_a = \frac{18.6(120 - 9.046/0.1419)}{120 + 18.6} = 7.55\ \text{kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{7.55}{0.832P} = 2 \quad \Rightarrow \quad P = 4.54\ \text{kip} \quad \textit{Ans.}$$

(b) Eq. (8-42) for Gerber criterion

$$S_a = \frac{1}{2(18.6)}\left[120\sqrt{120^2 + 4(18.6)\left(18.6 + \frac{9.046}{0.1419}\right)} - 120^2 - 2\left(\frac{9.046}{0.1419}\right)18.6\right]$$

$$= 11.32\ \text{kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{11.32}{0.832P} = 2$$

From which

$$P = \frac{11.32}{2(0.832)} = 6.80\ \text{kip} \quad \textit{Ans.}$$

(c) $\sigma_a = 0.832P = 0.832(6.80) = 5.66\ \text{kpsi}$

$\sigma_m = S_a + \sigma_a = 11.32 + 63.75 = 75.07\ \text{kpsi}$

Load factor, Eq. (8-28)

$$n = \frac{S_pA_t - F_i}{CP} = \frac{85(0.1419) - 9.046}{0.236(6.80)} = 1.88 \quad \textit{Ans.}$$

Separation load factor, Eq. (8-29)

$$n = \frac{F_i}{(1 - C)P} = \frac{9.046}{6.80(1 - 0.236)} = 1.74 \quad \textit{Ans.}$$

8-36 Table 8-2: $A_t = 0.969\ \text{in}^2$ (coarse)

$A_t = 1.073\ \text{in}^2$ (fine)

Table 8-9: $S_p = 74\ \text{kpsi}, \quad S_{ut} = 105\ \text{kpsi}$

Table 8-17: $S_e = 16.3\ \text{kpsi}$

Coarse thread, UNC

$$F_i = 0.75(0.969)(74) = 53.78 \text{ kip}$$

$$\sigma_i = \frac{F_i}{A_t} = \frac{53.78}{0.969} = 55.5 \text{ kpsi}$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.30P}{2(0.969)} = 0.155P \text{ kpsi}$$

Eq. (8-42):

$$S_a = \frac{1}{2(16.3)}\left[105\sqrt{105^2 + 4(16.3)(16.3 + 55.5)} - 105^2 - 2(55.5)(16.3)\right] = 9.96 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{9.96}{0.155P} = 2$$

From which

$$P = \frac{9.96}{0.155(2)} = 32.13 \text{ kip} \quad Ans.$$

Fine thread, UNF

$$F_i = 0.75(1.073)(74) = 59.55 \text{ kip}$$

$$\sigma_i = \frac{59.55}{1.073} = 55.5 \text{ kpsi}$$

$$\sigma_a = \frac{0.32P}{2(1.073)} = 0.149P \text{ kpsi}$$

$$S_a = 9.96 \quad \text{(as before)}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{9.96}{0.149P} = 2$$

From which

$$P = \frac{9.96}{0.149(2)} = 33.42 \text{ kip} \quad Ans.$$

Percent improvement

$$\frac{33.42 - 32.13}{32.13}(100) \doteq 4\% \quad Ans.$$

8-37 For a M 30 × 3.5 ISO 8.8 bolt with $P = 80$ kN/bolt and $C = 0.33$

Table 8-1: $A_t = 561 \text{ mm}^2$

Table 8-11: $S_p = 600$ MPa

$S_{ut} = 830$ MPa

Table 8-17: $S_e = 129$ MPa

$$F_i = 0.75(561)(10^{-3})(600) = 252.45 \text{ kN}$$

$$\sigma_i = \frac{252.45(10^{-3})}{561} = 450 \text{ MPa}$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.33(80)(10^3)}{2(561)} = 23.53 \text{ MPa}$$

Eq. (8-42):

$$S_a = \frac{1}{2(129)}\left[830\sqrt{830^2 + 4(129)(129 + 450)} - 830^2 - 2(450)(129)\right] = 77.0 \text{ MPa}$$

Fatigue factor of safety

$$n_f = \frac{S_a}{\sigma_a} = \frac{77.0}{23.53} = 3.27 \quad Ans.$$

Load factor from Eq. (8-28),

$$n = \frac{S_p A_t - F_i}{CP} = \frac{600(10^{-3})(561) - 252.45}{0.33(80)} = 3.19 \quad Ans.$$

Separation load factor from Eq. (8-29),

$$n = \frac{F_i}{(1 - C)P} = \frac{252.45}{(1 - 0.33)(80)} = 4.71 \quad Ans.$$

8-38

(a) Table 8-2: $A_t = 0.0775 \text{ in}^2$

Table 8-9: $S_p = 85 \text{ kpsi}, \quad S_{ut} = 120 \text{ kpsi}$

Table 8-17: $S_e = 18.6 \text{ kpsi}$

Unthreaded grip

$$k_b = \frac{A_d E}{l} = \frac{\pi(0.375)^2(30)}{4(13.5)} = 0.245 \text{ Mlbf/in per bolt} \quad Ans.$$

$$A_m = \frac{\pi}{4}[(D + 2t)^2 - D^2] = \frac{\pi}{4}(4.75^2 - 4^2) = 5.154 \text{ in}^2$$

$$k_m = \frac{A_m E}{l} = \frac{5.154(30)}{12}\left(\frac{1}{6}\right) = 2.148 \text{ Mlbf/in/bolt.} \quad Ans.$$

(b)

$$F_i = 0.75(0.0775)(85) = 4.94 \text{ kip}$$

$$\sigma_i = 0.75(85) = 63.75 \text{ kpsi}$$

$$P = pA = \frac{2000}{6}\left[\frac{\pi}{4}(4)^2\right] = 4189 \text{ lbf/bolt}$$

$$C = \frac{0.245}{0.245 + 2.148} = 0.102$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.102(4.189)}{2(0.0775)} = 2.77 \text{ kpsi}$$

Eq. (8-40) for Goodman

$$S_a = \frac{18.6(120 - 63.75)}{120 + 18.6} = 7.55 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{7.55}{2.77} = 2.73 \quad \textit{Ans.}$$

(c) From Eq. (8-42) for Gerber fatigue criterion,

$$S_a = \frac{1}{2(18.6)}\left[120\sqrt{120^2 + 4(18.6)(18.6 + 63.75)} - 120^2 - 2(63.75)(18.6)\right]$$

$$= 11.32 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{11.32}{2.77} = 4.09 \quad \textit{Ans.}$$

(d) Pressure causing joint separation from Eq. (8-29)

$$n = \frac{F_i}{(1 - C)P} = 1$$

$$P = \frac{F_i}{1 - C} = \frac{4.94}{1 - 0.102} = 5.50 \text{ kip}$$

$$p = \frac{P}{A} = \frac{5500}{\pi(4^2)/4}6 = 2626 \text{ psi} \quad \textit{Ans.}$$

8-39 This analysis is important should the initial bolt tension fail. Members: $S_y = 71$ kpsi, $S_{sy} = 0.577(71) = 41.0$ kpsi. Bolts: SAE grade 8, $S_y = 130$ kpsi, $S_{sy} = 0.577(130) = 75.01$ kpsi

Shear in bolts

$$A_s = 2\left[\frac{\pi(0.375^2)}{4}\right] = 0.221 \text{ in}^2$$

$$F_s = \frac{A_s S_{sy}}{n} = \frac{0.221(75.01)}{3} = 5.53 \text{ kip}$$

Bearing on bolts

$$A_b = 2(0.375)(0.25) = 0.188 \text{ in}^2$$

$$F_b = \frac{A_b S_{yc}}{n} = \frac{0.188(130)}{2} = 12.2 \text{ kip}$$

Bearing on member

$$F_b = \frac{0.188(71)}{2.5} = 5.34 \text{ kip}$$

Tension of members

$$A_t = (1.25 - 0.375)(0.25) = 0.219 \text{ in}^2$$

$$F_t = \frac{0.219(71)}{3} = 5.18 \text{ kip}$$

$$F = \min(5.53, 12.2, 5.34, 5.18) = 5.18 \text{ kip} \quad \textit{Ans.}$$

The tension in the members controls the design.

8-40 Members: $S_y = 32$ kpsi

Bolts: $S_y = 92$ kpsi, $S_{sy} = (0.577)92 = 53.08$ kpsi

Shear of bolts

$$A_s = 2\left[\frac{\pi(0.375)^2}{4}\right] = 0.221 \text{ in}^2$$

$$\tau = \frac{F_s}{A_s} = \frac{4}{0.221} = 18.1 \text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau} = \frac{53.08}{18.1} = 2.93 \quad Ans.$$

Bearing on bolts

$$A_b = 2(0.25)(0.375) = 0.188 \text{ in}^2$$

$$\sigma_b = \frac{-4}{0.188} = -21.3 \text{ kpsi}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{92}{|-21.3|} = 4.32 \quad Ans.$$

Bearing on members

$$n = \frac{S_{yc}}{|\sigma_b|} = \frac{32}{|-21.3|} = 1.50 \quad Ans.$$

Tension of members

$$A_t = (2.375 - 0.75)(1/4) = 0.406 \text{ in}^2$$

$$\sigma_t = \frac{4}{0.406} = 9.85 \text{ kpsi}$$

$$n = \frac{S_y}{A_t} = \frac{32}{9.85} = 3.25 \quad Ans.$$

8-41 Members: $S_y = 71$ kpsi

Bolts: $S_y = 92$ kpsi, $S_{sy} = 0.577(92) = 53.08$ kpsi

Shear of bolts

$$F = S_{sy}A/n$$

$$F_s = \frac{53.08(2)(\pi/4)(7/8)^2}{1.8} = 35.46 \text{ kip}$$

Bearing on bolts

$$F_b = \frac{2(7/8)(3/4)(92)}{2.2} = 54.89 \text{ kip}$$

Bearing on members

$$F_b = \frac{2(7/8)(3/4)(71)}{2.4} = 38.83 \text{ kip}$$

Tension in members

$$F_t = \frac{(3 - 0.875)(3/4)(71)}{2.6} = 43.52 \text{ kip}$$

$$F = \min(35.46, 54.89, 38.83, 43.52) = 35.46 \text{ kip} \quad Ans.$$

8-42 Members: $S_y = 47$ kpsi

Bolts: $S_y = 92$ kpsi, $S_{sy} = 0.577(92) = 53.08$ kpsi

Shear of bolts

$$A_d = \frac{\pi(0.75)^2}{4} = 0.442 \text{ in}^2$$

$$\tau_s = \frac{20}{3(0.442)} = 15.08 \text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau_s} = \frac{53.08}{15.08} = 3.52 \quad Ans.$$

Bearing on bolt

$$\sigma_b = -\frac{20}{3(3/4)(5/8)} = -14.22 \text{ kpsi}$$

$$n = -\frac{S_y}{\sigma_b} = -\left(\frac{92}{-14.22}\right) = 6.47 \quad Ans.$$

Bearing on members

$$\sigma_b = -\frac{F}{A_b} = -\frac{20}{3(3/4)(5/8)} = -14.22 \text{ kpsi}$$

$$n = -\frac{S_y}{\sigma_b} = -\frac{47}{14.22} = 3.31 \quad Ans.$$

Tension on members

$$\sigma_t = \frac{F}{A} = \frac{20}{(5/8)[7.5 - 3(3/4)]} = 6.10 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma_t} = \frac{47}{6.10} = 7.71 \quad Ans.$$

8-43 Members: $S_y = 57$ kpsi

Bolts: $S_y = 92$ kpsi, $S_{sy} = 0.577(92) = 53.08$ kpsi

Shear of bolts

$$A_s = 3\left[\frac{\pi(3/8)^2}{4}\right] = 0.3313 \text{ in}^2$$

$$\tau_s = \frac{F}{A} = \frac{5.4}{0.3313} = 16.3 \text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau_s} = \frac{53.08}{16.3} = 3.26 \quad Ans.$$

Bearing on bolt

$$A_b = 3\left(\frac{3}{8}\right)\left(\frac{5}{16}\right) = 0.3516 \text{ in}^2$$

$$\sigma_b = -\frac{F}{A_b} = -\frac{5.4}{0.3516} = -15.36 \text{ kpsi}$$

$$n = -\frac{S_y}{\sigma_b} = -\left(\frac{92}{-15.36}\right) = 5.99 \quad \textit{Ans.}$$

Bearing on members

$$A_b = 0.3516 \text{ in}^2 \text{ (From bearing on bolt calculations)}$$

$$\sigma_b = -15.36 \text{ kpsi (From bearing on bolt calculations)}$$

$$n = -\frac{S_y}{\sigma_b} = -\left(\frac{57}{-15.36}\right) = 3.71 \quad \textit{Ans.}$$

Tension in members

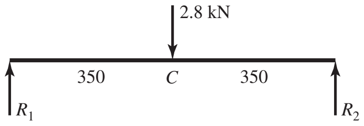
Failure across two bolts

$$A = \frac{5}{16}\left[2\frac{3}{8} - 2\left(\frac{3}{8}\right)\right] = 0.5078 \text{ in}^2$$

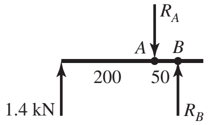
$$\sigma = \frac{F}{A} = \frac{5.4}{0.5078} = 10.63 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma_t} = \frac{57}{10.63} = 5.36 \quad \textit{Ans.}$$

8-44



By symmetry, $R_1 = R_2 = 1.4$ kN



$$\sum M_B = 0 \quad 1.4(250) - 50R_A = 0 \quad \Rightarrow \quad R_A = 7 \text{ kN}$$

$$\sum M_A = 0 \quad 200(1.4) - 50R_B = 0 \quad \Rightarrow \quad R_B = 5.6 \text{ kN}$$

Members: $S_y = 370$ MPa

Bolts: $S_y = 420$ MPa, $S_{sy} = 0.577(420) = 242.3$ MPa

Bolt shear:

$$A_s = \frac{\pi}{4}(10^2) = 78.54 \text{ mm}^2$$

$$\tau = \frac{7(10^3)}{78.54} = 89.13 \text{ MPa}$$

$$n = \frac{S_{sy}}{\tau} = \frac{242.3}{89.13} = 2.72$$

Bearing on member: $A_b = td = 10(10) = 100 \text{ mm}^2$

$$\sigma_b = \frac{-7(10^3)}{100} = -70 \text{ MPa}$$

$$n = -\frac{S_y}{\sigma} = \frac{-370}{-70} = 5.29$$

Strength of member

At A, $M = 1.4(200) = 280 \text{ N} \cdot \text{m}$

$$I_A = \frac{1}{12}[10(50^3) - 10(10^3)] = 103.3(10^3) \text{ mm}^4$$

$$\sigma_A = \frac{Mc}{I_A} = \frac{280(25)}{103.3(10^3)}(10^3) = 67.76 \text{ MPa}$$

$$n = \frac{S_y}{\sigma_A} = \frac{370}{67.76} = 5.46$$

At C, $M = 1.4(350) = 490 \text{ N} \cdot \text{m}$

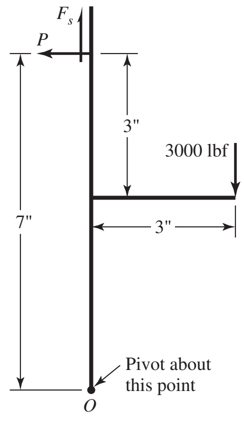
$$I_C = \frac{1}{12}(10)(50^3) = 104.2(10^3) \text{ mm}^4$$

$$\sigma_C = \frac{490(25)}{104.2(10^3)}(10^3) = 117.56 \text{ MPa}$$

$$n = \frac{S_y}{\sigma_C} = \frac{370}{117.56} = 3.15 < 5.46 \qquad C \text{ more critical}$$

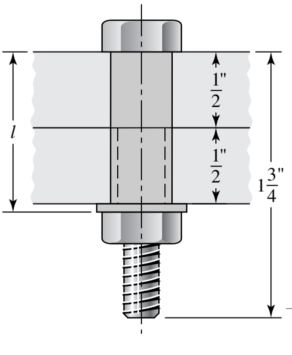
$$n = \min(2.72, 5.29, 3.15) = 2.72 \quad \textit{Ans.}$$

8-45



$$F_s = 3000 \text{ lbf}$$

$$P = \frac{3000(3)}{7} = 1286 \text{ lbf}$$



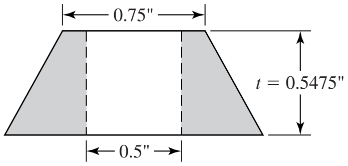
$$H = \frac{7}{16} \text{ in}$$

$$l = \frac{1}{2} + \frac{1}{2} + 0.095 = 1.095 \text{ in}$$

$$L \geq l + H = 1.095 + (7/16) = 1.532 \text{ in}$$

Use $1\frac{3}{4}''$ bolts

$$
\begin{aligned}
L_T &= 2D + \frac{1}{4} = 2(0.5) + 0.25 = 1.25 \text{ in} \\
l_d &= 1.75 - 1.25 = 0.5 \\
l_t &= 1.095 - 0.5 = 0.595 \\
A_d &= \frac{\pi(0.5)^2}{4} = 0.1963 \text{ in}^2 \\
A_t &= 0.1419 \text{ in} \\
k_b &= \frac{A_d A_t E}{A_d l_t + A_t l_d} \\
&= \frac{0.1963(0.1419)(30)}{0.1963(0.595) + 0.1419(0.5)} \\
&= 4.451 \text{ Mlbf/in}
\end{aligned}
$$



Two identical frusta

$$
\begin{aligned}
A &= 0.787\,15,\ B = 0.628\,73 \\
k_m &= EdA \exp\left(0.628\,73 \frac{d}{L_G}\right) \\
&= 30(0.5)(0.787\,15)\left[\exp\left(0.628\,73 \frac{0.5}{1.095}\right)\right] \\
k_m &= 15.733 \text{ Mlbf/in} \\
C &= \frac{4.451}{4.451 + 15.733} = 0.2205 \\
S_p &= 85 \text{ kpsi} \\
F_i &= 0.75(0.1419)(85) = 9.046 \text{ kip} \\
\sigma_i &= 0.75(85) = 63.75 \text{ kpsi} \\
\sigma_b &= \frac{CP + F_i}{A_t} = \frac{0.2205(1.286) + 9.046}{0.1419} = 65.75 \text{ kpsi} \\
\tau_s &= \frac{F_s}{A_s} = \frac{3}{0.1963} = 15.28 \text{ kpsi}
\end{aligned}
$$

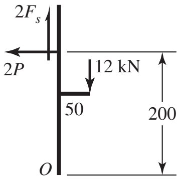
von Mises stress

$$\sigma' = \left(\sigma_b^2 + 3\tau_s^2\right)^{1/2} = [65.74^2 + 3(15.28^2)]^{1/2} = 70.87 \text{ kpsi}$$

Stress margin

$$m = S_p - \sigma' = 85 - 70.87 = 14.1 \text{ kpsi} \quad \textit{Ans.}$$

8-46



$$2P(200) = 12(50)$$

$$P = \frac{12(50)}{2(200)} = 1.5 \text{ kN per bolt}$$

$$F_s = 6 \text{ kN/bolt}$$

$$S_p = 380 \text{ MPa}$$

$$A_t = 245 \text{ mm}^2,\ A_d = \frac{\pi}{4}(20^2) = 314.2 \text{ mm}^2$$

$$F_i = 0.75(245)(380)(10^{-3}) = 69.83 \text{ kN}$$

$$\sigma_i = \frac{69.83(10^3)}{245} = 285 \text{ MPa}$$

$$\sigma_b = \frac{CP + F_i}{A_t} = \left(\frac{0.30(1.5) + 69.83}{245}\right)(10^3) = 287 \text{ MPa}$$

$$\tau = \frac{F_s}{A_d} = \frac{6(10^3)}{314.2} = 19.1 \text{ MPa}$$

$$\sigma' = [287^2 + 3(19.1^2)]^{1/2} = 289 \text{ MPa}$$

$$m = S_p - \sigma' = 380 - 289 = 91 \text{ MPa}$$

Thus the bolt will *not* exceed the proof stress. *Ans.*

8-47 Using the result of Prob. 5-31 for lubricated assembly

$$F_x = \frac{2\pi f T}{0.18d}$$

With a design factor of n_d gives

$$T = \frac{0.18 n_d F_x d}{2\pi f} = \frac{0.18(3)(1000)d}{2\pi(0.12)} = 716d$$

or $T/d = 716$. Also

$$\frac{T}{d} = K(0.75 S_p A_t)$$
$$= 0.18(0.75)(85\,000)A_t$$
$$= 11\,475 A_t$$

Form a table

Size	A_t	$T/d = 11\,475 A_t$	n
$\frac{1}{4} - 28$	0.0364	417.7	1.75
$\frac{5}{16} - 24$	0.058	665.55	2.8
$\frac{3}{8} - 24$	0.0878	1007.5	4.23

The factor of safety in the last column of the table comes from

$$n = \frac{2\pi f(T/d)}{0.18F_x} = \frac{2\pi(0.12)(T/d)}{0.18(1000)} = 0.0042(T/d)$$

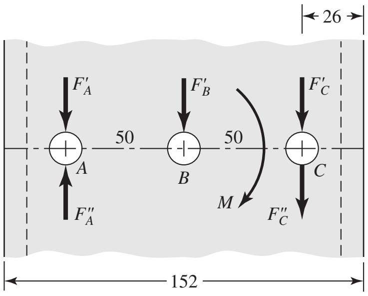
Select a $\frac{3}{8}$" − 24 UNF capscrew. The setting is given by

$$T = (11\,475 A_t)d = 1007.5(0.375) = 378 \text{ lbf} \cdot \text{in}$$

Given the coarse scale on a torque wrench, specify a torque wrench setting of 400 lbf · in. Check the factor of safety

$$n = \frac{2\pi f T}{0.18 F_x d} = \frac{2\pi(0.12)(400)}{0.18(1000)(0.375)} = 4.47$$

8-48



Bolts: $S_p = 380$ MPa, $S_y = 420$ MPa
Channel: $t = 6.4$ mm, $S_y = 170$ MPa
Cantilever: $S_y = 190$ MPa
Nut: $H = 10.8$ mm

$$F'_A = F'_B = F'_C = F/3$$

$$M = (50 + 26 + 125)F = 201F$$

$$F''_A = F''_C = \frac{201F}{2(50)} = 2.01F$$

$$F_C = F'_C + F''_C = \left(\frac{1}{3} + 2.01\right) F = 2.343F$$

Bolts:
The shear bolt area is $A = \pi(12^2)/4 = 113.1 \text{ mm}^2$

$$S_{sy} = 0.577(420) = 242.3 \text{ MPa}$$

$$F = \frac{S_{sy}}{n}\left(\frac{A}{2.343}\right) = \frac{242.3(113.1)(10^{-3})}{2.8(2.343)} = 4.18 \text{ kN}$$

Bearing on bolt: For a 12-mm bolt, at the channel,

$$A_b = td = (6.4)(12) = 76.8 \text{ mm}^2$$

$$F = \frac{S_y}{n}\left(\frac{A_b}{2.343}\right) = \frac{420}{2.8}\left[\frac{76.8(10^{-3})}{2.343}\right] = 4.92 \text{ kN}$$

Bearing on channel: $A_b = 76.8 \text{ mm}^2$, $S_y = 170$ MPa

$$F = \frac{170}{2.8}\left[\frac{76.8(10^{-3})}{2.343}\right] = 1.99 \text{ kN}$$

Bearing on cantilever:

$$A_b = 12(12) = 144\ \text{mm}^2$$

$$F = \frac{190}{2.8}\left[\frac{(144)(10^{-3})}{2.343}\right] = 4.17\ \text{kN}$$

Bending of cantilever:

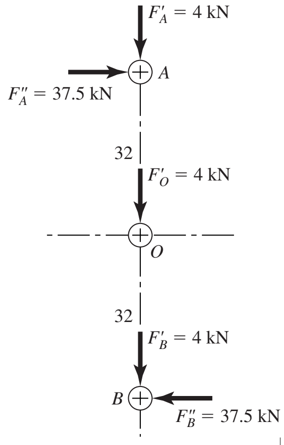
$$I = \frac{1}{12}(12)(50^3 - 12^3) = 1.233(10^5)\ \text{mm}^4$$

$$\frac{I}{c} = \frac{1.233(10^5)}{25} = 4932$$

$$F = \frac{M}{151} = \frac{4932(190)}{2.8(151)(10^3)} = 2.22\ \text{kN}$$

So $F = 1.99$ kN based on bearing on channel *Ans.*

8-49 $F' = 4$ kN; $M = 12(200) = 2400\ \text{N}\cdot\text{m}$



$$F''_A = F''_B = \frac{2400}{64} = 37.5\ \text{kN}$$

$$F_A = F_B = \sqrt{(4)^2 + (37.5)^2} = 37.7\ \text{kN} \quad \textit{Ans.}$$

$$F_O = 4\ \text{kN} \quad \textit{Ans.}$$

Bolt shear:

$$A_s = \frac{\pi(12)^2}{4} = 113\ \text{mm}^2$$

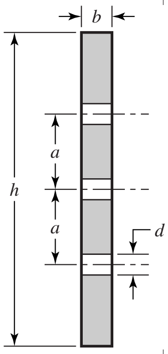
$$\tau = \frac{37.7(10)^3}{113} = 334\ \text{MPa} \quad \textit{Ans.}$$

Bearing on member:

$$A_b = 12(8) = 96\ \text{mm}^2$$

$$\sigma = -\frac{37.7(10)^3}{96} = -393\ \text{MPa} \quad \textit{Ans.}$$

Bending stress in plate:



$$I = \frac{bh^3}{12} - \frac{bd^3}{12} - 2\left(\frac{bd^3}{12} + a^2bd\right)$$

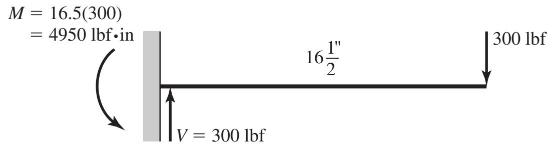
$$= \frac{8(136)^3}{12} - \frac{8(12)^3}{12} - 2\left[\frac{8(12)^3}{12} + (32)^2(8)(12)\right]$$

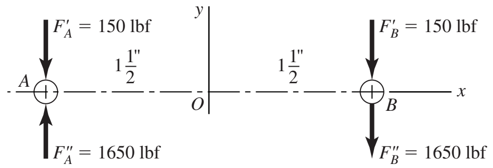
$$= 1.48(10)^6\ \text{mm}^4 \quad \textit{Ans.}$$

$$M = 12(200) = 2400\ \text{N}\cdot\text{m}$$

$$\sigma = \frac{Mc}{I} = \frac{2400(68)}{1.48(10)^6}(10)^3 = 110\ \text{MPa} \quad \textit{Ans.}$$

8-50





Shear of bolt:

$$A_s = \frac{\pi}{4}(0.5)^2 = 0.1963 \text{ in}^2$$

$$\tau = \frac{F}{A} = \frac{1800}{0.1963} = 9170 \text{ psi}$$

$$S_{sy} = 0.577(92) = 53.08 \text{ kpsi}$$

$$n = \frac{53.08}{9.17} = 5.79 \quad \textit{Ans.}$$

$$F''_A = F''_B = \frac{4950}{3} = 1650 \text{ lbf}$$

$$F_A = 1500 \text{ lbf}, \qquad F_B = 1800 \text{ lbf}$$

Bearing on bolt:

$$A_b = \frac{1}{2}\left(\frac{3}{8}\right) = 0.1875 \text{ in}^2$$

$$\sigma = -\frac{F}{A} = -\frac{1800}{0.1875} = -9600 \text{ psi}$$

$$n = \frac{92}{9.6} = 9.58 \quad \textit{Ans.}$$

Bearing on members: $S_y = 54$ kpsi, $n = \dfrac{54}{9.6} = 5.63$ *Ans.*

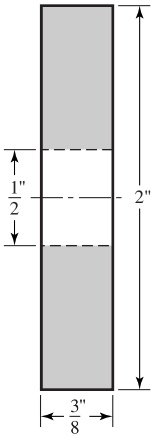
Bending of members: Considering the right-hand bolt

$$M = 300(15) = 4500 \text{ lbf} \cdot \text{in}$$

$$I = \frac{0.375(2)^3}{12} - \frac{0.375(0.5)^3}{12} = 0.246 \text{ in}^4$$

$$\sigma = \frac{Mc}{I} = \frac{4500(1)}{0.246} = 18\,300 \text{ psi}$$

$$n = \frac{54(10)^3}{18\,300} = 2.95 \quad \textit{Ans.}$$



8-51 The direct shear load per bolt is $F' = 2500/6 = 417$ lbf. The moment is taken only by the four outside bolts. This moment is $M = 2500(5) = 12\,500$ lbf · in.

Thus $F'' = \dfrac{12\,500}{2(5)} = 1250$ lbf and the resultant bolt load is

$$F = \sqrt{(417)^2 + (1250)^2} = 1318 \text{ lbf}$$

Bolt strength, $S_y = 57$ kpsi; Channel strength, $S_y = 46$ kpsi; Plate strength, $S_y = 45.5$ kpsi

Shear of bolt: $A_s = \pi(0.625)^2/4 = 0.3068 \text{ in}^2$

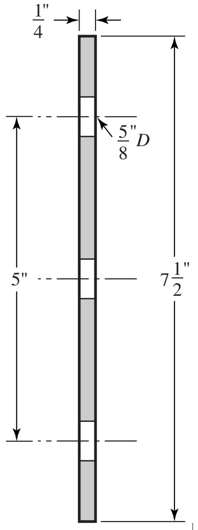
$$n = \frac{S_{sy}}{\tau} = \frac{(0.577)(57\,000)}{1318/0.3068} = 7.66 \quad \textit{Ans.}$$

Bearing on bolt: Channel thickness is $t = 3/16$ in;

$$A_b = (0.625)(3/16) = 0.117\ \text{in}^2;\ n = \frac{57\,000}{1318/0.117} = 5.07 \quad \textit{Ans.}$$

Bearing on channel:
$$n = \frac{46\,000}{1318/0.117} = 4.08 \quad \textit{Ans.}$$

Bearing on plate:
$$A_b = 0.625(1/4) = 0.1563\ \text{in}^2$$
$$n = \frac{45\,500}{1318/0.1563} = 5.40 \quad \textit{Ans.}$$



Bending of plate:

$$I = \frac{0.25(7.5)^3}{12} - \frac{0.25(0.625)^3}{12}$$
$$-2\left[\frac{0.25(0.625)^3}{12} + \left(\frac{1}{4}\right)\left(\frac{5}{8}\right)(2.5)^2\right] = 6.821\ \text{in}^4$$

$$M = 6250\ \text{lbf} \cdot \text{in per plate}$$

$$\sigma = \frac{Mc}{I} = \frac{6250(3.75)}{6.821} = 3436\ \text{psi}$$

$$n = \frac{45\,500}{3436} = 13.2 \quad \textit{Ans.}$$

8-52 Specifying bolts, screws, dowels and rivets is the way a student learns about such components. However, choosing an array a priori is based on experience. Here is a chance for students to build some experience.

8-53 Now that the student can put an a priori decision of an array together with the specification of fasteners.

8-54 A computer program will vary with computer language or software application.

Chapter 9

9-1 Eq. (9-3):

$$F = 0.707hl\tau = 0.707(5/16)(4)(20) = 17.7 \text{ kip} \quad \textit{Ans.}$$

9-2 Table 9-6: $\tau_{\text{all}} = 21.0$ kpsi

$$\begin{aligned} f &= 14.85h \text{ kip/in} \\ &= 14.85(5/16) = 4.64 \text{ kip/in} \\ F &= fl = 4.64(4) = 18.56 \text{ kip} \quad \textit{Ans.} \end{aligned}$$

9-3 Table A-20:

1018 HR: $S_{ut} = 58$ kpsi, $S_y = 32$ kpsi
1018 CR: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned} \tau_{\text{all}} &= \min(0.30S_{ut}, 0.40S_y) \\ &= \min[0.30(58), 0.40(32)] \\ &= \min(17.4, 12.8) = 12.8 \text{ kpsi} \end{aligned}$$

for both materials.

Eq. (9-3):

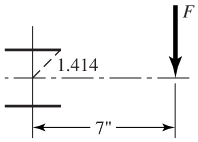
$$F = 0.707hl\tau_{\text{all}}$$

$$F = 0.707(5/16)(4)(12.8) = 11.3 \text{ kip} \quad \textit{Ans.}$$

9-4 Eq. (9-3)

$$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(32)}{(5/16)(4)(2)} = 18.1 \text{ kpsi} \quad \textit{Ans.}$$

9-5 $b = d = 2$ in



(a) *Primary shear* Table 9-1

$$\tau_y' = \frac{V}{A} = \frac{F}{1.414(5/16)(2)} = 1.13F \text{ kpsi}$$

Secondary shear Table 9-1

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{2[(3)(2^2) + 2^2]}{6} = 5.333\text{ in}^3$$

$$J = 0.707hJ_u = 0.707(5/16)(5.333) = 1.18\text{ in}^4$$

$$\tau_x'' = \tau_y'' = \frac{Mr_y}{J} = \frac{7F(1)}{1.18} = 5.93F\text{ kpsi}$$

Maximum shear

$$\tau_{\max} = \sqrt{\tau_x''^2 + (\tau_y' + \tau_y'')^2} = F\sqrt{5.93^2 + (1.13 + 5.93)^2} = 9.22F\text{ kpsi}$$

$$F = \frac{\tau_{\text{all}}}{9.22} = \frac{20}{9.22} = 2.17\text{ kip} \quad \textit{Ans.} \tag{1}$$

(b) For E7010 from Table 9-6, $\tau_{\text{all}} = 21$ kpsi

Table A-20:

HR 1020 Bar: $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi

HR 1015 Support: $S_{ut} = 50$ kpsi, $S_y = 27.5$ kpsi

Table 9-5, E7010 Electrode: $S_{ut} = 70$ kpsi, $S_y = 57$ kpsi

The support controls the design.

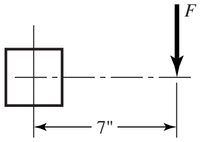
Table 9-4:

$$\tau_{\text{all}} = \min[0.30(50), 0.40(27.5)] = \min[15, 11] = 11\text{ kpsi}$$

The allowable load from Eq. (1) is

$$F = \frac{\tau_{\text{all}}}{9.22} = \frac{11}{9.22} = 1.19\text{ kip} \quad \textit{Ans.}$$

9-6 $b = d = 2$ in



Primary shear

$$\tau_y' = \frac{V}{A} = \frac{F}{1.414(5/16)(2 + 2)} = 0.566F$$

Secondary shear

Table 9-1: $$J_u = \frac{(b + d)^3}{6} = \frac{(2 + 2)^3}{6} = 10.67\text{ in}^3$$

$$J = 0.707hJ_u = 0.707(5/16)(10.67) = 2.36\text{ in}^4$$

$$\tau_x'' = \tau_y'' = \frac{Mr_y}{J} = \frac{(7F)(1)}{2.36} = 2.97F$$

Maximum shear

$$\tau_{\max} = \sqrt{\tau_x''^2 + (\tau_y' + \tau_y'')^2} = F\sqrt{2.97^2 + (0.556 + 2.97)^2} = 4.61F \text{ kpsi}$$

$$F = \frac{\tau_{\text{all}}}{4.61} \quad \textit{Ans.}$$

which is twice $\tau_{\max}/9.22$ of Prob. 9-5.

9-7 Weldment, subjected to alternating fatigue, has throat area of

$$A = 0.707(6)(60 + 50 + 60) = 721 \text{ mm}^2$$

Members' endurance limit: AISI 1010 steel

$$S_{ut} = 320 \text{ MPa}, \qquad S_e' = 0.5(320) = 160 \text{ MPa}$$

$$k_a = 272(320)^{-0.995} = 0.875$$

$$k_b = 1 \quad \text{(direct shear)}$$

$$k_c = 0.59 \quad \text{(shear)}$$

$$k_d = 1$$

$$k_f = \frac{1}{K_{fs}} = \frac{1}{2.7} = 0.370$$

$$S_{se} = 0.875(1)(0.59)(0.37)(160) = 30.56 \text{ MPa}$$

Electrode's endurance: 6010

$$S_{ut} = 62(6.89) = 427 \text{ MPa}$$

$$S_e' = 0.5(427) = 213.5 \text{ MPa}$$

$$k_a = 272(427)^{-0.995} = 0.657$$

$$k_b = 1 \quad \text{(direct shear)}$$

$$k_c = 0.59 \quad \text{(shear)}$$

$$k_d = 1$$

$$k_f = 1/K_{fs} = 1/2.7 = 0.370$$

$$S_{se} = 0.657(1)(0.59)(0.37)(213.5) = 30.62 \text{ MPa} \doteq 30.56$$

Thus, the members and the electrode are of equal strength. For a factor of safety of 1,

$$F_a = \tau_a A = 30.6(721)(10^{-3}) = 22.1 \text{ kN} \quad \textit{Ans.}$$

9-8 *Primary shear* $\tau' = 0$ (why?)

Secondary shear

Table 9-1:

$$J_u = 2\pi r^3 = 2\pi(4)^3 = 402 \text{ cm}^3$$

$$J = 0.707hJ_u = 0.707(0.5)(402) = 142 \text{ cm}^4$$

$$M = 200F \text{ N} \cdot \text{m} \quad (F \text{ in kN})$$

$$\tau'' = \frac{Mr}{2J} = \frac{(200F)(4)}{2(142)} = 2.82F \quad \text{(2 welds)}$$

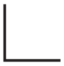
$$F = \frac{\tau_{\text{all}}}{\tau''} = \frac{140}{2.82} = 49.2 \text{ kN} \quad \textit{Ans.}$$

9-9

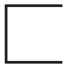
Rank

$$\text{fom}' = \frac{J_u}{lh} = \frac{a^3/12}{ah} = \frac{a^2}{12h} = 0.0833\left(\frac{a^2}{h}\right)$$ ⑤

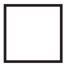
$$\text{fom}' = \frac{a(3a^2 + a^2)}{6(2a)h} = \frac{a^2}{3h} = 0.3333\left(\frac{a^2}{h}\right)$$ ①



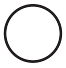
$$\text{fom}' = \frac{(2a)^4 - 6a^2a^2}{12(a+a)2ah} = \frac{5a^2}{24h} = 0.2083\left(\frac{a^2}{h}\right)$$ ④



$$\text{fom}' = \frac{1}{3ah}\left[\frac{8a^3 + 6a^3 + a^3}{12} - \frac{a^4}{2a+a}\right] = \frac{11}{36}\frac{a^2}{h} = 0.3056\left(\frac{a^2}{h}\right)$$ ②



$$\text{fom}' = \frac{(2a)^3}{6h}\frac{1}{4a} = \frac{8a^3}{24ah} = \frac{a^2}{3h} = 0.3333\left(\frac{a^2}{h}\right)$$ ①



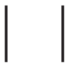
$$\text{fom}' = \frac{2\pi(a/2)^3}{\pi ah} = \frac{a^3}{4ah} = \frac{a^2}{4h} = 0.25\left(\frac{a^2}{h}\right)$$ ③

These rankings apply to fillet weld patterns in torsion that have a square area $a \times a$ in which to place weld metal. The object is to place as much metal as possible to the border. If your area is rectangular, your goal is the same but the rankings may change.

Students will be surprised that the circular weld bead does not rank first.

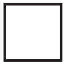
9-10

$$\text{fom}' = \frac{I_u}{lh} = \frac{1}{a}\left(\frac{a^3}{12}\right)\left(\frac{1}{h}\right) = \frac{1}{12}\left(\frac{a^2}{h}\right) = 0.0833\left(\frac{a^2}{h}\right)$$ ⑤

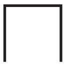


$$\text{fom}' = \frac{I_u}{lh} = \frac{1}{2ah}\left(\frac{a^3}{6}\right) = 0.0833\left(\frac{a^2}{h}\right)$$ ⑤

$$\text{fom}' = \frac{I_u}{lh} = \frac{1}{2ah}\left(\frac{a^2}{2}\right) = \frac{1}{4}\left(\frac{a^2}{h}\right) = 0.25\left(\frac{a^2}{h}\right)$$ ①



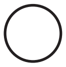
$$\text{fom}' = \frac{I_u}{lh} = \frac{1}{[2(2a)]h}\left(\frac{a^2}{6}\right)(3a+a) = \frac{1}{6}\left(\frac{a^2}{h}\right) = 0.1667\left(\frac{a^2}{h}\right) \quad ②$$



$$\bar{x} = \frac{b}{2} = \frac{a}{2}, \quad \bar{y} = \frac{d^2}{b+2d} = \frac{a^2}{3a} = \frac{a}{3}$$

$$I_u = \frac{2d^3}{3} - 2d^2\left(\frac{a}{3}\right) + (b+2d)\left(\frac{a^2}{9}\right) = \frac{2a^3}{3} - \frac{2a^3}{3} + 3a\left(\frac{a^2}{9}\right) = \frac{a^3}{3}$$

$$\text{fom}' = \frac{I_u}{lh} = \frac{a^3/3}{3ah} = \frac{1}{9}\left(\frac{a^2}{h}\right) = 0.1111\left(\frac{a^2}{h}\right) \quad ④$$



$$I_u = \pi r^3 = \frac{\pi a^3}{8}$$

$$\text{fom}' = \frac{I_u}{lh} = \frac{\pi a^3/8}{\pi a h} = \frac{a^2}{8h} = 0.125\left(\frac{a^2}{h}\right) \quad ③$$

The CEE-section pattern was not ranked because the deflection of the beam is out-of-plane.

If you have a square area in which to place a fillet weldment pattern under bending, your objective is to place as much material as possible away from the *x*-axis. If your area is rectangular, your goal is the same, but the rankings may change.

9-11 Materials:

Attachment (1018 HR) $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

Member (A36) $S_y = 36$ kpsi, S_{ut} ranges from 58 to 80 kpsi, use 58.

The member and attachment are weak compared to the E60XX electrode.

Decision Specify E6010 electrode

Controlling property: $\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = \min(16.6, 12.8) = 12.8$ kpsi

For a static load the parallel and transverse fillets are the same. If *n* is the number of beads,

$$\tau = \frac{F}{n(0.707)hl} = \tau_{\text{all}}$$

$$nh = \frac{F}{0.707 l \tau_{\text{all}}} = \frac{25}{0.707(3)(12.8)} = 0.921$$

Make a table.

Number of beads n	Leg size h
1	0.921
2	0.460 → 1/2"
3	0.307 → 5/16"
4	0.230 → 1/4"

Decision: Specify 1/4" leg size

Decision: Weld all-around

Weldment Specifications:

Pattern: All-around square
Electrode: E6010
Type: Two parallel fillets *Ans.*
Two transverse fillets
Length of bead: 12 in
Leg: 1/4 in

For a figure of merit of, in terms of weldbead volume, is this design optimal?

9-12 *Decision:* Choose a parallel fillet weldment pattern. By so-doing, we've chosen an optimal pattern (see Prob. 9-9) and have thus reduced a synthesis problem to an analysis problem:

Table 9-1: $A = 1.414hd = 1.414(h)(3) = 4.24h \text{ in}^3$

Primary shear

$$\tau'_y = \frac{V}{A} = \frac{3000}{4.24h} = \frac{707}{h}$$

Secondary shear

Table 9-1: $$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{3[3(3^2) + 3^2]}{6} = 18 \text{ in}^3$$

$$J = 0.707(h)(18) = 12.7h \text{ in}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{3000(7.5)(1.5)}{12.7h} = \frac{2657}{h} = \tau''_y$$

$$\tau_{\max} = \sqrt{\tau''^2_x + (\tau'_y + \tau''_y)^2} = \frac{1}{h}\sqrt{2657^2 + (707 + 2657)^2} = \frac{4287}{h}$$

Attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

Member (A36): $S_y = 36$ kpsi

The attachment is weaker

Decision: Use E60XX electrode

$$\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = 12.8 \text{ kpsi}$$

$$\tau_{\max} = \tau_{\text{all}} = \frac{4287}{h} = 12\,800 \text{ psi}$$

$$h = \frac{4287}{12\,800} = 0.335 \text{ in}$$

Decision: Specify 3/8" leg size

Weldment Specifications:
Pattern: Parallel fillet welds
Electrode: E6010
Type: Fillet *Ans.*
Length of bead: 6 in
Leg size: 3/8 in

9-13 An optimal square space (3" × 3") weldment pattern is $||$ or $\overline{\underline{\quad}}$ or □. In Prob. 9-12, there was roundup of leg size to $3/8$ in. Consider the member material to be structural A36 steel.

Decision: Use a parallel horizontal weld bead pattern for welding optimization and convenience.

Materials:

Attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

Member (A36): $S_y = 36$ kpsi, S_{ut} 58–80 kpsi; use 58 kpsi

From Table 9-4 AISC welding code,

$$\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = \min(16.6, 12.8) = 12.8 \text{ kpsi}$$

Select a stronger electrode material from Table 9-3.

Decision: Specify E6010

Throat area and other properties:

$$A = 1.414hd = 1.414(h)(3) = 4.24h \text{ in}^2$$

$$\bar{x} = b/2 = 3/2 = 1.5 \text{ in}$$

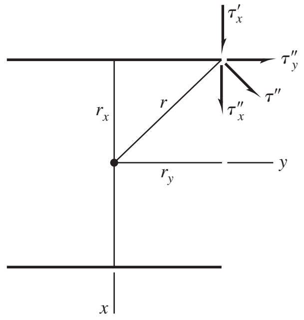
$$\bar{y} = d/2 = 3/2 = 1.5 \text{ in}$$

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{3[3(3^2) + 3^2]}{6} = 18 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(h)(18) = 12.73h \text{ in}^4$$

Primary shear:

$$\tau'_x = \frac{V}{A} = \frac{3000}{4.24h} = \frac{707.5}{h}$$



Secondary shear:

$$\tau'' = \frac{Mr}{J}$$

$$\tau''_x = \tau'' \cos 45^\circ = \frac{Mr}{J} \cos 45^\circ = \frac{Mr_x}{J}$$

$$\tau''_x = \frac{3000(6 + 1.5)(1.5)}{12.73h} = \frac{2651}{h}$$

$$\tau''_y = \tau''_x = \frac{2651}{h}$$

$$\tau_{\max} = \sqrt{(\tau_x'' + \tau_x')^2 + \tau_y''^2}$$

$$= \frac{1}{h}\sqrt{(2651 + 707.5)^2 + 2651^2}$$

$$= \frac{4279}{h} \text{ psi}$$

Relate stress and strength:

$$\tau_{\max} = \tau_{\text{all}}$$

$$\frac{4279}{h} = 12\,800$$

$$h = \frac{4279}{12\,800} = 0.334 \text{ in} \rightarrow 3/8 \text{ in}$$

Weldment Specifications:

Pattern: Horizontal parallel weld tracks
Electrode: E6010
Type of weld: Two parallel fillet welds
Length of bead: 6 in
Leg size: 3/8 in

Additional thoughts:

Since the round-up in leg size was substantial, why not investigate a backward C ⊐ weld pattern. One might then expect shorter horizontal weld beads which will have the advantage of allowing a shorter member (assuming the member has not yet been designed). This will show the inter-relationship between attachment design and supporting members.

9-14 *Materials:*

Member (A36): $S_y = 36$ kpsi, $S_{ut} = 58$ to 80 kpsi; use $S_{ut} = 58$ kpsi

Attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

$$\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = 12.8 \text{ kpsi}$$

Decision: Use E6010 electrode. From Table 9-3: $S_y = 50$ kpsi, $S_{ut} = 62$ kpsi, $\tau_{\text{all}} = \min[0.3(62), 0.4(50)] = 20$ kpsi

Decision: Since A36 and 1018 HR are weld metals to an unknown extent, use $\tau_{\text{all}} = 12.8$ kpsi

Decision: Use the most efficient weld pattern–square, weld-all-around. Choose 6" × 6" size.

Attachment length:

$$l_1 = 6 + a = 6 + 6.25 = 12.25 \text{ in}$$

Throat area and other properties:

$$A = 1.414h(b + d) = 1.414(h)(6 + 6) = 17.0h$$

$$\bar{x} = \frac{b}{2} = \frac{6}{2} = 3 \text{ in}, \quad \bar{y} = \frac{d}{2} = \frac{6}{2} = 3 \text{ in}$$

Primary shear

$$\tau_y' = \frac{V}{A} = \frac{F}{A} = \frac{20\,000}{17h} = \frac{1176}{h} \text{ psi}$$

Secondary shear

$$J_u = \frac{(b+d)^3}{6} = \frac{(6+6)^3}{6} = 288 \text{ in}^3$$

$$J = 0.707h(288) = 203.6h \text{ in}^4$$

$$\tau_x'' = \tau_y'' = \frac{Mr_y}{J} = \frac{20\,000(6.25+3)(3)}{203.6h} = \frac{2726}{h} \text{ psi}$$

$$\tau_{\max} = \sqrt{\tau_x''^2 + (\tau_y'' + \tau_y')^2} = \frac{1}{h}\sqrt{2726^2 + (2726+1176)^2} = \frac{4760}{h} \text{ psi}$$

Relate stress to strength

$$\tau_{\max} = \tau_{\text{all}}$$

$$\frac{4760}{h} = 12\,800$$

$$h = \frac{4760}{12\,800} = 0.372 \text{ in}$$

Decision:
Specify 3/8 in leg size

Specifications:
Pattern: All-around square weld bead track
Electrode: E6010
Type of weld: Fillet
Weld bead length: 24 in
Leg size: 3/8 in
Attachment length: 12.25 in

9-15 This is a good analysis task to test the students' understanding

(1) Solicit information related to a priori decisions.

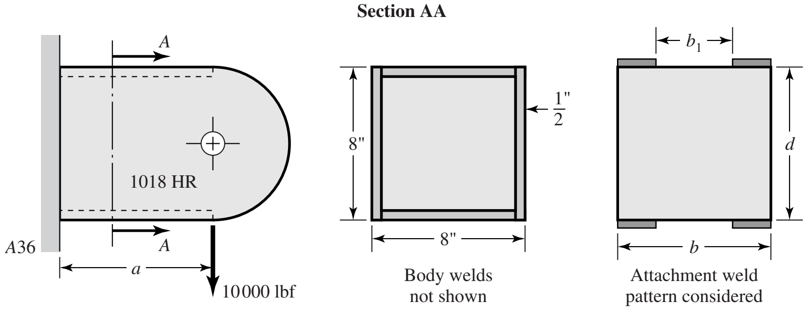
(2) Solicit design variables *b* and *d*.

(3) Find *h* and round and output all parameters on a single screen. Allow return to Step 1 or Step 2.

(4) When the iteration is complete, the final display can be the bulk of your adequacy assessment.

Such a program can teach too.

9-16 The objective of this design task is to have the students teach themselves that the weld patterns of Table 9-3 can be added or subtracted to obtain the properties of a comtemplated weld pattern. The instructor can control the level of complication. I have left the presentation of the drawing to you. Here is one possibility. Study the problem's opportunities, then present this (or your sketch) with the problem assignment.



Use b_1 as the design variable. Express properties as a function of b_1. From Table 9-3, category 3:

$$A = 1.414h(b - b_1)$$

$$\bar{x} = b/2, \quad \bar{y} = d/2$$

$$I_u = \frac{bd^2}{2} - \frac{b_1 d^2}{2} = \frac{(b - b_1)d^2}{2}$$

$$I = 0.707hI_u$$

$$\tau' = \frac{V}{A} = \frac{F}{1.414h(b - b_1)}$$

$$\tau'' = \frac{Mc}{I} = \frac{Fa(d/2)}{0.707hI_u}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2}$$

Parametric study

Let $a = 10$ in, $b = 8$ in, $d = 8$ in, $b_1 = 2$ in, $\tau_{\text{all}} = 12.8$ kpsi, $l = 2(8 - 2) = 12$ in

$$A = 1.414h(8 - 2) = 8.48h \text{ in}^2$$

$$I_u = (8 - 2)(8^2/2) = 192 \text{ in}^3$$

$$I = 0.707(h)(192) = 135.7h \text{ in}^4$$

$$\tau' = \frac{10\,000}{8.48h} = \frac{1179}{h} \text{ psi}$$

$$\tau'' = \frac{10\,000(10)(8/2)}{135.7h} = \frac{2948}{h} \text{ psi}$$

$$\tau_{\max} = \frac{1}{h}\sqrt{1179^2 + 2948^2} = \frac{3175}{h} = 12\,800$$

from which $h = 0.248$ in. Do not round off the leg size – something to learn.

$$\text{fom}' = \frac{I_u}{hl} = \frac{192}{0.248(12)} = 64.5$$

$$A = 8.48(0.248) = 2.10 \text{ in}^2$$

$$I = 135.7(0.248) = 33.65 \text{ in}^4$$

$$\text{vol} = \frac{h^2}{2}l = \frac{0.248^2}{2}12 = 0.369\,\text{in}^3$$

$$\frac{I}{\text{vol}} = \frac{33.65}{0.369} = 91.2 = \text{eff}$$

$$\tau' = \frac{1179}{0.248} = 4754\,\text{psi}$$

$$\tau'' = \frac{2948}{0.248} = 11\,887\,\text{psi}$$

$$\tau_{\max} = \frac{4127}{0.248} \doteq 12\,800\,\text{psi}$$

Now consider the case of uninterrupted welds,

$$b_1 = 0$$

$$A = 1.414(h)(8-0) = 11.31h$$

$$I_u = (8-0)(8^2/2) = 256\,\text{in}^3$$

$$I = 0.707(256)h = 181h\,\text{in}^4$$

$$\tau' = \frac{10\,000}{11.31h} = \frac{884}{h}$$

$$\tau'' = \frac{10\,000(10)(8/2)}{181h} = \frac{2210}{h}$$

$$\tau_{\max} = \frac{1}{h}\sqrt{884^2 + 2210^2} = \frac{2380}{h} = \tau_{\text{all}}$$

$$h = \frac{\tau_{\max}}{\tau_{\text{all}}} = \frac{2380}{12\,800} = 0.186\,\text{in}$$

Do not round off *h*.

$$A = 11.31(0.186) = 2.10\,\text{in}^2$$

$$I = 181(0.186) = 33.67$$

$$\tau' = \frac{884}{0.186} = 4753\,\text{psi}, \quad \text{vol} = \frac{0.186^2}{2}16 = 0.277\,\text{in}^3$$

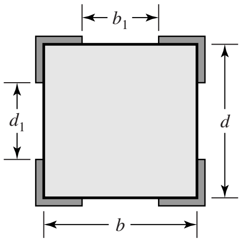
$$\tau'' = \frac{2210}{0.186} = 11\,882\,\text{psi}$$

$$\text{fom}' = \frac{I_u}{hl} = \frac{256}{0.186(16)} = 86.0$$

$$\text{eff} = \frac{I}{(h^2/2)l} = \frac{33.67}{(0.186^2/2)16} = 121.7$$

Conclusions: To meet allowable stress limitations, *I* and *A* do not change, nor do τ and σ. To meet the shortened bead length, *h* is increased proportionately. However, volume of bead laid down increases as h^2. The uninterrupted bead is superior. In this example, we did not round *h* and as a result we learned something. Our measures of merit are also sensitive to rounding. When the design decision is made, rounding to the next larger standard weld fillet size will decrease the merit.

Had the weld bead gone around the corners, the situation would change. Here is a followup task analyzing an alternative weld pattern.



9-17 From Table 9-2

For the box $A = 1.414h(b + d)$

Subtracting b_1 from b and d_1 from d

$$A = 1.414\,h(b - b_1 + d - d_1)$$

$$I_u = \frac{d^2}{6}(3b + d) - \frac{d_1^3}{6} - \frac{b_1 d^2}{2}$$

$$= \frac{1}{2}(b - b_1)d^2 + \frac{1}{6}\left(d^3 - d_1^3\right)$$

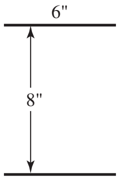
length of bead $l = 2(b - b_1 + d - d_1)$

$$\text{fom} = I_u/hl$$

9-18 Computer programs will vary.

9-19 $\tau_{\text{all}} = 12\,800$ psi. Use Fig. 9-17(a) for general geometry, but employ = beads and then || beads.

Horizontal parallel weld bead pattern



$b = 6$ in
$d = 8$ in

From Table 9-2, category 3

$$A = 1.414\,hb = 1.414(h)(6) = 8.48\,h\text{ in}^2$$

$$\bar{x} = b/2 = 6/2 = 3\text{ in}, \qquad \bar{y} = d/2 = 8/2 = 4\text{ in}$$

$$I_u = \frac{bd^2}{2} = \frac{6(8)^2}{2} = 192\text{ in}^3$$

$$I = 0.707hI_u = 0.707(h)(192) = 135.7h\text{ in}^4$$

$$\tau' = \frac{10\,000}{8.48h} = \frac{1179}{h}\text{ psi}$$

$$\tau'' = \frac{Mc}{I} = \frac{10\,000(10)(8/2)}{135.7h} = \frac{2948}{h}\text{ psi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h}(1179^2 + 2948^2)^{1/2} = \frac{3175}{h}\text{ psi}$$

Equate the maximum and allowable shear stresses.

$$\tau_{\max} = \tau_{\text{all}} = \frac{3175}{h} = 12\,800$$

from which $h = 0.248$ in. It follows that

$$I = 135.7(0.248) = 33.65\text{ in}^4$$

The volume of the weld metal is

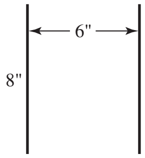
$$\text{vol} = \frac{h^2 l}{2} = \frac{0.248^2(6+6)}{2} = 0.369\text{ in}^3$$

The effectiveness, $(\text{eff})_{\text{H}}$, is

$$(\text{eff})_{\text{H}} = \frac{I}{\text{vol}} = \frac{33.65}{0.369} = 91.2\text{ in}$$

$$(\text{fom}')_{\text{H}} = \frac{I_u}{hl} = \frac{192}{0.248(6+6)} = 64.5\text{ in}$$

Vertical parallel weld beads



$b = 6$ in
$d = 8$ in

From Table 9-2, category 2

$$A = 1.414hd = 1.414(h)(8) = 11.31h\text{ in}^2$$

$$\bar{x} = b/2 = 6/2 = 3\text{ in}, \qquad \bar{y} = d/2 = 8/2 = 4\text{ in}$$

$$I_u = \frac{d^3}{6} = \frac{8^3}{6} = 85.33\text{ in}^3$$

$$I = 0.707hI_u = 0.707(h)(85.33) = 60.3h$$

$$\tau' = \frac{10\,000}{11.31h} = \frac{884}{h}\text{ psi}$$

$$\tau'' = \frac{Mc}{I} = \frac{10\,000(10)(8/2)}{60.3\,h} = \frac{6633}{h}\text{ psi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h}(884^2 + 6633^2)^{1/2}$$

$$= \frac{6692}{h}\text{ psi}$$

Equating $\tau_{\max}$ to τ_{all} gives $h = 0.523$ in. It follows that

$$I = 60.3(0.523) = 31.5\ \text{in}^4$$

$$\text{vol} = \frac{h^2 l}{2} = \frac{0.523^2}{2}(8+8) = 2.19\ \text{in}^3$$

$$(\text{eff})_\text{V} = \frac{I}{\text{vol}} = \frac{31.6}{2.19} = 14.4\ \text{in}$$

$$(\text{fom}')_\text{V} = \frac{I_u}{hl} = \frac{85.33}{0.523(8+8)} = 10.2\ \text{in}$$

The ratio of $(\text{eff})_\text{V}/(\text{eff})_\text{H}$ is $14.4/91.2 = 0.158$. The ratio $(\text{fom}')_\text{V}/(\text{fom}')_\text{H}$ is $10.2/64.5 = 0.158$. This is not surprising since

$$\text{eff} = \frac{I}{\text{vol}} = \frac{I}{(h^2/2)l} = \frac{0.707\,hI_u}{(h^2/2)l} = 1.414\,\frac{I_u}{hl} = 1.414\ \text{fom}'$$

The ratios $(\text{eff})_\text{V}/(\text{eff})_\text{H}$ and $(\text{fom}')_\text{V}/(\text{fom}')_\text{H}$ give the same information.

9-20 Because the loading is pure torsion, there is no primary shear. From Table 9-1, category 6:

$$J_u = 2\pi r^3 = 2\pi(1)^3 = 6.28\ \text{in}^3$$

$$J = 0.707\,hJ_u = 0.707(0.25)(6.28)$$

$$= 1.11\ \text{in}^4$$

$$\tau = \frac{Tr}{J} = \frac{20(1)}{1.11} = 18.0\ \text{kpsi} \quad \textit{Ans.}$$

9-21

$$h = 0.375\ \text{in}, \quad d = 8\ \text{in}, \quad b = 1\ \text{in}$$

From Table 9-2, category 2:

$$A = 1.414(0.375)(8) = 4.24\ \text{in}^2$$

$$I_u = \frac{d^3}{6} = \frac{8^3}{6} = 85.3\ \text{in}^3$$

$$I = 0.707hI_u = 0.707(0.375)(85.3) = 22.6\ \text{in}^4$$

$$\tau' = \frac{F}{A} = \frac{5}{4.24} = 1.18\ \text{kpsi}$$

$$M = 5(6) = 30\ \text{kip}\cdot\text{in}$$

$$c = (1+8+1-2)/2 = 4\ \text{in}$$

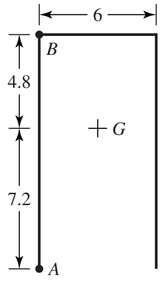
$$\tau'' = \frac{Mc}{I} = \frac{30(4)}{22.6} = 5.31\ \text{kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{1.18^2 + 5.31^2}$$

$$= 5.44\ \text{kpsi} \quad \textit{Ans.}$$

9-22 $h = 0.6\,\text{cm}, \quad b = 6\,\text{cm}, \quad d = 12\,\text{cm}.$

Table 9-3, category 5:



$$A = 0.707h(b + 2d)$$

$$= 0.707(0.6)[6 + 2(12)] = 12.7\,\text{cm}^2$$

$$\bar{y} = \frac{d^2}{b + 2d} = \frac{12^2}{6 + 2(12)} = 4.8\,\text{cm}$$

$$I_u = \frac{2d^3}{3} - 2d^2\bar{y} + (b + 2d)\bar{y}^2$$

$$= \frac{2(12)^3}{3} - 2(12^2)(4.8) + [6 + 2(12)]4.8^2$$

$$= 461\,\text{cm}^3$$

$$I = 0.707hI_u = 0.707(0.6)(461) = 196\,\text{cm}^4$$

$$\tau' = \frac{F}{A} = \frac{7.5(10^3)}{12.7(10^2)} = 5.91\text{ MPa}$$

$$M = 7.5(120) = 900\,\text{N}\cdot\text{m}$$

$$c_A = 7.2\,\text{cm}, \qquad c_B = 4.8\,\text{cm}$$

The critical location is at *A*.

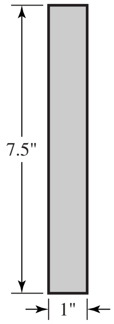
$$\tau''_A = \frac{Mc_A}{I} = \frac{900(7.2)}{196} = 33.1\,\text{MPa}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = (5.91^2 + 33.1^2)^{1/2} = 33.6\,\text{MPa}$$

$$n = \frac{\tau_{\text{all}}}{\tau_{\max}} = \frac{120}{33.6} = 3.57 \quad \textit{Ans.}$$

9-23 The largest possible weld size is 1/16 in. This is a small weld and thus difficult to accomplish. The bracket's load-carrying capability is not known. There are geometry problems associated with sheet metal folding, load-placement and location of the center of twist. This is not available to us. We will identify the strongest possible weldment.

Use a rectangular, weld-all-around pattern – Table 9-2, category 6:



$$A = 1.414\,h(b + d)$$

$$= 1.414(1/16)(1 + 7.5)$$

$$= 0.751\,\text{in}^2$$

$$\bar{x} = b/2 = 0.5\,\text{in}$$

$$\bar{y} = \frac{d}{2} = \frac{7.5}{2} = 3.75\,\text{in}$$

$$I_u = \frac{d^2}{6}(3b + d) = \frac{7.5^2}{6}[3(1) + 7.5] = 98.4\,\text{in}^3$$

$$I = 0.707hI_u = 0.707(1/16)(98.4) = 4.35\,\text{in}^4$$

$$M = (3.75 + 0.5)W = 4.25W$$

$$\tau' = \frac{V}{A} = \frac{W}{0.751} = 1.332W$$

$$\tau'' = \frac{Mc}{I} = \frac{4.25W(7.5/2)}{4.35} = 3.664W$$

$$\tau_{\text{max}} = \sqrt{\tau'^2 + \tau''^2} = W\sqrt{1.332^2 + 3.664^2} = 3.90W$$

Material properties: The allowable stress given is low. Let's demonstrate that.

For the A36 structural steel member, $S_y = 36\,\text{kpsi}$ and $S_{ut} = 58\,\text{kpsi}$. For the 1020 CD attachment, use HR properties of $S_y = 30\,\text{kpsi}$ and $S_{ut} = 55$. The E6010 electrode has strengths of $S_y = 50$ and $S_{ut} = 62\,\text{kpsi}$.

Allowable stresses:

A36: $$\tau_{\text{all}} = \min[0.3(58), 0.4(36)] = \min(17.4, 14.4) = 14.4\,\text{kpsi}$$

1020: $$\tau_{\text{all}} = \min[0.3(55), 0.4(30)]$$
$$\tau_{\text{all}} = \min(16.5, 12) = 12\,\text{kpsi}$$

E6010: $$\tau_{\text{all}} = \min[0.3(62), 0.4(50)] = \min(18.6, 20) = 18.6\,\text{kpsi}$$

Since Table 9-6 gives 18.0 kpsi as the allowable shear stress, use this lower value.

Therefore, the allowable shear stress is

$$\tau_{\text{all}} = \min(14.4, 12, 18.0) = 12\,\text{kpsi}$$

However, the allowable stress in the problem statement is 0.9 kpsi which is low from the weldment perspective. The load associated with this strength is

$$\tau_{\text{max}} = \tau_{\text{all}} = 3.90W = 900$$

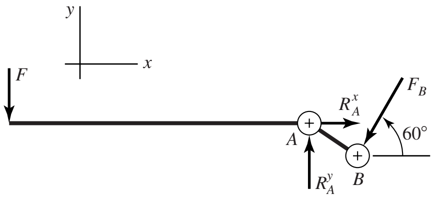
$$W = \frac{900}{3.90} = 231\,\text{lbf}$$

If the welding can be accomplished (1/16 leg size is a small weld), the weld strength is 12 000 psi and the load $W = 3047\,\text{lbf}$. Can the bracket carry such a load?

There are geometry problems associated with sheet metal folding. Load placement is important and the center of twist has not been identified. Also, the load-carrying capability of the top bend is unknown.

These uncertainties may require the use of a different weld pattern. Our solution provides the best weldment and thus insight for comparing a welded joint to one which employs screw fasteners.

9-24



$$F = 100\,\text{lbf}, \qquad \tau_{\text{all}} = 3\,\text{kpsi}$$

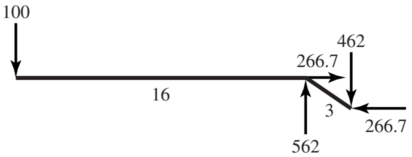
$$F_B = 100(16/3) = 533.3\,\text{lbf}$$

$$F_B^x = -533.3\cos 60^\circ = -266.7\,\text{lbf}$$

$$F_B^y = -533.3\cos 30^\circ = -462\,\text{lbf}$$

It follows that $R_A^y = 562\,\text{lbf}$ and $R_A^x = 266.7\,\text{lbf}$, $R_A = 622\,\text{lbf}$

$$M = 100(16) = 1600\,\text{lbf}\cdot\text{in}$$



The OD of the tubes is 1 in. From Table 9-1, category 6:

$$\begin{aligned} A &= 1.414(\pi h r)(2) \\ &= 2(1.414)(\pi h)(1/2) = 4.44h\,\text{in}^2 \\ J_u &= 2\pi r^3 = 2\pi(1/2)^3 = 0.785\,\text{in}^3 \\ J &= 2(0.707)hJ_u = 1.414(0.785)h = 1.11h\,\text{in}^4 \\ \tau' &= \frac{V}{A} = \frac{622}{4.44h} = \frac{140}{h} \\ \tau'' &= \frac{Tc}{J} = \frac{Mc}{J} = \frac{1600(0.5)}{1.11h} = \frac{720.7}{h} \end{aligned}$$

The shear stresses, τ' and τ'', are additive algebraically

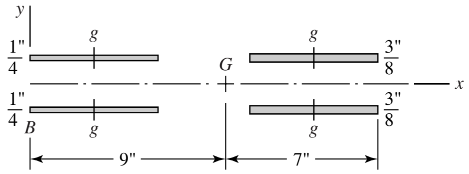
$$\tau_{\max} = \frac{1}{h}(140 + 720.7) = \frac{861}{h}\ \text{psi}$$

$$\tau_{\max} = \tau_{\text{all}} = \frac{861}{h} = 3000$$

$$h = \frac{861}{3000} = 0.287 \to 5/16"$$

Decision: Use 5/16 in fillet welds *Ans.*

9-25



For the pattern in bending shown, find the centroid G of the weld group.

$$\begin{aligned}\bar{x} &= \frac{6(0.707)(1/4)(3) + 6(0.707)(3/8)(13)}{6(0.707)(1/4) + 6(0.707)(3/8)} \\ &= 9\text{ in} \\ I_{1/4} &= 2\left(I_G + A_{\bar{x}}^{2}\right) \\ &= 2\left[\frac{0.707(1/4)(6^3)}{12} + 0.707(1/4)(6)(6^2)\right] \\ &= 82.7\text{ in}^4 \\ I_{3/8} &= 2\left[\frac{0.707(3/8)(6^3)}{12} + 0.707(3/8)(6)(4^2)\right] \\ &= 60.4\text{ in}^4 \\ I &= I_{1/4} + I_{3/8} = 82.7 + 60.4 = 143.1\text{ in}^4\end{aligned}$$

The critical location is at B. From Eq. (9-3),

$$\begin{aligned}\tau' &= \frac{F}{2[6(0.707)(3/8 + 1/4)]} = 0.189F \\ \tau'' &= \frac{Mc}{I} = \frac{(8F)(9)}{143.1} = 0.503F \\ \tau_{\max} &= \sqrt{\tau'^2 + \tau''^2} = F\sqrt{0.189^2 + 0.503^2} = 0.537F\end{aligned}$$

Materials:

A36 Member: $S_y = 36$ kpsi
1015 HR Attachment: $S_y = 27.5$ kpsi
E6010 Electrode: $S_y = 50$ kpsi

$$\tau_{\text{all}} = 0.577\min(36, 27.5, 50) = 15.9\text{ kpsi}$$

$$F = \frac{\tau_{\text{all}}/n}{0.537} = \frac{15.9/2}{0.537} = 14.8\text{ kip} \quad \textit{Ans.}$$

9-26 Figure P9-26*b* is a free-body diagram of the bracket. Forces and moments that act on the welds are equal, but of opposite sense.

(a) $M = 1200(0.366) = 439\text{ lbf}\cdot\text{in}$ *Ans.*

(b) $F_y = 1200\sin 30^\circ = 600\text{ lbf}$ *Ans.*

(c) $F_x = 1200\cos 30^\circ = 1039\text{ lbf}$ *Ans.*

(d) From Table 9-2, category 6:

$$A = 1.414(0.25)(0.25 + 2.5) = 0.972 \text{ in}^2$$

$$I_u = \frac{d^2}{6}(3b + d) = \frac{2.5^2}{6}[3(0.25) + 2.5] = 3.39 \text{ in}^3$$

The second area moment about an axis through G and parallel to z is

$$I = 0.707hI_u = 0.707(0.25)(3.39) = 0.599 \text{ in}^4 \quad \textit{Ans.}$$

(e) Refer to Fig. P.9-26*b*. The shear stress due to F_y is

$$\tau_1 = \frac{F_y}{A} = \frac{600}{0.972} = 617 \text{ psi}$$

The shear stress along the throat due to F_x is

$$\tau_2 = \frac{F_x}{A} = \frac{1039}{0.972} = 1069 \text{ psi}$$

The resultant of τ_1 and τ_2 is in the throat plane

$$\tau' = \left(\tau_1^2 + \tau_2^2\right)^{1/2} = (617^2 + 1069^2)^{1/2} = 1234 \text{ psi}$$

The bending of the throat gives

$$\tau'' = \frac{Mc}{I} = \frac{439(1.25)}{0.599} = 916 \text{ psi}$$

The maximum shear stress is

$$\tau_{\max} = (\tau'^2 + \tau''^2)^{1/2} = (1234^2 + 916^2)^{1/2} = 1537 \text{ psi} \quad \textit{Ans.}$$

(f) *Materials:*

1018 HR Member: $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi (Table A-20)

E6010 Electrode: $S_y = 50$ kpsi (Table 9-3)

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{0.577S_y}{\tau_{\max}} = \frac{0.577(32)}{1.537} = 12.0 \quad \textit{Ans.}$$

(g) Bending in the attachment near the base. The cross-sectional area is approximately equal to *bh.*

$$A_1 \doteq bh = 0.25(2.5) = 0.625 \text{ in}^2$$

$$\tau_{xy} = \frac{F_x}{A_1} = \frac{1039}{0.625} = 1662 \text{ psi}$$

$$\frac{I}{c} = \frac{bd^2}{6} = \frac{0.25(2.5)^2}{6} = 0.260 \text{ in}^3$$

At location A

$$\sigma_y = \frac{F_y}{A_1} + \frac{M}{I/c}$$

$$\sigma_y = \frac{600}{0.625} + \frac{439}{0.260} = 2648 \text{ psi}$$

The von Mises stress σ' is

$$\sigma' = \left(\sigma_y^2 + 3\tau_{xy}^2\right)^{1/2} = [2648^2 + 3(1662)^2]^{1/2} = 3912 \text{ psi}$$

Thus, the factor of safety is,

$$n = \frac{S_y}{\sigma'} = \frac{32}{3.912} = 8.18 \quad Ans.$$

The clip on the mooring line bears against the side of the 1/2-in hole. If the clip fills the hole

$$\sigma = \frac{F}{td} = \frac{-1200}{0.25(0.50)} = -9600 \text{ psi}$$

$$n = -\frac{S_y}{\sigma'} = -\frac{32(10^3)}{-9600} = 3.33 \quad Ans.$$

Further investigation of this situation requires more detail than is included in the task statement.

(h) In shear fatigue, the weakest constituent of the weld melt is 1018 with $S_{ut} = 58$ kpsi

$$S_e' = 0.5S_{ut} = 0.5(58) = 29 \text{ kpsi}$$

Table 7-4:

$$k_a = 14.4(58)^{-0.718} = 0.780$$

For the size factor estimate, we first employ Eq. (7-24) for the equivalent diameter.

$$d_e = 0.808\sqrt{0.707hb} = 0.808\sqrt{0.707(2.5)(0.25)} = 0.537 \text{ in}$$

Eq. (7-19) is used next to find k_b

$$k_b = \left(\frac{d_e}{0.30}\right)^{-0.107} = \left(\frac{0.537}{0.30}\right)^{-0.107} = 0.940$$

The load factor for shear k_c, is

$$k_c = 0.59$$

The endurance strength in shear is

$$S_{se} = 0.780(0.940)(0.59)(29) = 12.5 \text{ kpsi}$$

From Table 9-5, the shear stress-concentration factor is $K_{fs} = 2.7$. The loading is repeatedly-applied.

$$\tau_a = \tau_m = K_{fs}\frac{\tau_{\max}}{2} = 2.7\frac{1.537}{2} = 2.07 \text{ kpsi}$$

Table 7-10: Gerber factor of safety n_f, adjusted for shear, with $S_{su} = 0.67S_{ut}$

$$n_f = \frac{1}{2}\left[\frac{0.67(58)}{2.07}\right]^2\left(\frac{2.07}{12.5}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(2.07)(12.5)}{0.67(58)(2.07)}\right]^2}\right\} = 5.52 \quad Ans.$$

Attachment metal should be checked for bending fatigue.

9-27 Use $b = d = 4$ in. Since $h = 5/8$ in, the primary shear is

$$\tau' = \frac{F}{1.414(5/8)(4)} = 0.283F$$

The secondary shear calculations, for a moment arm of 14 in give

$$J_u = \frac{4[3(4^2) + 4^2]}{6} = 42.67 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(5/8)42.67 = 18.9 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{14F(2)}{18.9} = 1.48F$$

Thus, the maximum shear and allowable load are:

$$\tau_{\max} = F\sqrt{1.48^2 + (0.283 + 1.48)^2} = 2.30F$$

$$F = \frac{\tau_{\text{all}}}{2.30} = \frac{20}{2.30} = 8.70 \text{ kip} \quad \textit{Ans.}$$

From Prob. 9-5*b*, $\tau_{\text{all}} = 11$ kpsi

$$F_{\text{all}} = \frac{\tau_{\text{all}}}{2.30} = \frac{11}{2.30} = 4.78 \text{ kip}$$

The allowable load has thus increased by a factor of 1.8 *Ans.*

9-28 Purchase the hook having the design shown in Fig. P9-28*b*. Referring to text Fig. 9-32*a*, this design reduces peel stresses.

9-29 **(a)**

$$\bar{\tau} = \frac{1}{l}\int_{-l/2}^{l/2} \frac{P\omega\cosh(\omega x)}{4b\sinh(\omega l/2)}\,dx$$

$$= A_1\int_{-l/2}^{l/2} \cosh(\omega x)\,dx$$

$$= \frac{A_1}{\omega}\sinh(\omega x)\Big|_{-l/2}^{l/2}$$

$$= \frac{A_1}{\omega}[\sinh(\omega l/2) - \sinh(-\omega l/2)]$$

$$= \frac{A_1}{\omega}[\sinh(\omega l/2) - (-\sinh(\omega l/2))]$$

$$= \frac{2A_1\sinh(\omega l/2)}{\omega}$$

$$= \frac{P\omega}{4bl\sinh(\omega l/2)}[2\sinh(\omega l/2)]$$

$$\bar{\tau} = \frac{P}{2bl} \quad \textit{Ans.}$$

(b)

$$\tau(l/2) = \frac{P\omega \cosh(\omega l/2)}{4b \sinh(\omega l/2)} = \frac{P\omega}{4b \tanh(\omega l/2)} \quad Ans.$$

(c)

$$K = \frac{\tau(l/2)}{\bar{\tau}} = \frac{P\omega}{4b \sinh(\omega l/2)} \left(\frac{2bl}{P}\right)$$

$$K = \frac{\omega l/2}{\tanh(\omega l/2)} \quad Ans.$$

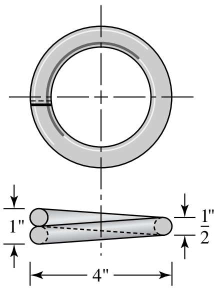
For computer programming, it can be useful to express the hyperbolic tangent in terms of exponentials:

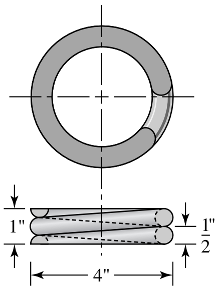
$$K = \frac{\omega l}{2} \frac{\exp(\omega l/2) - \exp(-\omega l/2)}{\exp(\omega l/2) + \exp(-\omega l/2)} \quad Ans.$$

9-30 This is a computer programming exercise. All programs will vary.

Chapter 10

10-1





10-2 $A = Sd^m$

$$\dim(A_{\text{uscu}}) = \dim(S)\,\dim(d^m) = \text{kpsi} \cdot \text{in}^m$$

$$\dim(A_{\text{SI}}) = \dim(S_1)\,\dim\left(d_1^m\right) = \text{MPa} \cdot \text{mm}^m$$

$$A_{\text{SI}} = \frac{\text{MPa}}{\text{kpsi}} \cdot \frac{\text{mm}^m}{\text{in}^m} A_{\text{uscu}} = 6.894\,757(25.40)^m A_{\text{uscu}} \doteq 6.895(25.4)^m A_{\text{uscu}} \quad \textit{Ans.}$$

For music wire, from Table 10-4:

$$A_{\text{uscu}} = 201, \quad m = 0.145; \quad \text{what is } A_{\text{SI}}?$$

$$A_{\text{SI}} = 6.89(25.4)^{0.145}(201) = 2214 \text{ MPa} \cdot \text{mm}^m \quad \textit{Ans.}$$

10-3 Given: Music wire, $d = 0.105$ in, OD $= 1.225$ in, plain ground ends, $N_t = 12$ coils.

Table 10-1: $$N_a = N_t - 1 = 12 - 1 = 11$$

$$L_s = dN_t = 0.105(12) = 1.26 \text{ in}$$

Table 10-4: $$A = 201, \quad m = 0.145$$

(a) Eq. (10-14): $$S_{ut} = \frac{201}{(0.105)^{0.145}} = 278.7 \text{ kpsi}$$

Table 10-6: $$S_{sy} = 0.45(278.7) = 125.4 \text{ kpsi}$$

$$D = 1.225 - 0.105 = 1.120 \text{ in}$$

$$C = \frac{D}{d} = \frac{1.120}{0.105} = 10.67$$

Eq. (10-6): $$K_B = \frac{4(10.67) + 2}{4(10.67) - 3} = 1.126$$

Eq. (10-3): $$F|_{S_{sy}} = \frac{\pi d^3 S_{sy}}{8K_B D} = \frac{\pi(0.105)^3(125.4)(10^3)}{8(1.126)(1.120)} = 45.2 \text{ lbf}$$

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{(0.105)^4(11.75)(10^6)}{8(1.120)^3(11)} = 11.55 \text{ lbf/in}$$

$$L_0 = \frac{F|_{S_{sy}}}{k} + L_s = \frac{45.2}{11.55} + 1.26 = 5.17 \text{ in} \quad \textit{Ans.}$$

(b) $F|_{S_{sy}} = 45.2$ lbf *Ans.*

(c) $k = 11.55$ lbf/in *Ans.*

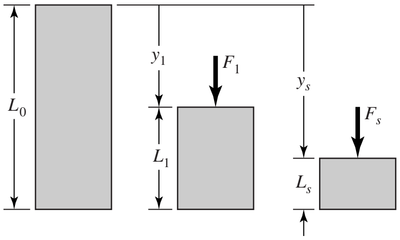
(d) $(L_0)_{\text{cr}} = \dfrac{2.63D}{\alpha} = \dfrac{2.63(1.120)}{0.5} = 5.89$ in

Many designers provide $(L_0)_{\text{cr}}/L_0 \geq 5$ or more; therefore, plain ground ends are not often used in machinery due to buckling uncertainty.

10-4 Referring to Prob. 10-3 solution, $C = 10.67$, $N_a = 11$, $S_{sy} = 125.4$ kpsi, $(L_0)_{\text{cr}} = 5.89$ in and $F = 45.2$ lbf (at yield).

Eq. (10-18): $4 \leq C \leq 12$ $C = 10.67$ *O.K.*

Eq. (10-19): $3 \leq N_a \leq 15$ $N_a = 11$ *O.K.*



$$L_0 = 5.17 \text{ in}, \quad L_s = 1.26 \text{ in}$$

$$y_1 = \frac{F_1}{k} = \frac{30}{11.55} = 2.60 \text{ in}$$

$$L_1 = L_0 - y_1 = 5.17 - 2.60 = 2.57 \text{ in}$$

$$\xi = \frac{y_s}{y_1} - 1 = \frac{5.17 - 1.26}{2.60} - 1 = 0.50$$

Eq. (10-20): $\xi \geq 0.15$, $\xi = 0.50$ *O.K.*

From Eq. (10-3) for static service

$$\tau_1 = K_B\left(\frac{8F_1D}{\pi d^3}\right) = 1.126\left[\frac{8(30)(1.120)}{\pi(0.105)^3}\right] = 83\,224 \text{ psi}$$

$$n_s = \frac{S_{sy}}{\tau_1} = \frac{125.4(10^3)}{83\,224} = 1.51$$

Eq. (10-21): $n_s \geq 1.2$, $n_s = 1.51$ *O.K.*

$$\tau_s = \tau_1\left(\frac{45.2}{30}\right) = 83\,224\left(\frac{45.2}{30}\right) = 125\,391 \text{ psi}$$

$$S_{sy}/\tau_s = 125.4(10^3)/125\,391 \doteq 1$$

$S_{sy}/\tau_s \geq (n_s)_d$: Not solid-safe. *Not O.K.*

$L_0 \leq (L_0)_{\text{cr}}$: 5.17 ≤ 5.89 Margin could be higher, *Not O.K.*

Design is unsatisfactory. Operate over a rod? *Ans.*

10-5 Static service spring with: HD steel wire, $d = 2$ mm, OD $= 22$ mm, $N_t = 8.5$ turns plain and ground ends.

Preliminaries

Table 10-5: $A = 1783\ \text{MPa} \cdot \text{mm}^m, \quad m = 0.190$

Eq. (10-14): $S_{ut} = \dfrac{1783}{(2)^{0.190}} = 1563\ \text{MPa}$

Table 10-6: $S_{sy} = 0.45(1563) = 703.4\ \text{MPa}$

Then,

$$D = \text{OD} - d = 22 - 2 = 20\ \text{mm}$$
$$C = 20/2 = 10$$
$$K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$$
$$N_a = 8.5 - 1 = 7.5\ \text{turns}$$
$$L_s = 2(8.5) = 17\ \text{mm}$$

Eq. (10-21): Use $n_s = 1.2$ for solid-safe property.

$$F_s = \frac{\pi d^3 S_{sy}/n_s}{8K_B D} = \frac{\pi(2)^3(703.4/1.2)}{8(1.135)(20)}\left[\frac{(10^{-3})^3(10^6)}{10^{-3}}\right] = 81.12\ \text{N}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(2)^4(79.3)}{8(20)^3(7.5)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.002\,643(10^6) = 2643\ \text{N/m}$$

$$y_s = \frac{F_s}{k} = \frac{81.12}{2643(10^{-3})} = 30.69\ \text{mm}$$

(a) $L_0 = y + L_s = 30.69 + 17 = 47.7$ mm *Ans.*

(b) Table 10-1: $p = \dfrac{L_0}{N_t} = \dfrac{47.7}{8.5} = 5.61$ mm *Ans.*

(c) $F_s = 81.12$ N (from above) *Ans.*

(d) $k = 2643$ N/m (from above) *Ans.*

(e) Table 10-2 and Eq. (10-13):

$$(L_0)_{\text{cr}} = \frac{2.63D}{\alpha} = \frac{2.63(20)}{0.5} = 105.2\ \text{mm}$$
$$(L_0)_{\text{cr}}/L_0 = 105.2/47.7 = 2.21$$

This is less than 5. Operate over a rod?

Plain and ground ends have a poor eccentric footprint. *Ans.*

10-6 Referring to Prob. 10-5 solution: $C = 10$, $N_a = 7.5$, $k = 2643$ N/m, $d = 2$ mm, $D = 20$ mm, $F_s = 81.12$ N and $N_t = 8.5$ turns.

Eq. (10-18): $4 \le C \le 12, \quad C = 10 \quad$ *O.K.*

Eq. (10-19): $3 \le N_a \le 15, \quad N_a = 7.5 \quad O.K.$

$$y_1 = \frac{F_1}{k} = \frac{75}{2643(10^{-3})} = 28.4 \text{ mm}$$

$$(y)_{\text{for yield}} = \frac{81.12(1.2)}{2643(10^{-3})} = 36.8 \text{ mm}$$

$$y_s = \frac{81.12}{2643(10^{-3})} = 30.69 \text{ mm}$$

$$\xi = \frac{(y)_{\text{for yield}}}{y_1} - 1 = \frac{36.8}{28.4} - 1 = 0.296$$

Eq. (10-20): $\xi \ge 0.15, \quad \xi = 0.296 \quad O.K.$

Table 10-6: $S_{sy} = 0.45 S_{ut} \quad O.K.$

As-wound

$$\tau_s = K_B\left(\frac{8F_s D}{\pi d^3}\right) = 1.135\left[\frac{8(81.12)(20)}{\pi(2)^3}\right]\left[\frac{10^{-3}}{(10^{-3})^3(10^6)}\right] = 586 \text{ MPa}$$

Eq. (10-21): $\dfrac{S_{sy}}{\tau_s} = \dfrac{703.4}{586} = 1.2 \quad O.K.$ (Basis for Prob. 10-5 solution)

Table 10-1: $L_s = N_t d = 8.5(2) = 17$ mm

$$L_0 = \frac{F_s}{k} + L_s = \frac{81.12}{2.643} + 17 = 47.7 \text{ mm}$$

$$\frac{2.63D}{\alpha} = \frac{2.63(20)}{0.5} = 105.2 \text{ mm}$$

$$\frac{(L_0)_{\text{cr}}}{L_0} = \frac{105.2}{47.7} = 2.21$$

which is less than 5. Operate over a rod? *Not O.K.*

Plain and ground ends have a poor eccentric footprint. *Ans.*

10-7 Given: A228 (music wire), SQ&GRD ends, $d = 0.006$ in, OD $= 0.036$ in, $L_0 = 0.63$ in, $N_t = 40$ turns.

Table 10-4: $A = 201 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.145$

$$D = \text{OD} - d = 0.036 - 0.006 = 0.030 \text{ in}$$

$$C = D/d = 0.030/0.006 = 5$$

$$K_B = \frac{4(5) + 2}{4(5) - 3} = 1.294$$

Table 10-1: $N_a = N_t - 2 = 40 - 2 = 38$ turns

$$S_{ut} = \frac{201}{(0.006)^{0.145}} = 422.1 \text{ kpsi}$$

$$S_{sy} = 0.45(422.1) = 189.9 \text{ kpsi}$$

$$k = \frac{Gd^4}{8D^3 N_a} = \frac{12(10^6)(0.006)^4}{8(0.030)^3(38)} = 1.895 \text{ lbf/in}$$

Table 10-1: $L_s = N_t d = 40(0.006) = 0.240$ in

Now $F_s = ky_s$ where $y_s = L_0 - L_s = 0.390$ in. Thus,

$$\tau_s = K_B \left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.294\left[\frac{8(1.895)(0.39)(0.030)}{\pi(0.006)^3}\right](10^{-3}) = 338.2 \text{ kpsi} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $338.2 > 189.9$ kpsi; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y_s' = \frac{(\tau_s/n_s)(\pi d^3)}{8K_B kD} = \frac{(189\,900/1.2)(\pi)(0.006)^3}{8(1.294)(1.895)(0.030)} = 0.182 \text{ in}$$

Using a design factor of 1.2,

$$L_0' = L_s + y_s' = 0.240 + 0.182 = 0.422 \text{ in}$$

The spring should be wound to a free length of 0.422 in. *Ans.*

10-8 Given: B159 (phosphor bronze), SQ&GRD ends, $d = 0.012$ in, OD $= 0.120$ in, $L_0 = 0.81$ in, $N_t = 15.1$ turns.

Table 10-4: $A = 145 \text{ kpsi} \cdot \text{in}^m, \quad m = 0$

Table 10-5: $G = 6$ Mpsi

$$D = \text{OD} - d = 0.120 - 0.012 = 0.108 \text{ in}$$

$$C = D/d = 0.108/0.012 = 9$$

$$K_B = \frac{4(9)+2}{4(9)-3} = 1.152$$

Table 10-1: $N_a = N_t - 2 = 15.1 - 2 = 13.1$ turns

$$S_{ut} = \frac{145}{0.012^0} = 145 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.35(145) = 50.8$ kpsi

$$k = \frac{Gd^4}{8D^3N_a} = \frac{6(10^6)(0.012)^4}{8(0.108)^3(13.1)} = 0.942 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.012(15.1) = 0.181$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 0.81 - 0.181 = 0.629$ in

$$\tau_s = K_B \left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.152\left[\frac{8(0.942)(0.6)(0.108)}{\pi(0.012)^3}\right](10^{-3}) = 108.6 \text{ kpsi} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $108.6 > 50.8$ kpsi; the spring is not solid safe. Solving Eq. (1) for y_s' gives

$$y_s' = \frac{(S_{sy}/n)\pi d^3}{8K_B kD} = \frac{(50.8/1.2)(\pi)(0.012)^3(10^3)}{8(1.152)(0.942)(0.108)} = 0.245 \text{ in}$$

$$L_0' = L_s + y_s' = 0.181 + 0.245 = 0.426 \text{ in}$$

Wind the spring to a free length of 0.426 in. *Ans.*

10-9 Given: A313 (stainless steel), SQ&GRD ends, $d = 0.040$ in, OD $= 0.240$ in, $L_0 = 0.75$ in, $N_t = 10.4$ turns.

Table 10-4: $A = 169\,\text{kpsi}\cdot\text{in}^m, \quad m = 0.146$

Table 10-5: $G = 10(10^6)$ psi

$$D = \text{OD} - d = 0.240 - 0.040 = 0.200 \text{ in}$$

$$C = D/d = 0.200/0.040 = 5$$

$$K_B = \frac{4(5)+2}{4(5)-3} = 1.294$$

Table 10-6: $N_a = N_t - 2 = 10.4 - 2 = 8.4$ turns

$$S_{ut} = \frac{169}{(0.040)^{0.146}} = 270.4 \text{ kpsi}$$

Table 10-13: $S_{sy} = 0.35(270.4) = 94.6$ kpsi

$$k = \frac{Gd^4}{8D^3N_a} = \frac{10(10^6)(0.040)^4}{8(0.2)^3(8.4)} = 47.62 \text{ lbf/in}$$

Table 10-6: $L_s = dN_t = 0.040(10.4) = 0.416$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 0.75 - 0.416 = 0.334$ in

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.294\left[\frac{8(47.62)(0.334)(0.2)}{\pi(0.040)^3}\right](10^{-3}) = 163.8 \text{ kpsi} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $163.8 > 94.6$ kpsi; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_BkD} = \frac{(94\,600/1.2)(\pi)(0.040)^3}{8(1.294)(47.62)(0.2)} = 0.161 \text{ in}$$

$$L_0' = L_s + y_s' = 0.416 + 0.161 = 0.577 \text{ in}$$

Wind the spring to a free length 0.577 in. *Ans.*

10-10 Given: A227 (hard drawn steel), $d = 0.135$ in, OD $= 2.0$ in, $L_0 = 2.94$ in, $N_t = 5.25$ turns.

Table 10-4: $A = 140\text{ kpsi}\cdot\text{in}^m, \quad m = 0.190$

Table 10-5: $G = 11.4(10^6)$ psi

$$D = \text{OD} - d = 2 - 0.135 = 1.865 \text{ in}$$

$$C = D/d = 1.865/0.135 = 13.81$$

$$K_B = \frac{4(13.81)+2}{4(13.81)-3} = 1.096$$

$$N_a = N_t - 2 = 5.25 - 2 = 3.25 \text{ turns}$$

$$S_{ut} = \frac{140}{(0.135)^{0.190}} = 204.8 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45(204.8) = 92.2$ kpsi

$$k = \frac{Gd^4}{8D^3N_a} = \frac{11.4(10^6)(0.135)^4}{8(1.865)^3(3.25)} = 22.45 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.135(5.25) = 0.709$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 2.94 - 0.709 = 2.231$ in

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.096\left[\frac{8(22.45)(2.231)(1.865)}{\pi(0.135)^3}\right](10^{-3}) = 106.0 \text{ kpsi} \quad (1)$$

$\tau_s > S_{sy}$, that is, $106 > 92.2$ kpsi; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_BkD} = \frac{(92\,200/1.2)(\pi)(0.135)^3}{8(1.096)(22.45)(1.865)} = 1.612 \text{ in}$$

$$L_0' = L_s + y_s' = 0.709 + 1.612 = 2.321 \text{ in}$$

Wind the spring to a free length of 2.32 in. *Ans.*

10-11 Given: A229 (OQ&T steel), SQ&GRD ends, $d = 0.144$ in, OD $= 1.0$ in, $L_0 = 3.75$ in, $N_t = 13$ turns.

Table 10-4: $A = 147 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.187$

Table 10-5: $G = 11.4(10^6)$ psi

$$D = \text{OD} - d = 1.0 - 0.144 = 0.856 \text{ in}$$

$$C = D/d = 0.856/0.144 = 5.944$$

$$K_B = \frac{4(5.944) + 2}{4(5.944) - 3} = 1.241$$

Table 10-1: $N_a = N_t - 2 = 13 - 2 = 11$ turns

$$S_{ut} = \frac{147}{(0.144)^{0.187}} = 211.2 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50(211.2) = 105.6$ kpsi

$$k = \frac{Gd^4}{8D^3N_a} = \frac{11.4(10^6)(0.144)^4}{8(0.856)^3(11)} = 88.8 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.144(13) = 1.872$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 3.75 - 1.872 = 1.878$ in

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.241\left[\frac{8(88.8)(1.878)(0.856)}{\pi(0.144)^3}\right](10^{-3}) = 151.1 \text{ kpsi} \quad (1)$$

$\tau_s > S_{sy}$, that is, $151.1 > 105.6$ kpsi; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_BkD} = \frac{(105\,600/1.2)(\pi)(0.144)^3}{8(1.241)(88.8)(0.856)} = 1.094 \text{ in}$$

$$L_0' = L_s + y_s' = 1.878 + 1.094 = 2.972 \text{ in}$$

Wind the spring to a free length 2.972 in. *Ans.*

10-12 Given: A232 (Cr-V steel), SQ&GRD ends, $d = 0.192$ in, OD = 3 in, $L_0 = 9$ in, $N_t =$ 8 turns.

Table 10-4: $A = 169 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.168$

Table 10-5: $G = 11.2(10^6)$ psi

$$D = \text{OD} - d = 3 - 0.192 = 2.808 \text{ in}$$

$$C = D/d = 2.808/0.192 = 14.625 \quad \text{(large)}$$

$$K_B = \frac{4(14.625) + 2}{4(14.625) - 3} = 1.090$$

Table 10-1: $N_a = N_t - 2 = 8 - 2 = 6$ turns

$$S_{ut} = \frac{169}{(0.192)^{0.168}} = 223.0 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50(223.0) = 111.5$ kpsi

$$k = \frac{Gd^4}{8D^3N_a} = \frac{11.2(10^6)(0.192)^4}{8(2.808)^3(6)} = 14.32 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.192(8) = 1.536$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 9 - 1.536 = 7.464$ in

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.090\left[\frac{8(14.32)(7.464)(2.808)}{\pi(0.192)^3}\right](10^{-3}) = 117.7 \text{ kpsi} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $117.7 > 111.5$ kpsi; the spring is not solid safe. Solving Eq. (1) for y_s gives

$$y'_s = \frac{(S_{sy}/n)(\pi d^3)}{8K_BkD} = \frac{(111\,500/1.2)(\pi)(0.192)^3}{8(1.090)(14.32)(2.808)} = 5.892 \text{ in}$$

$$L'_0 = L_s + y'_s = 1.536 + 5.892 = 7.428 \text{ in}$$

Wind the spring to a free length of 7.428 in. *Ans.*

10-13 Given: A313 (stainless steel) SQ&GRD ends, $d = 0.2$ mm, OD = 0.91 mm, $L_0 =$ 15.9 mm, $N_t = 40$ turns.

Table 10-4: $A = 1867 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.146$

Table 10-5: $G = 69.0$ GPa

$$D = \text{OD} - d = 0.91 - 0.2 = 0.71 \text{ mm}$$

$$C = D/d = 0.71/0.2 = 3.55 \quad \text{(small)}$$

$$K_B = \frac{4(3.55) + 2}{4(3.55) - 3} = 1.446$$

$$N_a = N_t - 2 = 40 - 2 = 38 \text{ turns}$$

$$S_{ut} = \frac{1867}{(0.2)^{0.146}} = 2361.5 \text{ MPa}$$

Table 10-6:

$$S_{sy} = 0.35(2361.5) = 826.5 \text{ MPa}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(0.2)^4(69.0)}{8(0.71)^3(38)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right]$$

$$= 1.0147(10^{-3})(10^6) = 1014.7 \text{ N/m} \quad \text{or} \quad 1.0147 \text{ N/mm}$$

$$L_s = dN_t = 0.2(40) = 8 \text{ mm}$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 15.9 - 8 = 7.9$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.446\left[\frac{8(1.0147)(7.9)(0.71)}{\pi(0.2)^3}\right]\left[\frac{10^{-3}(10^{-3})(10^{-3})}{(10^{-3})^3}\right]$$

$$= 2620(1) = 2620 \text{ MPa} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $2620 > 826.5$ MPa; the spring is not solid safe. Solve Eq. (1) for y_s giving

$$y'_s = \frac{(S_{sy}/n)(\pi d^3)}{8K_B kD} = \frac{(826.5/1.2)(\pi)(0.2)^3}{8(1.446)(1.0147)(0.71)} = 2.08 \text{ mm}$$

$$L'_0 = L_s + y'_s = 8.0 + 2.08 = 10.08 \text{ mm}$$

Wind the spring to a free length of 10.08 mm. This only addresses the solid-safe criteria. There are additional problems. *Ans.*

10-14 Given: A228 (music wire), SQ&GRD ends, $d = 1$ mm, OD $= 6.10$ mm, $L_0 = 19.1$ mm, $N_t = 10.4$ turns.

Table 10-4: $A = 2211 \text{ MPa} \cdot \text{mm}^m$, $m = 0.145$

Table 10-5: $G = 81.7$ GPa

$$D = \text{OD} - d = 6.10 - 1 = 5.1 \text{ mm}$$

$$C = D/d = 5.1/1 = 5.1$$

$$N_a = N_t - 2 = 10.4 - 2 = 8.4 \text{ turns}$$

$$K_B = \frac{4(5.1)+2}{4(5.1)-3} = 1.287$$

$$S_{ut} = \frac{2211}{(1)^{0.145}} = 2211 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.45(2211) = 995$ MPa

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(1)^4(81.7)}{8(5.1)^3(8.4)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.009\,165(10^6)$$

$$= 9165 \text{ N/m} \quad \text{or} \quad 9.165 \text{ N/mm}$$

$$L_s = dN_t = 1(10.4) = 10.4 \text{ mm}$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 19.1 - 10.4 = 8.7 \text{ mm}$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.287\left[\frac{8(9.165)(8.7)(5.1)}{\pi(1)^3}\right] = 1333 \text{ MPa} \qquad (1)$$

$\tau_s > S_{sy}$, that is, 1333 > 995 MPa; the spring is not solid safe. Solve Eq. (1) for y_s giving

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_B kD} = \frac{(995/1.2)(\pi)(1)^3}{8(1.287)(9.165)(5.1)} = 5.43 \text{ mm}$$

$$L_0' = L_s + y_s' = 10.4 + 5.43 = 15.83 \text{ mm}$$

Wind the spring to a free length of 15.83 mm. *Ans.*

10-15 Given: A229 (OQ&T spring steel), SQ&GRD ends, $d = 3.4$ mm, OD = 50.8 mm, $L_0 =$ 74.6 mm, $N_t = 5.25$.

Table 10-4: $A = 1855 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.187$

Table 10-5: $G = 77.2$ GPa

$$D = \text{OD} - d = 50.8 - 3.4 = 47.4 \text{ mm}$$

$$C = D/d = 47.4/3.4 = 13.94 \quad \text{(large)}$$

$$N_a = N_t - 2 = 5.25 - 2 = 3.25 \text{ turns}$$

$$K_B = \frac{4(13.94) + 2}{4(13.94) - 3} = 1.095$$

$$S_{ut} = \frac{1855}{(3.4)^{0.187}} = 1476 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.50(1476) = 737.8$ MPa

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(3.4)^4(77.2)}{8(47.4)^3(3.25)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.003\,75(10^6)$$

$$= 3750 \text{ N/m} \quad \text{or} \quad 3.750 \text{ N/mm}$$

$$L_s = dN_t = 3.4(5.25) = 17.85$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 74.6 - 17.85 = 56.75 \text{ mm}$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right]$$

$$= 1.095\left[\frac{8(3.750)(56.75)(47.4)}{\pi(3.4)^3}\right] = 720.2 \text{ MPa} \qquad (1)$$

$\tau_s < S_{sy}$, that is, 720.2 < 737.8 MPa

$\therefore$ The spring is solid safe. With $n_s = 1.2$,

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_B k D} = \frac{(737.8/1.2)(\pi)(3.4)^3}{8(1.095)(3.75)(47.4)} = 48.76 \text{ mm}$$

$$L_0' = L_s + y_s' = 17.85 + 48.76 = 66.61 \text{ mm}$$

Wind the spring to a free length of 66.61 mm. *Ans.*

10-16 Given: B159 (phosphor bronze), SQ&GRD ends, $d = 3.7$ mm, OD $= 25.4$ mm, $L_0 = 95.3$ mm, $N_t = 13$ turns.

Table 10-4: $A = 932 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.064$

Table 10-5: $G = 41.4$ GPa

$$D = \text{OD} - d = 25.4 - 3.7 = 21.7 \text{ mm}$$

$$C = D/d = 21.7/3.7 = 5.865$$

$$K_B = \frac{4(5.865) + 2}{4(5.865) - 3} = 1.244$$

$$N_a = N_t - 2 = 13 - 2 = 11 \text{ turns}$$

$$S_{ut} = \frac{932}{(3.7)^{0.064}} = 857.1 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.35(857.1) = 300$ MPa

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(3.7)^4(41.4)}{8(21.7)^3(11)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.008\,629(10^6)$$

$$= 8629 \text{ N/m} \quad \text{or} \quad 8.629 \text{ N/mm}$$

$$L_s = dN_t = 3.7(13) = 48.1 \text{ mm}$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 95.3 - 48.1 = 47.2 \text{ mm}$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right]$$

$$= 1.244\left[\frac{8(8.629)(47.2)(21.7)}{\pi(3.7)^3}\right] = 553 \text{ MPa} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $553 > 300$ MPa; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_B k D} = \frac{(300/1.2)(\pi)(3.7)^3}{8(1.244)(8.629)(21.7)} = 21.35 \text{ mm}$$

$$L_0' = L_s + y_s' = 48.1 + 21.35 = 69.45 \text{ mm}$$

Wind the spring to a free length of 69.45 mm. *Ans.*

10-17 Given: A232 (Cr-V steel), SQ&GRD ends, $d = 4.3$ mm, OD $= 76.2$ mm, $L_0 = 228.6$ mm, $N_t = 8$ turns.

Table 10-4: $A = 2005 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.168$

Table 10-5: $G = 77.2$ GPa

$$D = \text{OD} - d = 76.2 - 4.3 = 71.9 \text{ mm}$$

$$C = D/d = 71.9/4.3 = 16.72 \quad \text{(large)}$$

$$K_B = \frac{4(16.72) + 2}{4(16.72) - 3} = 1.078$$

$$N_a = N_t - 2 = 8 - 2 = 6 \text{ turns}$$

$$S_{ut} = \frac{2005}{(4.3)^{0.168}} = 1569 \text{ MPa}$$

Table 10-6:

$$S_{sy} = 0.50(1569) = 784.5 \text{ MPa}$$

$$k = \frac{d^4G}{8D^3N_a} = \frac{(4.3)^4(77.2)}{8(71.9)^3(6)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.001\,479(10^6)$$

$$= 1479 \text{ N/m} \quad \text{or} \quad 1.479 \text{ N/mm}$$

$$L_s = dN_t = 4.3(8) = 34.4 \text{ mm}$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 228.6 - 34.4 = 194.2 \text{ mm}$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.078\left[\frac{8(1.479)(194.2)(71.9)}{\pi(4.3)^3}\right] = 713.0 \text{ MPa} \qquad (1)$$

$\tau_s < S_{sy}$, that is, $713.0 < 784.5$; the spring is solid safe. With $n_s = 1.2$

Eq. (1) becomes

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_BkD} = \frac{(784.5/1.2)(\pi)(4.3)^3}{8(1.078)(1.479)(71.9)} = 178.1 \text{ mm}$$

$$L_0' = L_s + y_s' = 34.4 + 178.1 = 212.5 \text{ mm}$$

Wind the spring to a free length of $L_0' = 212.5$ mm. *Ans.*

10-18 For the wire diameter analyzed, $G = 11.75$ Mpsi per Table 10-5. Use squared and ground ends. The following is a spread-sheet study using Fig. 10-3 for parts (*a*) and (*b*). For N_a, $k = 20/2 = 10$ lbf/in.

(a) Spring over a Rod					(b) Spring in a Hole				
Source	Parameter Values				Source	Parameter Values			
	d	0.075	0.08	0.085		d	0.075	0.08	0.085
	D	0.875	0.88	0.885		D	0.875	0.870	0.865
	ID	0.800	0.800	0.800		ID	0.800	0.790	0.780
	OD	0.950	0.960	0.970		OD	0.950	0.950	0.950
Eq. (10-2)	C	11.667	11.000	10.412	Eq. (10-2)	C	11.667	10.875	10.176
Eq. (10-9)	N_a	6.937	8.828	11.061	Eq. (10-9)	N_a	6.937	9.136	11.846
Table 10-1	N_t	8.937	10.828	13.061	Table 10-1	N_t	8.937	11.136	13.846
Table 10-1	L_s	0.670	0.866	1.110	Table 10-1	L_s	0.670	0.891	1.177
$1.15y + L_s$	L_0	2.970	3.166	3.410	$1.15y + L_s$	L_0	2.970	3.191	3.477
Eq. (10-13)	$(L_0)_{\mathrm{cr}}$	4.603	4.629	4.655	Eq. (10-13)	$(L_0)_{\mathrm{cr}}$	4.603	4.576	4.550
Table 10-4	A	201.000	201.000	201.000	Table 10-4	A	201.000	201.000	201.000
Table 10-4	m	0.145	0.145	0.145	Table 10-4	m	0.145	0.145	0.145
Eq. (10-14)	S_{ut}	292.626	289.900	287.363	Eq. (10-14)	S_{ut}	292.626	289.900	287.363
Table 10-6	S_{sy}	131.681	130.455	129.313	Table 10-6	S_{sy}	131.681	130.455	129.313
Eq. (10-6)	K_B	1.115	1.122	1.129	Eq. (10-6)	K_B	1.115	1.123	1.133
Eq. (10-3)	n_s	0.973	1.155	1.357	Eq. (10-3)	n_s	0.973	1.167	1.384
Eq. (10-22)	fom	−0.282	−0.391	−0.536	Eq. (10-22)	fom	−0.282	−0.398	−0.555

For $n_s \geq 1.2$, the optimal size is $d = 0.085$ in for both cases.

10-19 From the figure: $L_0 = 120$ mm, OD = 50 mm, and $d = 3.4$ mm. Thus

$$D = \mathrm{OD} - d = 50 - 3.4 = 46.6 \text{ mm}$$

(a) By counting, $N_t = 12.5$ turns. Since the ends are squared along 1/4 turn on each end,

$$N_a = 12.5 - 0.5 = 12 \text{ turns} \quad \textit{Ans.}$$

$$p = 120/12 = 10 \text{ mm} \quad \textit{Ans.}$$

The solid stack is 13 diameters across the top and 12 across the bottom.

$$L_s = 13(3.4) = 44.2 \text{ mm} \quad \textit{Ans.}$$

(b) $d = 3.4/25.4 = 0.1339$ in and from Table 10-5, $G = 78.6$ GPa

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(3.4)^4(78.6)(10^9)}{8(46.6)^3(12)}(10^{-3}) = 1080 \text{ N/m} \quad \textit{Ans.}$$

(c) $F_s = k(L_0 - L_s) = 1080(120 - 44.2)(10^{-3}) = 81.9 \text{ N} \quad \textit{Ans.}$

(d) $C = D/d = 46.6/3.4 = 13.71$

$$K_B = \frac{4(13.71) + 2}{4(13.71) - 3} = 1.096$$

$$\tau_s = \frac{8K_B F_s D}{\pi d^3} = \frac{8(1.096)(81.9)(46.6)}{\pi(3.4)^3} = 271 \text{ MPa} \quad \textit{Ans.}$$

10-20 One approach is to select A227-47 HD steel for its low cost. Then, for $y_1 \leq 3/8$ at $F_1 = 10$ lbf, $k \geq 10/0.375 = 26.67$ lbf/in. Try $d = 0.080$ in #14 gauge

For a clearance of 0.05 in: ID $= (7/16) + 0.05 = 0.4875$ in; OD $= 0.4875 + 0.16 = 0.6475$ in

$$D = 0.4875 + 0.080 = 0.5675 \text{ in}$$

$$C = 0.5675/0.08 = 7.094$$

$$G = 11.5 \text{ Mpsi}$$

$$N_a = \frac{d^4G}{8kD^3} = \frac{(0.08)^4(11.5)(10^6)}{8(26.67)(0.5675)^3} = 12.0 \text{ turns}$$

$$N_t = 12 + 2 = 14 \text{ turns}, \quad L_s = dN_t = 0.08(14) = 1.12 \text{ in} \quad O.K.$$

$$L_0 = 1.875 \text{ in}, \quad y_s = 1.875 - 1.12 = 0.755 \text{ in}$$

$$F_s = ky_s = 26.67(0.755) = 20.14 \text{ lbf}$$

$$K_B = \frac{4(7.094) + 2}{4(7.094) - 3} = 1.197$$

$$\tau_s = K_B\left(\frac{8F_sD}{\pi d^3}\right) = 1.197\left[\frac{8(20.14)(0.5675)}{\pi(0.08)^3}\right] = 68\,046 \text{ psi}$$

Table 10-4: $\quad A = 140 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.190$

$$S_{sy} = 0.45\frac{140}{(0.080)^{0.190}} = 101.8 \text{ kpsi}$$

$$n = \frac{101.8}{68.05} = 1.50 > 1.2 \quad O.K.$$

$$\tau_1 = \frac{F_1}{F_s}\tau_s = \frac{10}{20.14}(68.05) = 33.79 \text{ kpsi},$$

$$n_1 = \frac{101.8}{33.79} = 3.01 > 1.5 \quad O.K.$$

There is much latitude for reducing the amount of material. Iterate on y_1 using a spread sheet. The final results are: $y_1 = 0.32$ in, $k = 31.25$ lbf/in, $N_a = 10.3$ turns, $N_t = 12.3$ turns, $L_s = 0.985$ in, $L_0 = 1.820$ in, $y_s = 0.835$ in, $F_s = 26.1$ lbf, $K_B = 1.197$, $\tau_s = 88\,190$ kpsi, $n_s = 1.15$, and $n_1 = 3.01$.

$$\text{ID} = 0.4875 \text{ in}, \quad \text{OD} = 0.6475 \text{ in}, \quad d = 0.080 \text{ in}$$

Try other sizes and/or materials.

10-21 A stock spring catalog may have over two hundred pages of compression springs with up to 80 springs per page listed.

- Students should be aware that such catalogs exist.
- Many springs are selected from catalogs rather than designed.
- The wire size you want may not be listed.
- Catalogs may also be available on disk or the web through search routines. For example, disks are available from Century Spring at

  1 − (800) − 237 − 5225

  www.centuryspring.com
- It is better to familiarize yourself with vendor resources rather than invent them yourself.
- Sample catalog pages can be given to students for study.

10-22 For a coil radius given by:

$$R = R_1 + \frac{R_2 - R_1}{2\pi N}\theta$$

The torsion of a section is $T = PR$ where $dL = R\,d\theta$

$$\delta_p = \frac{\partial U}{\partial P} = \frac{1}{GJ}\int T\frac{\partial T}{\partial P}\,dL = \frac{1}{GJ}\int_0^{2\pi N} PR^3\,d\theta$$

$$= \frac{P}{GJ}\int_0^{2\pi N}\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^3 d\theta$$

$$= \frac{P}{GJ}\left(\frac{1}{4}\right)\left(\frac{2\pi N}{R_2 - R_1}\right)\left[\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^4\right]\Bigg|_0^{2\pi N}$$

$$= \frac{\pi PN}{2GJ(R_2 - R_1)}\left(R_2^4 - R_1^4\right) = \frac{\pi PN}{2GJ}(R_1 + R_2)\left(R_1^2 + R_2^2\right)$$

$$J = \frac{\pi}{32}d^4 \quad \therefore \delta_p = \frac{16PN}{Gd^4}(R_1 + R_2)\left(R_1^2 + R_2^2\right)$$

$$k = \frac{P}{\delta_p} = \frac{d^4 G}{16N(R_1 + R_2)\left(R_1^2 + R_2^2\right)} \quad Ans.$$

10-23 For a food service machinery application select A313 Stainless wire.

$$G = 10(10^6)\text{ psi}$$

Note that for $0.013 \le d \le 0.10$ in $\quad A = 169, \quad m = 0.146$

$0.10 < d \le 0.20$ in $\quad A = 128, \quad m = 0.263$

$$F_a = \frac{18 - 4}{2} = 7\text{ lbf}, \quad F_m = \frac{18 + 4}{2} = 11\text{ lbf}, \quad r = 7/11$$

Try $\quad d = 0.080$ in, $\quad S_{ut} = \dfrac{169}{(0.08)^{0.146}} = 244.4$ kpsi

$$S_{su} = 0.67S_{ut} = 163.7\text{ kpsi}, \qquad S_{sy} = 0.35S_{ut} = 85.5\text{ kpsi}$$

Try unpeened using Zimmerli's endurance data: $S_{sa} = 35$ kpsi, $S_{sm} = 55$ kpsi

Gerber: $\quad S_{se} = \dfrac{S_{sa}}{1 - (S_{sm}/S_{su})^2} = \dfrac{35}{1 - (55/163.7)^2} = 39.5$ kpsi

$$S_{sa} = \frac{(7/11)^2(163.7)^2}{2(39.5)}\left\{-1 + \sqrt{1 + \left[\frac{2(39.5)}{(7/11)(163.7)}\right]^2}\right\} = 35.0\text{ kpsi}$$

$$\alpha = S_{sa}/n_f = 35.0/1.5 = 23.3\text{ kpsi}$$

$$\beta = \frac{8F_a}{\pi d^2}(10^{-3}) = \left[\frac{8(7)}{\pi(0.08^2)}\right](10^{-3}) = 2.785\text{ kpsi}$$

$$C = \frac{2(23.3) - 2.785}{4(2.785)} + \sqrt{\left[\frac{2(23.3) - 2.785}{4(2.785)}\right]^2 - \frac{3(23.3)}{4(2.785)}} = 6.97$$

$$D = Cd = 6.97(0.08) = 0.558\text{ in}$$

$$K_B = \frac{4(6.97)+2}{4(6.97)-3} = 1.201$$

$$\tau_a = K_B\left(\frac{8F_aD}{\pi d^3}\right) = 1.201\left[\frac{8(7)(0.558)}{\pi(0.08^3)}(10^{-3})\right] = 23.3 \text{ kpsi}$$

$$n_f = 35/23.3 = 1.50 \quad \text{checks}$$

$$N_a = \frac{Gd^4}{8kD^3} = \frac{10(10^6)(0.08)^4}{8(9.5)(0.558)^3} = 31.02 \text{ turns}$$

$$N_t = 31+2 = 33 \text{ turns}, \quad L_s = dN_t = 0.08(33) = 2.64 \text{ in}$$

$$y_{\max} = F_{\max}/k = 18/9.5 = 1.895 \text{ in},$$

$$y_s = (1+\xi)y_{\max} = (1+0.15)(1.895) = 2.179 \text{ in}$$

$$L_0 = 2.64 + 2.179 = 4.819 \text{ in}$$

$$(L_0)_{\text{cr}} = 2.63\frac{D}{\alpha} = \frac{2.63(0.558)}{0.5} = 2.935 \text{ in}$$

$$\tau_s = 1.15(18/7)\tau_a = 1.15(18/7)(23.3) = 68.9 \text{ kpsi}$$

$$n_s = S_{sy}/\tau_s = 85.5/68.9 = 1.24$$

$$f = \sqrt{\frac{kg}{\pi^2 d^2 D N_a \gamma}} = \sqrt{\frac{9.5(386)}{\pi^2(0.08^2)(0.558)(31.02)(0.283)}} = 109 \text{ Hz}$$

These steps are easily implemented on a spreadsheet, as shown below, for different diameters.

	d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263
A	169.000	169.000	128.000	128.000
S_{ut}	244.363	239.618	231.257	223.311
S_{su}	163.723	160.544	154.942	149.618
S_{sy}	85.527	83.866	80.940	78.159
S_{se}	39.452	39.654	40.046	40.469
S_{sa}	35.000	35.000	35.000	35.000
α	23.333	23.333	23.333	23.333
β	2.785	2.129	1.602	1.228
C	6.977	9.603	13.244	17.702
D	0.558	0.879	1.397	2.133
K_B	1.201	1.141	1.100	1.074
τ_a	23.333	23.333	23.333	23.333
n_f	1.500	1.500	1.500	1.500
N_a	30.893	13.594	5.975	2.858
N_t	32.993	15.594	7.975	4.858
L_s	2.639	1.427	0.841	0.585
y_s	2.179	2.179	2.179	2.179
L_0	4.818	3.606	3.020	2.764
$(L_0)_{\text{cr}}$	2.936	4.622	7.350	11.220
τ_s	69.000	69.000	69.000	69.000
n_s	1.240	1.215	1.173	1.133
f (Hz)	108.895	114.578	118.863	121.775

The shaded areas depict conditions outside the recommended design conditions. Thus, one spring is satisfactory–A313, as wound, unpeened, squared and ground,

$$d = 0.0915\,\text{in}, \quad \text{OD} = 0.879 + 0.092 = 0.971\,\text{in}, \quad N_t = 15.59 \text{ turns}$$

10-24 The steps are the same as in Prob. 10-23 except that the Gerber-Zimmerli criterion is replaced with Goodman-Zimmerli:

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})}$$

The problem then proceeds as in Prob. 10-23. The results for the wire sizes are shown below (see solution to Prob. 10-23 for additional details).

Iteration of d for the first trial

	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.151	1.108	1.078	1.058
A	169.000	169.000	128.000	128.000	τ_a	29.008	29.040	29.090	29.127
S_{ut}	244.363	239.618	231.257	223.311	n_f	1.500	1.500	1.500	1.500
S_{su}	163.723	160.544	154.942	149.618	N_a	14.191	6.456	2.899	1.404
S_{sy}	85.527	83.866	80.940	78.159	N_t	16.191	8.456	4.899	3.404
S_{se}	52.706	53.239	54.261	55.345	L_s	1.295	0.774	0.517	0.410
S_{sa}	43.513	43.560	43.634	43.691	y_s	2.179	2.179	2.179	2.179
α	29.008	29.040	29.090	29.127	L_0	3.474	2.953	2.696	2.589
β	2.785	2.129	1.602	1.228	$(L_0)_{\text{cr}}$	3.809	5.924	9.354	14.219
C	9.052	12.309	16.856	22.433	τ_s	85.782	85.876	86.022	86.133
D	0.724	1.126	1.778	2.703	n_s	0.997	0.977	0.941	0.907
					f (Hz)	141.284	146.853	151.271	154.326

Without checking all of the design conditions, it is obvious that none of the wire sizes satisfy $n_s \geq 1.2$. Also, the Gerber line is closer to the yield line than the Goodman. Setting $n_f = 1.5$ for Goodman makes it impossible to reach the yield line $(n_s < 1)$. The table below uses $n_f = 2$.

Iteration of d for the second trial

	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.221	1.154	1.108	1.079
A	169.000	169.000	128.000	128.000	τ_a	21.756	21.780	21.817	21.845
S_{ut}	244.363	239.618	231.257	223.311	n_f	2.000	2.000	2.000	2.000
S_{su}	163.723	160.544	154.942	149.618	N_a	40.243	17.286	7.475	3.539
S_{sy}	85.527	83.866	80.940	78.159	N_t	42.243	19.286	9.475	5.539
S_{se}	52.706	53.239	54.261	55.345	L_s	3.379	1.765	1.000	0.667
S_{sa}	43.513	43.560	43.634	43.691	y_s	2.179	2.179	2.179	2.179
α	21.756	21.780	21.817	21.845	L_0	5.558	3.944	3.179	2.846
β	2.785	2.129	1.602	1.228	$(L_0)_{\text{cr}}$	2.691	4.266	6.821	10.449
C	6.395	8.864	12.292	16.485	τ_s	64.336	64.407	64.517	64.600
D	0.512	0.811	1.297	1.986	n_s	1.329	1.302	1.255	1.210
					f (Hz)	99.816	105.759	110.312	113.408

The satisfactory spring has design specifications of: A313, as wound, unpeened, squared and ground, $d = 0.0915$ in, $\text{OD} = 0.811 + 0.092 = 0.903$ in, $N_t = 19.3$ turns.

10-25 This is the same as Prob. 10-23 since $S_{se} = S_{sa} = 35$ kpsi. Therefore, design the spring using: A313, as wound, un-peened, squared and ground, $d = 0.915$ in, OD $= 0.971$ in, $N_t = 15.59$ turns.

10-26 For the Gerber fatigue-failure criterion, $S_{su} = 0.67 S_{ut}$,

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})^2}, \qquad S_{sa} = \frac{r^2 S_{su}^2}{2 S_{se}} \left[-1 + \sqrt{1 + \left(\frac{2 S_{se}}{r S_{su}} \right)^2} \right]$$

The equation for S_{sa} is the basic difference. The last 2 columns of diameters of Ex. 10-5 are presented below with additional calculations.

	$d = 0.105$	$d = 0.112$		$d = 0.105$	$d = 0.112$
S_{ut}	278.691	276.096	N_a	8.915	6.190
S_{su}	186.723	184.984	L_s	1.146	0.917
S_{se}	38.325	38.394	L_0	3.446	3.217
S_{sy}	125.411	124.243	$(L_0)_{\text{cr}}$	6.630	8.160
S_{sa}	34.658	34.652	K_B	1.111	1.095
α	23.105	23.101	τ_a	23.105	23.101
β	1.732	1.523	n_f	1.500	1.500
C	12.004	13.851	τ_s	70.855	70.844
D	1.260	1.551	n_s	1.770	1.754
ID	1.155	1.439	f_n	105.433	106.922
OD	1.365	1.663	fom	−0.973	−1.022

There are only slight changes in the results.

10-27 As in Prob. 10-26, the basic change is S_{sa}.

For Goodman,
$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})}$$

Recalculate S_{sa} with
$$S_{sa} = \frac{r S_{se} S_{su}}{r S_{su} + S_{se}}$$

Calculations for the last 2 diameters of Ex. 10-5 are given below.

	$d = 0.105$	$d = 0.112$		$d = 0.105$	$d = 0.112$
S_{ut}	278.691	276.096	N_a	9.153	6.353
S_{su}	186.723	184.984	L_s	1.171	0.936
S_{se}	49.614	49.810	L_0	3.471	3.236
S_{sy}	125.411	124.243	$(L_0)_{\text{cr}}$	6.572	8.090
S_{sa}	34.386	34.380	K_B	1.112	1.096
α	22.924	22.920	τ_a	22.924	22.920
β	1.732	1.523	n_f	1.500	1.500
C	11.899	13.732	τ_s	70.301	70.289
D	1.249	1.538	n_s	1.784	1.768
ID	1.144	1.426	f_n	104.509	106.000
OD	1.354	1.650	fom	−0.986	−1.034

There are only slight differences in the results.

10-28 Use: $E = 28.6$ Mpsi, $G = 11.5$ Mpsi, $A = 140$ kpsi $\cdot$ in^m, $m = 0.190$, rel cost $= 1$.

Try $$d = 0.067 \text{ in}, \quad S_{ut} = \frac{140}{(0.067)^{0.190}} = 234.0 \text{ kpsi}$$

Table 10-6: $$S_{sy} = 0.45 S_{ut} = 105.3 \text{ kpsi}$$

Table 10-7: $$S_y = 0.75 S_{ut} = 175.5 \text{ kpsi}$$

Eq. (10-34) with $D/d = C$ and $C_1 = C$

$$\sigma_A = \frac{F_{\max}}{\pi d^2}[(K)_A(16C) + 4] = \frac{S_y}{n_y}$$

$$\frac{4C^2 - C - 1}{4C(C-1)}(16C) + 4 = \frac{\pi d^2 S_y}{n_y F_{\max}}$$

$$4C^2 - C - 1 = (C-1)\left(\frac{\pi d^2 S_y}{4 n_y F_{\max}} - 1\right)$$

$$C^2 - \frac{1}{4}\left(1 + \frac{\pi d^2 S_y}{4 n_y F_{\max}} - 1\right)C + \frac{1}{4}\left(\frac{\pi d^2 S_y}{4 n_y F_{\max}} - 2\right) = 0$$

$$C = \frac{1}{2}\left[\frac{\pi d^2 S_y}{16 n_y F_{\max}} \pm \sqrt{\left(\frac{\pi d^2 S_y}{16 n_y F_{\max}}\right)^2 - \frac{\pi d^2 S_y}{4 n_y F_{\max}} + 2}\,\right] \quad \text{take positive root}$$

$$= \frac{1}{2}\left\{\frac{\pi(0.067^2)(175.5)(10^3)}{16(1.5)(18)} + \sqrt{\left[\frac{\pi(0.067)^2(175.5)(10^3)}{16(1.5)(18)}\right]^2 - \frac{\pi(0.067)^2(175.5)(10^3)}{4(1.5)(18)} + 2}\right\} = 4.590$$

$$D = Cd = 0.3075 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range. This results in the best fom.

$$F_i = \frac{\pi(0.067)^3}{8(0.3075)}\left\{\frac{33\,500}{\exp[0.105(4.590)]} - 1000\left(4 - \frac{4.590 - 3}{6.5}\right)\right\} = 6.505 \text{ lbf}$$

For simplicity, we will round up to the next integer or half integer;
therefore, use $F_i = 7$ lbf

$$k = \frac{18 - 7}{0.5} = 22 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.067)^4(11.5)(10^6)}{8(22)(0.3075)^3} = 45.28 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 45.28 - \frac{11.5}{28.6} = 44.88 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.590) - 1 + 44.88](0.067) = 3.555 \text{ in}$$

$$L_{18\,\text{lbf}} = 3.555 + 0.5 = 4.055 \text{ in}$$

$$\text{Body:}\quad K_B = \frac{4C+2}{4C-3} = \frac{4(4.590)+2}{4(4.590)-3} = 1.326$$

$$\tau_{\max} = \frac{8K_B F_{\max} D}{\pi d^3} = \frac{8(1.326)(18)(0.3075)}{\pi(0.067)^3}(10^{-3}) = 62.1 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{S_{sy}}{\tau_{\max}} = \frac{105.3}{62.1} = 1.70$$

$$r_2 = 2d = 2(0.067) = 0.134 \text{ in},\quad C_2 = \frac{2r_2}{d} = \frac{2(0.134)}{0.067} = 4$$

$$(K)_B = \frac{4C_2-1}{4C_2-4} = \frac{4(4)-1}{4(4)-4} = 1.25$$

$$\tau_B = (K)_B\left[\frac{8F_{\max}D}{\pi d^3}\right] = 1.25\left[\frac{8(18)(0.3075)}{\pi(0.067)^3}\right](10^{-3}) = 58.58 \text{ kpsi}$$

$$(n_y)_B = \frac{S_{sy}}{\tau_B} = \frac{105.3}{58.58} = 1.80$$

$$\text{fom} = -(1)\frac{\pi^2 d^2(N_b+2)D}{4} = -\frac{\pi^2(0.067)^2(44.88+2)(0.3075)}{4} = -0.160$$

Several diameters, evaluated using a spreadsheet, are shown below.

d:	0.067	0.072	0.076	0.081	0.085	0.09	0.095	0.104
S_{ut}	233.977	230.799	228.441	225.692	223.634	221.219	218.958	215.224
S_{sy}	105.290	103.860	102.798	101.561	100.635	99.548	98.531	96.851
S_y	175.483	173.100	171.331	169.269	167.726	165.914	164.218	161.418
C	4.589	5.412	6.099	6.993	7.738	8.708	9.721	11.650
D	0.307	0.390	0.463	0.566	0.658	0.784	0.923	1.212
F_i (calc)	6.505	5.773	5.257	4.675	4.251	3.764	3.320	2.621
F_i (rd)	7.0	6.0	5.5	5.0	4.5	4.0	3.5	3.0
k	22.000	24.000	25.000	26.000	27.000	28.000	29.000	30.000
N_a	45.29	27.20	19.27	13.10	9.77	7.00	5.13	3.15
N_b	44.89	26.80	18.86	12.69	9.36	6.59	4.72	2.75
L_0	3.556	2.637	2.285	2.080	2.026	2.071	2.201	2.605
$L_{18\,\text{lbf}}$	4.056	3.137	2.785	2.580	2.526	2.571	2.701	3.105
K_B	1.326	1.268	1.234	1.200	1.179	1.157	1.139	1.115
$\tau_{\max}$	62.118	60.686	59.707	58.636	57.875	57.019	56.249	55.031
$(n_y)_{\text{body}}$	1.695	1.711	1.722	1.732	1.739	1.746	1.752	1.760
τ_B	58.576	59.820	60.495	61.067	61.367	61.598	61.712	61.712
$(n_y)_B$	1.797	1.736	1.699	1.663	1.640	1.616	1.597	1.569
$(n_y)_A$	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500
fom	−0.160	−0.144	−0.138	−0.135	−0.133	−0.135	−0.138	−0.154

Except for the 0.067 in wire, all springs satisfy the requirements of length and number of coils. The 0.085 in wire has the highest fom.

10-29 Given: $N_b = 84$ coils, $F_i = 16$ lbf, OQ&T steel, OD = 1.5 in, $d = 0.162$ in.

$$D = 1.5 - 0.162 = 1.338 \text{ in}$$

(a) Eq. (10-39):

$$L_0 = 2(D - d) + (N_b + 1)d$$

$$= 2(1.338 - 0.162) + (84 + 1)(0.162) = 16.12 \text{ in} \quad Ans.$$

or $$2d + L_0 = 2(0.162) + 16.12 = 16.45 \text{ in overall.}$$

(b)

$$C = \frac{D}{d} = \frac{1.338}{0.162} = 8.26$$

$$K_B = \frac{4(8.26) + 2}{4(8.26) - 3} = 1.166$$

$$\tau_i = K_B \left[\frac{8F_i D}{\pi d^3}\right] = 1.166 \left[\frac{8(16)(1.338)}{\pi(0.162)^3}\right] = 14\,950 \text{ psi} \quad Ans.$$

(c) From Table 10-5 use: $G = 11.4(10^6)$ psi and $E = 28.5(10^6)$ psi

$$N_a = N_b + \frac{G}{E} = 84 + \frac{11.4}{28.5} = 84.4 \text{ turns}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(0.162)^4(11.4)(10^6)}{8(1.338)^3(84.4)} = 4.855 \text{ lbf/in} \quad Ans.$$

(d) Table 10-4: $A = 147 \text{ psi} \cdot \text{in}^m, \quad m = 0.187$

$$S_{ut} = \frac{147}{(0.162)^{0.187}} = 207.1 \text{ kpsi}$$

$$S_y = 0.75(207.1) = 155.3 \text{ kpsi}$$

$$S_{sy} = 0.50(207.1) = 103.5 \text{ kpsi}$$

Body

$$F = \frac{\pi d^3 S_{sy}}{\pi K_B D}$$

$$= \frac{\pi(0.162)^3(103.5)(10^3)}{8(1.166)(1.338)} = 110.8 \text{ lbf}$$

Torsional stress on hook point B

$$C_2 = \frac{2r_2}{d} = \frac{2(0.25 + 0.162/2)}{0.162} = 4.086$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4.086) - 1}{4(4.086) - 4} = 1.243$$

$$F = \frac{\pi(0.162)^3(103.5)(10^3)}{8(1.243)(1.338)} = 103.9 \text{ lbf}$$

Normal stress on hook point A

$$C_1 = \frac{2r_1}{d} = \frac{1.338}{0.162} = 8.26$$

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} = \frac{4(8.26)^2 - 8.26 - 1}{4(8.26)(8.26 - 1)} = 1.099$$

$$S_{yt} = \sigma = F\left[\frac{16(K)_A D}{\pi d^3} + \frac{4}{\pi d^2}\right]$$

$$F = \frac{155.3(10^3)}{[16(1.099)(1.338)]/[\pi(0.162)^3] + \{4/[\pi(0.162)^2]\}} = 85.8 \text{ lbf}$$

$$= \min(110.8, 103.9, 85.8) = 85.8 \text{ lbf} \quad \textit{Ans.}$$

(e) Eq. (10-48):

$$y = \frac{F - F_i}{k} = \frac{85.8 - 16}{4.855} = 14.4 \text{ in} \quad \textit{Ans.}$$

10-30 $F_{\min} = 9$ lbf, $F_{\max} = 18$ lbf

$$F_a = \frac{18 - 9}{2} = 4.5 \text{ lbf}, \qquad F_m = \frac{18 + 9}{2} = 13.5 \text{ lbf}$$

A313 stainless: $0.013 \le d \le 0.1$ $A = 169 \text{ kpsi} \cdot \text{in}^m$, $m = 0.146$

$0.1 \le d \le 0.2$ $A = 128 \text{ kpsi} \cdot \text{in}^m$, $m = 0.263$

$E = 28$ Mpsi, $G = 10$ Gpsi

Try $d = 0.081$ in and refer to the discussion following Ex. 10-7

$$S_{ut} = \frac{169}{(0.081)^{0.146}} = 243.9 \text{ kpsi}$$

$$S_{su} = 0.67 S_{ut} = 163.4 \text{ kpsi}$$

$$S_{sy} = 0.35 S_{ut} = 85.4 \text{ kpsi}$$

$$S_y = 0.55 S_{ut} = 134.2 \text{ kpsi}$$

Table 10-8: $S_r = 0.45 S_{ut} = 109.8$ kpsi

$$S_e = \frac{S_r/2}{1 - [S_r/(2S_{ut})]^2} = \frac{109.8/2}{1 - [(109.8/2)/243.9]^2} = 57.8 \text{ kpsi}$$

$$r = F_a/F_m = 4.5/13.5 = 0.333$$

Table 6-7: $$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{r S_{ut}}\right)^2}\right]$$

$$S_a = \frac{(0.333)^2(243.9^2)}{2(57.8)}\left[-1 + \sqrt{1 + \left[\frac{2(57.8)}{0.333(243.9)}\right]^2}\right] = 42.2 \text{ kpsi}$$

Hook bending

$$(\sigma_a)_A = F_a\left[(K)_A \frac{16C}{\pi d^2} + \frac{4}{\pi d^2}\right] = \frac{S_a}{(n_f)_A} = \frac{S_a}{2}$$

$$\frac{4.5}{\pi d^2}\left[\frac{(4C^2 - C - 1)16C}{4C(C - 1)} + 4\right] = \frac{S_a}{2}$$

This equation reduces to a quadratic in C—see Prob. 10-28

The useable root for C is

$$C = 0.5\left[\frac{\pi d^2 S_a}{144} + \sqrt{\left(\frac{\pi d^2 S_a}{144}\right)^2 - \frac{\pi d^2 S_a}{36} + 2}\,\right]$$

$$= 0.5\left\{\frac{\pi(0.081)^2(42.2)(10^3)}{144} + \sqrt{\left[\frac{\pi(0.081)^2(42.2)(10^3)}{144}\right]^2 - \frac{\pi(0.081)^2(42.2)(10^3)}{36} + 2}\right\}$$

$$= 4.91$$

$$D = Cd = 0.398 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range.

$$F_i = \frac{\pi(0.081)^3}{8(0.398)}\left\{\frac{33\,500}{\exp[0.105(4.91)]} - 1000\left(4 - \frac{4.91-3}{6.5}\right)\right\}$$

$$= 8.55 \text{ lbf}$$

For simplicity we will round up to next $1/4$ integer.

$$F_i = 8.75 \text{ lbf}$$

$$k = \frac{18-9}{0.25} = 36 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.081)^4(10)(10^6)}{8(36)(0.398)^3} = 23.7 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 23.7 - \frac{10}{28} = 23.3 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.91) - 1 + 23.3](0.081) = 2.602 \text{ in}$$

$$L_{\max} = L_0 + (F_{\max} - F_i)/k = 2.602 + (18 - 8.75)/36 = 2.859 \text{ in}$$

$$(\sigma_a)_A = \frac{4.5(4)}{\pi d^2}\left(\frac{4C^2 - C - 1}{C-1} + 1\right)$$

$$= \frac{18(10^{-3})}{\pi(0.081^2)}\left[\frac{4(4.91^2) - 4.91 - 1}{4.91 - 1} + 1\right] = 21.1 \text{ kpsi}$$

$$(n_f)_A = \frac{S_a}{(\sigma_a)_A} = \frac{42.2}{21.1} = 2 \quad \text{checks}$$

Body: $$K_B = \frac{4C+2}{4C-3} = \frac{4(4.91)+2}{4(4.91)-3} = 1.300$$

$$\tau_a = \frac{8(1.300)(4.5)(0.398)}{\pi(0.081)^3}(10^{-3}) = 11.16 \text{ kpsi}$$

$$\tau_m = \frac{F_m}{F_a}\tau_a = \frac{13.5}{4.5}(11.16) = 33.47 \text{ kpsi}$$

The repeating allowable stress from Table 7-8 is

$$S_{sr} = 0.30S_{ut} = 0.30(243.9) = 73.17 \text{ kpsi}$$

The Gerber intercept is

$$S_{se} = \frac{73.17/2}{1 - [(73.17/2)/163.4]^2} = 38.5 \text{ kpsi}$$

From Table 6-7,

$$(n_f)_{\text{body}} = \frac{1}{2}\left(\frac{163.4}{33.47}\right)^2\left(\frac{11.16}{38.5}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(33.47)(38.5)}{163.4(11.16)}\right]^2}\right\} = 2.53$$

Let $r_2 = 2d = 2(0.081) = 0.162$

$$C_2 = \frac{2r_2}{d} = 4, \qquad (K)_B = \frac{4(4) - 1}{4(4) - 4} = 1.25$$

$$(\tau_a)_B = \frac{(K)_B}{K_B}\tau_a = \frac{1.25}{1.30}(11.16) = 10.73 \text{ kpsi}$$

$$(\tau_m)_B = \frac{(K)_B}{K_B}\tau_m = \frac{1.25}{1.30}(33.47) = 32.18 \text{ kpsi}$$

Table 10-8: $(S_{sr})_B = 0.28S_{ut} = 0.28(243.9) = 68.3$ kpsi

$$(S_{se})_B = \frac{68.3/2}{1 - [(68.3/2)/163.4]^2} = 35.7 \text{ kpsi}$$

$$(n_f)_B = \frac{1}{2}\left(\frac{163.4}{32.18}\right)^2\left(\frac{10.73}{35.7}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(32.18)(35.7)}{163.4(10.73)}\right]^2}\right\} = 2.51$$

Yield

Bending:

$$(\sigma_A)_{\max} = \frac{4F_{\max}}{\pi d^2}\left[\frac{(4C^2 - C - 1)}{C - 1} + 1\right]$$

$$= \frac{4(18)}{\pi(0.081^2)}\left[\frac{4(4.91)^2 - 4.91 - 1}{4.91 - 1} + 1\right](10^{-3}) = 84.4 \text{ kpsi}$$

$$(n_y)_A = \frac{134.2}{84.4} = 1.59$$

Body:

$$\tau_i = (F_i/F_a)\tau_a = (8.75/4.5)(11.16) = 21.7 \text{ kpsi}$$

$$r = \tau_a/(\tau_m - \tau_i) = 11.16/(33.47 - 21.7) = 0.948$$

$$(S_{sa})_y = \frac{r}{r + 1}(S_{sy} - \tau_i) = \frac{0.948}{0.948 + 1}(85.4 - 21.7) = 31.0 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{(S_{sa})_y}{\tau_a} = \frac{31.0}{11.16} = 2.78$$

Hook shear:

$$S_{sy} = 0.3S_{ut} = 0.3(243.9) = 73.2 \text{ kpsi}$$

$$\tau_{\max} = (\tau_a)_B + (\tau_m)_B = 10.73 + 32.18 = 42.9 \text{ kpsi}$$

$$(n_y)_B = \frac{73.2}{42.9} = 1.71$$

$$\text{fom} = -\frac{7.6\pi^2 d^2(N_b+2)D}{4} = -\frac{7.6\pi^2(0.081)^2(23.3+2)(0.398)}{4} = -1.239$$

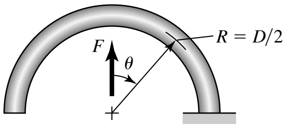
A tabulation of several wire sizes follow

d	0.081	0.085	0.092	0.098	0.105	0.12
S_{ut}	243.920	242.210	239.427	237.229	234.851	230.317
S_{su}	163.427	162.281	160.416	158.943	157.350	154.312
S_r	109.764	108.994	107.742	106.753	105.683	103.643
S_e	57.809	57.403	56.744	56.223	55.659	54.585
S_a	42.136	41.841	41.360	40.980	40.570	39.786
C	4.903	5.484	6.547	7.510	8.693	11.451
D	0.397	0.466	0.602	0.736	0.913	1.374
OD	0.478	0.551	0.694	0.834	1.018	1.494
F_i (calc)	8.572	7.874	6.798	5.987	5.141	3.637
F_i (rd)	8.75	9.75	10.75	11.75	12.75	13.75
k	36.000	36.000	36.000	36.000	36.000	36.000
N_a	23.86	17.90	11.38	8.03	5.55	2.77
N_b	23.50	17.54	11.02	7.68	5.19	2.42
L_0	2.617	2.338	2.127	2.126	2.266	2.918
$L_{18\,\text{lbf}}$	2.874	2.567	2.328	2.300	2.412	3.036
$(\sigma_a)_A$	21.068	20.920	20.680	20.490	20.285	19.893
$(n_f)_A$	2.000	2.000	2.000	2.000	2.000	2.000
K_B	1.301	1.264	1.216	1.185	1.157	1.117
$(\tau_a)_{\text{body}}$	11.141	10.994	10.775	10.617	10.457	10.177
$(\tau_m)_{\text{body}}$	33.424	32.982	32.326	31.852	31.372	30.532
S_{sr}	73.176	72.663	71.828	71.169	70.455	69.095
S_{se}	38.519	38.249	37.809	37.462	37.087	36.371
$(n_f)_{\text{body}}$	2.531	2.547	2.569	2.583	2.596	2.616
$(K)_B$	1.250	1.250	1.250	1.250	1.250	1.250
$(\tau_a)_B$	10.705	10.872	11.080	11.200	11.294	11.391
$(\tau_m)_B$	32.114	32.615	33.240	33.601	33.883	34.173
$(S_{sr})_B$	68.298	67.819	67.040	66.424	65.758	64.489
$(S_{se})_B$	35.708	35.458	35.050	34.728	34.380	33.717
$(n_f)_B$	2.519	2.463	2.388	2.341	2.298	2.235
S_y	134.156	133.215	131.685	130.476	129.168	126.674
$(\sigma_A)_{\text{max}}$	84.273	83.682	82.720	81.961	81.139	79.573
$(n_y)_A$	1.592	1.592	1.592	1.592	1.592	1.592
τ_i	21.663	23.820	25.741	27.723	29.629	31.097
r	0.945	1.157	1.444	1.942	2.906	4.703
$(S_{sy})_{\text{body}}$	85.372	84.773	83.800	83.030	82.198	80.611
$(S_{sa})_y$	30.958	32.688	34.302	36.507	39.109	40.832
$(n_y)_{\text{body}}$	2.779	2.973	3.183	3.438	3.740	4.012
$(S_{sy})_B$	73.176	72.663	71.828	71.169	70.455	69.095
$(\tau_B)_{\text{max}}$	42.819	43.486	44.321	44.801	45.177	45.564
$(n_y)_B$	1.709	1.671	1.621	1.589	1.560	1.516
fom	−1.246	−1.234	−1.245	−1.283	−1.357	−1.639

optimal fom

The shaded areas show the conditions not satisfied.

10-31 For the hook,



$$M = FR\sin\theta, \quad \partial M/\partial F = R\sin\theta$$

$$\delta_F = \frac{1}{EI}\int_0^{\pi/2} FR^2 \sin^2 R\, d\theta = \frac{\pi}{2}\frac{PR^3}{EI}$$

The total deflection of the body and the two hooks

$$\delta = \frac{8FD^3N_b}{d^4G} + 2\frac{\pi}{2}\frac{FR^3}{EI} = \frac{8FD^3N_b}{d^4G} + \frac{\pi F(D/2)^3}{E(\pi/64)(d^4)}$$

$$= \frac{8FD^3}{d^4G}\left(N_b + \frac{G}{E}\right) = \frac{8FD^3N_a}{d^4G}$$

$$\therefore \quad N_a = N_b + \frac{G}{E} \quad \text{QED}$$

10-32 Table 10-4 for A227:

$$A = 140 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.190$$

Table 10-5: $E = 28.5(10^6)$ psi

$$S_{ut} = \frac{140}{(0.162)^{0.190}} = 197.8 \text{ kpsi}$$

Eq. (10-57):

$$S_y = \sigma_{\text{all}} = 0.78(197.8) = 154.3 \text{ kpsi}$$

$$D = 1.25 - 0.162 = 1.088 \text{ in}$$

$$C = D/d = 1.088/0.162 = 6.72$$

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4(6.72)^2 - 6.72 - 1}{4(6.72)(6.72-1)} = 1.125$$

From $\sigma = K_i \dfrac{32M}{\pi d^3}$

Solving for M for the yield condition,

$$M_y = \frac{\pi d^3 S_y}{32K_i} = \frac{\pi(0.162)^3(154\,300)}{32(1.125)} = 57.2 \text{ lbf} \cdot \text{in}$$

Count the turns when $M = 0$

$$N = 2.5 - \frac{M_y}{d^4E/(10.8DN)}$$

from which

$$N = \frac{2.5}{1 + [10.8DM_y/(d^4E)]}$$

$$= \frac{2.5}{1 + \{[10.8(1.088)(57.2)]/[(0.162)^4(28.5)(10^6)]\}} = 2.417 \text{ turns}$$

This means $(2.5 - 2.417)(360^\circ)$ or 29.9° from closed. Treating the hand force as in the middle of the grip

$$r = 1 + \frac{3.5}{2} = 2.75 \text{ in}$$

$$F = \frac{M_y}{r} = \frac{57.2}{2.75} = 20.8 \text{ lbf} \quad \textit{Ans.}$$

10-33 The spring material and condition are unknown. Given $d = 0.081$ in and $\text{OD} = 0.500$,

(a) $D = 0.500 - 0.081 = 0.419$ in

Using $E = 28.6$ Mpsi for an estimate

$$k' = \frac{d^4 E}{10.8DN} = \frac{(0.081)^4(28.6)(10^6)}{10.8(0.419)(11)} = 24.7 \text{ lbf} \cdot \text{in/turn}$$

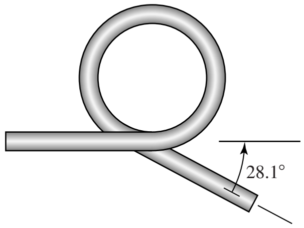
for each spring. The moment corresponding to a force of 8 lbf

$$Fr = (8/2)(3.3125) = 13.25 \text{ lbf} \cdot \text{in/spring}$$

The fraction windup turn is

$$n = \frac{Fr}{k'} = \frac{13.25}{24.7} = 0.536 \text{ turns}$$

The arm swings through an arc of slightly less than 180°, say 165°. This uses up $165/360$ or 0.458 turns. So $n = 0.536 - 0.458 = 0.078$ turns are left (or $0.078(360^\circ) = 28.1^\circ$). The original configuration of the spring was



Ans.

(b)

$$C = \frac{0.419}{0.081} = 5.17$$

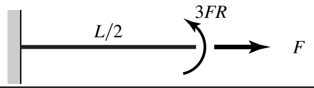
$$K_i = \frac{4(5.17)^2 - 5.17 - 1}{4(5.17)(5.17 - 1)} = 1.168$$

$$\sigma = K_i \frac{32M}{\pi d^3}$$

$$= 1.168\left[\frac{32(13.25)}{\pi(0.081)^3}\right] = 296\,623 \text{ psi} \quad \textit{Ans.}$$

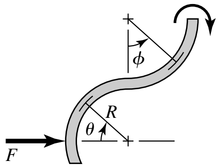
To achieve this stress level, the spring had to have set removed.

10-34 Consider half and double results

Straight section: 

$M = 3FR, \quad \dfrac{\partial M}{\partial P} = 3R$

Upper 180° section:



$$M = F[R + R(1 - \cos\phi)]$$

$$= FR(2 - \cos\phi), \quad \frac{\partial M}{\partial P} = R(2 - \cos\phi)$$

Lower section:

$$M = FR\sin\theta$$

$$\frac{\partial M}{\partial P} = R\sin\theta$$

Considering bending only:

$$\delta = \frac{2}{EI}\left[\int_0^{L/2} 9FR^2\,dx + \int_0^{\pi} FR^2(2-\cos\phi)^2 R\,d\phi + \int_0^{\pi/2} F(R\sin\theta)^2 R\,d\theta\right]$$

$$= \frac{2F}{EI}\left[\frac{9}{2}R^2 L + R^3\left(4\pi - 4\sin\phi\Big|_0^{\pi} + \frac{\pi}{2}\right) + R^3\left(\frac{\pi}{4}\right)\right]$$

$$= \frac{2FR^2}{EI}\left(\frac{19\pi}{4}R + \frac{9}{2}L\right) = \frac{FR^2}{2EI}(19\pi R + 18L) \quad \textit{Ans.}$$

10-35 Computer programs will vary.

10-36 Computer programs will vary.

Chapter 11

11-1 For the deep-groove 02-series ball bearing with $R = 0.90$, the design life x_D, in multiples of rating life, is

$$x_D = \frac{30\,000(300)(60)}{10^6} = 540 \quad \textit{Ans.}$$

The design radial load F_D is

$$F_D = 1.2(1.898) = 2.278 \text{ kN}$$

From Eq. (11-6),

$$\begin{aligned} C_{10} &= 2.278\left\{\frac{540}{0.02 + 4.439[\ln(1/0.9)]^{1/1.483}}\right\}^{1/3} \\ &= 18.59 \text{ kN} \quad \textit{Ans.} \end{aligned}$$

Table 11-2: Choose a 02-30 mm with $C_{10} = 19.5$ kN. *Ans.*

Eq. (11-18):

$$\begin{aligned} R &= \exp\left\{-\left[\frac{540(2.278/19.5)^3 - 0.02}{4.439}\right]^{1.483}\right\} \\ &= 0.919 \quad \textit{Ans.} \end{aligned}$$

11-2 For the Angular-contact 02-series ball bearing as described, the rating life multiple is

$$x_D = \frac{50\,000(480)(60)}{10^6} = 1440$$

The design load is radial and equal to

$$F_D = 1.4(610) = 854 \text{ lbf} = 3.80 \text{ kN}$$

Eq. (11-6):

$$\begin{aligned} C_{10} &= 854\left\{\frac{1440}{0.02 + 4.439[\ln(1/0.9)]^{1/1.483}}\right\}^{1/3} \\ &= 9665 \text{ lbf} = 43.0 \text{ kN} \end{aligned}$$

Table 11-2: Select a 02-55 mm with $C_{10} = 46.2$ kN. *Ans.*

Using Eq. (11-18),

$$\begin{aligned} R &= \exp\left\{-\left[\frac{1440(3.8/46.2)^3 - 0.02}{4.439}\right]^{1.483}\right\} \\ &= 0.927 \quad \textit{Ans.} \end{aligned}$$

11-3 For the straight-Roller 03-series bearing selection, $x_D = 1440$ rating lives from Prob. 11-2 solution.

$$F_D = 1.4(1650) = 2310 \text{ lbf} = 10.279 \text{ kN}$$

$$C_{10} = 10.279\left(\frac{1440}{1}\right)^{3/10} = 91.1 \text{ kN}$$

Table 11-3: Select a 03-55 mm with $C_{10} = 102$ kN. *Ans.*

Using Eq. (11-18),

$$R = \exp\left\{-\left[\frac{1440(10.28/102)^{10/3} - 0.02}{4.439}\right]^{1.483}\right\} = 0.942 \quad \textit{Ans.}$$

11-4 We can choose a reliability goal of $\sqrt{0.90} = 0.95$ for each bearing. We make the selections, find the existing reliabilities, multiply them together, and observe that the reliability goal is exceeded due to the roundup of capacity upon table entry.

Another possibility is to use the reliability of one bearing, say R_1. Then set the reliability goal of the second as

$$R_2 = \frac{0.90}{R_1}$$

or vice versa. This gives three pairs of selections to compare in terms of cost, geometry implications, etc.

11-5 Establish a reliability goal of $\sqrt{0.90} = 0.95$ for each bearing. For a 02-series angular contact ball bearing,

$$C_{10} = 854\left\{\frac{1440}{0.02 + 4.439[\ln(1/0.95)]^{1/1.483}}\right\}^{1/3}$$

$$= 11\,315 \text{ lbf} = 50.4 \text{ kN}$$

Select a 02-60 mm angular-contact bearing with $C_{10} = 55.9$ kN.

$$R_A = \exp\left\{-\left[\frac{1440(3.8/55.9)^3 - 0.02}{4.439}\right]^{1.483}\right\} = 0.969$$

For a 03-series straight-roller bearing,

$$C_{10} = 10.279\left\{\frac{1440}{0.02 + 4.439[\ln(1/0.95)]^{1/1.483}}\right\}^{3/10} = 105.2 \text{ kN}$$

Select a 03-60 mm straight-roller bearing with $C_{10} = 123$ kN.

$$R_B = \exp\left\{-\left[\frac{1440(10.28/123)^{10/3} - 0.02}{4.439}\right]^{1.483}\right\} = 0.977$$

The overall reliability is $R = 0.969(0.977) = 0.947,$ which exceeds the goal. Note, using R_A from this problem, and R_B from Prob. 11-3, $R = 0.969(0.942) = 0.913,$ which still exceeds the goal. Likewise, using R_B from this problem, and R_A from Prob. 11-2, $R = 0.927(0.977) = 0.906.$

The point is that the designer has choices. Discover them before making the selection decision. Did the answer to Prob. 11-4 uncover the possibilities?

11-6 Choose a 02-series ball bearing from manufacturer #2, having a service factor of 1. For $F_r = 8$ kN and $F_a = 4$ kN

$$x_D = \frac{5000(900)(60)}{10^6} = 270$$

Eq. (11-5):

$$C_{10} = 8\left\{\frac{270}{0.02 + 4.439[\ln(1/0.90)]^{1/1.483}}\right\}^{1/3} = 51.8 \text{ kN}$$

Trial #1: From Table (11-2) make a tentative selection of a deep-groove 02-70 mm with $C_0 = 37.5$ kN.

$$\frac{F_a}{C_0} = \frac{4}{37.5} = 0.107$$

Table 11-1:

$$F_a/(VF_r) = 0.5 > e$$

$$X_2 = 0.56, \quad Y_2 = 1.46$$

Eq. (11-9):

$$F_e = 0.56(1)(8) + 1.46(4) = 10.32 \text{ kN}$$

Eq. (11-6): For $R = 0.90$,

$$C_{10} = 10.32\left(\frac{270}{1}\right)^{1/3} = 66.7 \text{ kN} > 61.8 \text{ kN}$$

Trial #2: From Table 11-2 choose a 02-80 mm having $C_{10} = 70.2$ and $C_0 = 45.0$.

Check:

$$\frac{F_a}{C_0} = \frac{4}{45} = 0.089$$

Table 11-1: $X_2 = 0.56$, $Y_2 = 1.53$

$$F_e = 0.56(8) + 1.53(4) = 10.60 \text{ kN}$$

Eq. (11-6):

$$C_{10} = 10.60\left(\frac{270}{1}\right)^{1/3} = 68.51 \text{ kN} < 70.2 \text{ kN}$$

$\therefore$ Selection stands.

Decision: Specify a 02-80 mm deep-groove ball bearing. *Ans.*

11-7 From Prob. 11-6, $x_D = 270$ and the final value of F_e is 10.60 kN.

$$C_{10} = 10.6\left\{\frac{270}{0.02 + 4.439[\ln(1/0.96)]^{1/1.483}}\right\}^{1/3} = 84.47 \text{ kN}$$

Table 11-2: Choose a deep-groove ball bearing, based upon C_{10} load ratings.

Trial #1:

Tentatively select a 02-90 mm.

$$C_{10} = 95.6, \quad C_0 = 62 \text{ kN}$$

$$\frac{F_a}{C_0} = \frac{4}{62} = 0.0645$$

From Table 11-1, interpolate for Y_2.

F_a/C_0	Y_2
0.056	1.71
0.0645	Y_2
0.070	1.63

$$\frac{Y_2 - 1.71}{1.63 - 1.71} = \frac{0.0645 - 0.056}{0.070 - 0.056} = 0.607$$

$$Y_2 = 1.71 + 0.607(1.63 - 1.71) = 1.661$$

$$F_e = 0.56(8) + 1.661(4) = 11.12 \text{ kN}$$

$$C_{10} = 11.12\left\{\frac{270}{0.02 + 4.439[\ln(1/0.96)]^{1/1.483}}\right\}^{1/3}$$

$$= 88.61 \text{ kN} < 95.6 \text{ kN}$$

Bearing is OK.

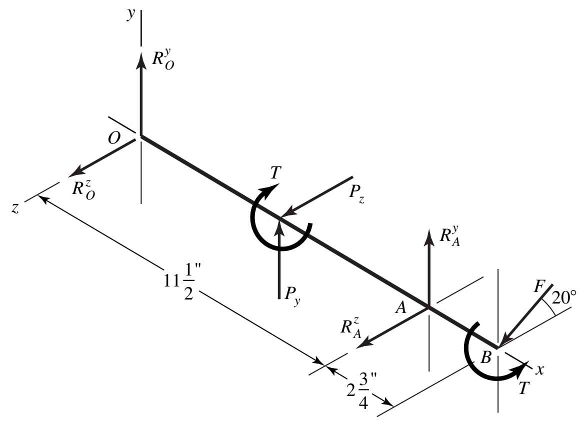
Decision: Specify a deep-groove 02-90 mm ball bearing. *Ans.*

11-8 For the straight cylindrical roller bearing specified with a service factor of 1, $R = 0.90$ and $F_r = 12$ kN

$$x_D = \frac{4000(750)(60)}{10^6} = 180$$

$$C_{10} = 12\left(\frac{180}{1}\right)^{3/10} = 57.0 \text{ kN} \quad \textit{Ans.}$$

11-9



Assume concentrated forces as shown.

$$P_z = 8(24) = 192 \text{ lbf}$$

$$P_y = 8(30) = 240 \text{ lbf}$$

$$T = 192(2) = 384 \text{ lbf} \cdot \text{in}$$

$$\sum T^x = -384 + 1.5F \cos 20^\circ = 0$$

$$F = \frac{384}{1.5(0.940)} = 272 \text{ lbf}$$

$$\sum M_O^z = 5.75P_y + 11.5R_A^y - 14.25F \sin 20^\circ = 0;$$

thus $\quad 5.75(240) + 11.5R_A^y - 14.25(272)(0.342) = 0$

$$R_A^y = -4.73 \text{ lbf}$$

$$\sum M_O^y = -5.75P_z - 11.5R_A^z - 14.25F \cos 20^\circ = 0;$$

thus $\quad -5.75(192) - 11.5R_A^z - 14.25(272)(0.940) = 0$

$$R_A^z = -413 \text{ lbf}; \qquad R_A = [(-413)^2 + (-4.73)^2]^{1/2} = 413 \text{ lbf}$$

$$\sum F^z = R_O^z + P_z + R_A^z + F \cos 20^\circ = 0$$

$$R_O^z + 192 - 413 + 272(0.940) = 0$$

$$R_O^z = -34.7 \text{ lbf}$$

$$\sum F^y = R_O^y + P_y + R_A^y - F \sin 20^\circ = 0$$

$$R_O^y + 240 - 4.73 - 272(0.342) = 0$$

$$R_O^y = -142 \text{ lbf}$$

$$R_O = [(-34.6)^2 + (-142)^2]^{1/2} = 146 \text{ lbf}$$

So the reaction at *A* governs.

Reliability Goal: $\sqrt{0.92} = 0.96$

$$F_D = 1.2(413) = 496 \text{ lbf}$$

$$x_D = 30\,000(300)(60/10^6) = 540$$

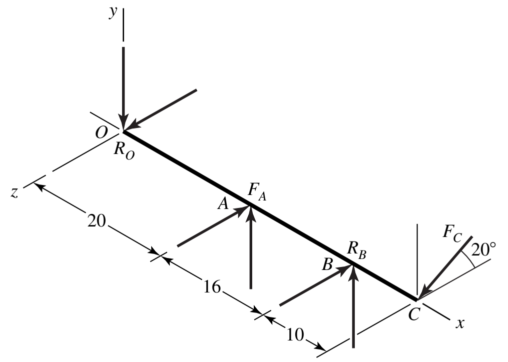
$$C_{10} = 496\left\{\frac{540}{0.02 + 4.439[\ln(1/0.96)]^{1/1.483}}\right\}^{1/3}$$

$$= 4980 \text{ lbf} = 22.16 \text{ kN}$$

A 02-35 bearing will do.

Decision: Specify an angular-contact 02-35 mm ball bearing for the locations at A and O. Check combined reliability. *Ans.*

11-10 For a combined reliability goal of 0.90, use $\sqrt{0.90} = 0.95$ for the individual bearings.



$$x_0 = \frac{50\,000(480)(60)}{10^6} = 1440$$

The resultant of the given forces are $R_O = [(-387)^2 + 467^2]^{1/2} = 607$ lbf and $R_B = [316^2 + (-1615)^2]^{1/2} = 1646$ lbf.

At O: $F_e = 1.4(607) = 850$ lbf

Ball:
$$C_{10} = 850\left\{\frac{1440}{0.02 + 4.439[\ln(1/0.95)]^{1/1.483}}\right\}^{1/3}$$

$$= 11\,262 \text{ lbf} \quad \text{or} \quad 50.1 \text{ kN}$$

Select a 02-60 mm angular-contact ball bearing with a basic load rating of 55.9 kN. *Ans.*

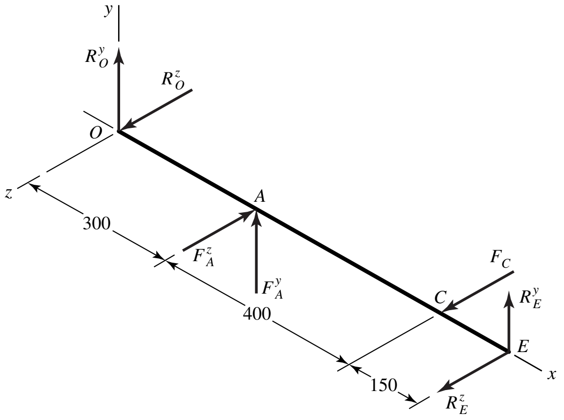
At B: $F_e = 1.4(1646) = 2304$ lbf

Roller:
$$C_{10} = 2304\left\{\frac{1440}{0.02 + 4.439[\ln(1/0.95)]^{1/1.483}}\right\}^{3/10}$$

$$= 23\,576 \text{ lbf} \quad \text{or} \quad 104.9 \text{ kN}$$

Select a 02-80 mm cylindrical roller or a 03-60 mm cylindrical roller. The 03-series roller has the same bore as the 02-series ball. *Ans.*

11-11 The reliability of the individual bearings is $R = \sqrt{0.999} = 0.9995$



From statics,

$$R_O^y = -163.4 \text{ N}, \quad R_O^z = 107 \text{ N}, \quad R_O = 195 \text{ N}$$

$$R_E^y = -89.2 \text{ N}, \quad R_E^z = -174.4 \text{ N}, \quad R_E = 196 \text{ N}$$

$$x_D = \frac{60\,000(1200)(60)}{10^6} = 4320$$

$$C_{10} = 0.196\left\{\frac{4340}{0.02 + 4.439[\ln(1/0.9995)]^{1/1.483}}\right\}^{1/3}$$

$$= 8.9 \text{ kN}$$

A 02-25 mm deep-groove ball bearing has a basic load rating of 14.0 kN which is ample. An extra-light bearing could also be investigated.

11-12 Given:

$$F_{rA} = 560 \text{ lbf} \quad \text{or} \quad 2.492 \text{ kN}$$

$$F_{rB} = 1095 \text{ lbf} \quad \text{or} \quad 4.873 \text{ kN}$$

Trial #1: Use $K_A = K_B = 1.5$ and from Table 11-6 choose an indirect mounting.

$$\frac{0.47F_{rA}}{K_A} < ? > \frac{0.47F_{rB}}{K_B} - (-1)(0)$$

$$\frac{0.47(2.492)}{1.5} < ? > \frac{0.47(4.873)}{1.5}$$

$0.781 < 1.527$ Therefore use the upper line of Table 11-6.

$$F_{aA} = F_{aB} = \frac{0.47F_{rB}}{K_B} = 1.527 \text{ kN}$$

$$P_A = 0.4F_{rA} + K_A F_{aA} = 0.4(2.492) + 1.5(1.527) = 3.29 \text{ kN}$$

$$P_B = F_{rB} = 4.873 \text{ kN}$$

Fig. 11-16: $f_T = 0.8$

Fig. 11-17: $f_V = 1.07$

Thus, $a_{3l} = f_T f_V = 0.8(1.07) = 0.856$

Individual reliability: $R_i = \sqrt{0.9} = 0.95$

Eq. (11-17):

$$(C_{10})_A = 1.4(3.29)\left[\frac{40\,000(400)(60)}{4.48(0.856)(1-0.95)^{2/3}(90)(10^6)}\right]^{0.3}$$

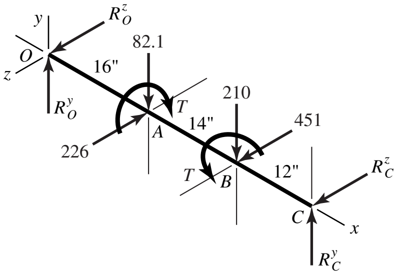
$$= 11.40 \text{ kN}$$

$$(C_{10})_B = 1.4(4.873)\left[\frac{40\,000(400)(60)}{4.48(0.856)(1-0.95)^{2/3}(90)(10^6)}\right]^{0.3}$$

$$= 16.88 \text{ kN}$$

From Fig. 11-15, choose cone 32 305 and cup 32 305 which provide $F_r = 17.4$ kN and $K = 1.95$. With $K = 1.95$ for both bearings, a second trial validates the choice of cone 32 305 and cup 32 305. *Ans.*

11-13



$$R = \sqrt{0.95} = 0.975$$

$$T = 240(12)(\cos 20^\circ) = 2706 \text{ lbf}\cdot\text{in}$$

$$F = \frac{2706}{6\cos 25^\circ} = 498 \text{ lbf}$$

In *xy*-plane:

$$\sum M_O = -82.1(16) - 210(30) + 42R_C^y = 0$$

$$R_C^y = 181 \text{ lbf}$$

$$R_O^y = 82 + 210 - 181 = 111 \text{ lbf}$$

In *xz*-plane:

$$\sum M_O = 226(16) - 452(30) - 42R_c^z = 0$$

$$R_C^z = -237 \text{ lbf}$$

$$R_O^z = 226 - 451 + 237 = 12 \text{ lbf}$$

$$R_O = (111^2 + 12^2)^{1/2} = 112 \text{ lbf} \quad \textit{Ans.}$$

$$R_C = (181^2 + 237^2)^{1/2} = 298 \text{ lbf} \quad \textit{Ans.}$$

$$F_{eO} = 1.2(112) = 134.4 \text{ lbf}$$

$$F_{eC} = 1.2(298) = 357.6 \text{ lbf}$$

$$x_D = \frac{40\,000(200)(60)}{10^6} = 480$$

$$(C_{10})_O = 134.4\left\{\frac{480}{0.02 + 4.439[\ln(1/0.975)]^{1/1.483}}\right\}^{1/3}$$

$$= 1438 \text{ lbf} \quad \text{or} \quad 6.398 \text{ kN}$$

$$(C_{10})_C = 357.6\left\{\frac{480}{0.02 + 4.439[\ln(1/0.975)]^{1/1.483}}\right\}^{1/3}$$

$$= 3825 \text{ lbf} \quad \text{or} \quad 17.02 \text{ kN}$$

Bearing at O: Choose a deep-groove 02-12 mm. *Ans.*

Bearing at C: Choose a deep-groove 02-30 mm. *Ans.*

There may be an advantage to the identical 02-30 mm bearings in a gear-reduction unit.

11-14 Shafts subjected to thrust can be constrained by bearings, one of which supports the thrust. The shaft floats within the endplay of the second (Roller) bearing. Since the thrust force here is larger than any radial load, the bearing absorbing the thrust is heavily loaded compared to the other bearing. The second bearing is thus oversized and does not contribute measurably to the chance of failure. This is predictable. The reliability goal is not $\sqrt{0.99}$, but 0.99 for the ball bearing. The reliability of the roller is 1. Beginning here saves effort.

Bearing at A (Ball)

$$F_r = (36^2 + 212^2)^{1/2} = 215 \text{ lbf} = 0.957 \text{ kN}$$
$$F_a = 555 \text{ lbf} = 2.47 \text{ kN}$$

Trial #1:
Tentatively select a 02-85 mm angular-contact with $C_{10} = 90.4$ kN and $C_0 = 63.0$ kN.

$$\frac{F_a}{C_0} = \frac{2.47}{63.0} = 0.0392$$

$$x_D = \frac{25\,000(600)(60)}{10^6} = 900$$

Table 11-1: $X_2 = 0.56$, $Y_2 = 1.88$

$$F_e = 0.56(0.957) + 1.88(2.47) = 5.18 \text{ kN}$$
$$F_D = f_A F_e = 1.3(5.18) = 6.73 \text{ kN}$$
$$C_{10} = 6.73\left\{\frac{900}{0.02 + 4.439[\ln(1/0.99)]^{1/1.483}}\right\}^{1/3}$$
$$= 107.7 \text{ kN} > 90.4 \text{ kN}$$

Trial #2:
Tentatively select a 02-95 mm angular-contact ball with $C_{10} = 121$ kN and $C_0 = 85$ kN.

$$\frac{F_a}{C_0} = \frac{2.47}{85} = 0.029$$

Table 11-1: $Y_2 = 1.98$

$$F_e = 0.56(0.957) + 1.98(2.47) = 5.43 \text{ kN}$$

$$F_D = 1.3(5.43) = 7.05 \text{ kN}$$

$$C_{10} = 7.05\left\{\frac{900}{0.02 + 4.439[\ln(1/0.99)]^{1/1.483}}\right\}^{1/3}$$

$$= 113 \text{ kN} < 121 \text{ kN} \quad O.K.$$

Select a 02-95 mm angular-contact ball bearing. *Ans.*

Bearing at B (Roller): Any bearing will do since $R = 1$. Let's prove it. From Eq. (11-18) when

$$\left(\frac{a_f F_D}{C_{10}}\right)^3 x_D < x_0 \qquad R = 1$$

The smallest 02-series roller has a $C_{10} = 16.8$ kN for a basic load rating.

$$\left(\frac{0.427}{16.8}\right)^3 (900) < ? > 0.02$$

$$0.0148 < 0.02 \qquad \therefore R = 1$$

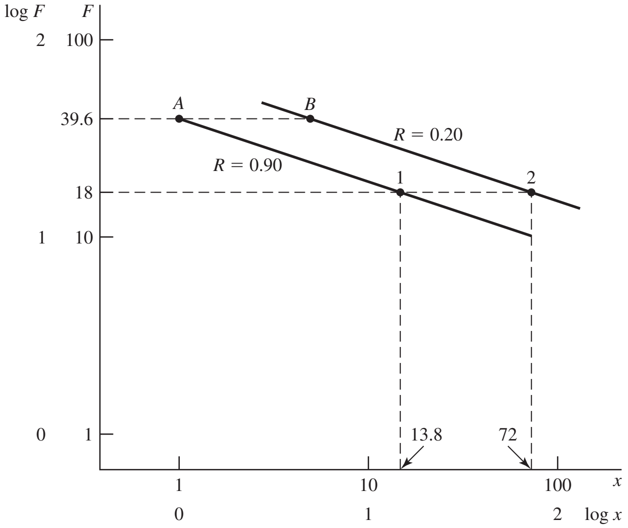
Spotting this early avoided rework from $\sqrt{0.99} = 0.995$.

Any 02-series roller bearing will do. Same bore or outside diameter is a common choice. (Why?) *Ans.*

11-15 Hoover Ball-bearing Division uses the same 2-parameter Weibull model as Timken: $b = 1.5, \theta = 4.48$. We have some data. Let's estimate parameters b and θ from it. In Fig. 11-5, we will use line AB. In this case, B is to the right of A.

For $F = 18$ kN, $$(x)_1 = \frac{115(2000)(16)}{10^6} = 13.8$$

This establishes point 1 on the $R = 0.90$ line.



The $R = 0.20$ locus is above and parallel to the $R = 0.90$ locus. For the two-parameter Weibull distribution, $x_0 = 0$ and points A and B are related by [see Eq. (20-25)]:

$$x_A = \theta[\ln(1/0.90)]^{1/b} \tag{1}$$

$$x_B = \theta[\ln(1/0.20)]^{1/b}$$

and x_B/x_A is in the same ratio as $600/115$. Eliminating θ

$$b = \frac{\ln[\ln(1/0.20)/\ln(1/0.90)]}{\ln(600/115)} = 1.65 \quad \textit{Ans.}$$

Solving for θ in Eq. (1)

$$\theta = \frac{x_A}{[\ln(1/R_A)]^{1/1.65}} = \frac{1}{[\ln(1/0.90)]^{1/1.65}} = 3.91 \quad \textit{Ans.}$$

Therefore, for the data at hand,

$$R = \exp\left[-\left(\frac{x}{3.91}\right)^{1.65}\right]$$

Check R at point B: $x_B = (600/115) = 5.217$

$$R = \exp\left[-\left(\frac{5.217}{3.91}\right)^{1.65}\right] = 0.20$$

Note also, for point 2 on the $R = 0.20$ line.

$$\log(5.217) - \log(1) = \log(x_m)_2 - \log(13.8)$$

$$(x_m)_2 = 72$$

11-16 This problem is rich in useful variations. Here is one.

Decision: Make straight roller bearings identical on a given shaft. Use a reliability goal of $(0.99)^{1/6} = 0.9983$.

Shaft a

$$F_A^r = (239^2 + 111^2)^{1/2} = 264 \text{ lbf} \quad \text{or} \quad 1.175 \text{ kN}$$

$$F_B^r = (502^2 + 1075^2)^{1/2} = 1186 \text{ lbf} \quad \text{or} \quad 5.28 \text{ kN}$$

Thus the bearing at B controls

$$x_D = \frac{10\,000(1200)(60)}{10^6} = 720$$

$$0.02 + 4.439[\ln(1/0.9983)]^{1/1.483} = 0.080\,26$$

$$C_{10} = 1.2(5.2)\left(\frac{720}{0.080\,26}\right)^{0.3} = 97.2 \text{ kN}$$

Select either a 02-80 mm with $C_{10} = 106$ kN or a 03-55 mm with $C_{10} = 102$ kN. *Ans.*

Shaft b

$$F_C^r = (874^2 + 2274^2)^{1/2} = 2436 \text{ lbf} \quad \text{or} \quad 10.84 \text{ kN}$$
$$F_D^r = (393^2 + 657^2)^{1/2} = 766 \text{ lbf} \quad \text{or} \quad 3.41 \text{ kN}$$

The bearing at C controls

$$x_D = \frac{10\,000(240)(60)}{10^6} = 144$$

$$C_{10} = 1.2(10.84)\left(\frac{144}{0.0826}\right)^{0.3} = 122 \text{ kN}$$

Select either a 02-90 mm with $C_{10} = 142$ kN or a 03-60 mm with $C_{10} = 123$ kN. *Ans.*

Shaft c

$$F_E^r = (1113^2 + 2385^2)^{1/2} = 2632 \text{ lbf} \quad \text{or} \quad 11.71 \text{ kN}$$
$$F_F^r = (417^2 + 895^2)^{1/2} = 987 \text{ lbf} \quad \text{or} \quad 4.39 \text{ kN}$$

The bearing at E controls

$$x_D = 10\,000(80)(60/10^6) = 48$$

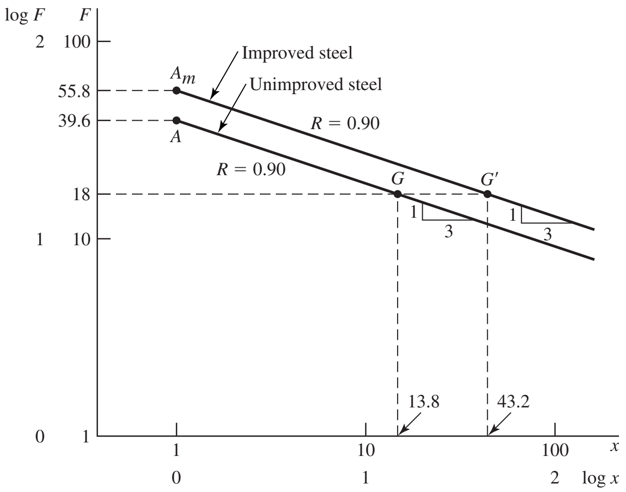
$$C_{10} = 1.2(11.71)\left(\frac{48}{0.0826}\right)^{0.3} = 94.8 \text{ kN}$$

Select a 02-80 mm with $C_{10} = 106$ kN or a 03-60 mm with $C_{10} = 123$ kN. *Ans.*

11-17 The horizontal separation of the $R = 0.90$ loci in a log F-log x plot such as Fig. 11-5 will be demonstrated. We refer to the solution of Prob. 11-15 to plot point G ($F =$ 18 kN, $x_G = 13.8$). We know that $(C_{10})_1 = 39.6$ kN, $x_1 = 1$. This establishes the unimproved steel $R = 0.90$ locus, line AG. For the improved steel

$$(x_m)_1 = \frac{360(2000)(60)}{10^6} = 43.2$$

We plot point G'($F = 18$ kN, $x_{G'} = 43.2$), and draw the $R = 0.90$ locus A_mG' parallel to AG



We can calculate $(C_{10})_m$ by similar triangles.

$$\frac{\log(C_{10})_m - \log 18}{\log 43.2 - \log 1} = \frac{\log 39.6 - \log 18}{\log 13.8 - \log 1}$$

$$\log(C_{10})_m = \frac{\log 43.2}{\log 13.8} \log\left(\frac{39.6}{18}\right) + \log 18$$

$$(C_{10})_m = 55.8 \text{ kN}$$

The usefulness of this plot is evident. The improvement is $43.2/13.8 = 3.13$ fold in life. This result is also available by $(L_{10})_m/(L_{10})_1$ as $360/115$ or 3.13 fold, but the plot shows the improvement is for all loading. Thus, the manufacturer's assertion that there is at least a 3-fold increase in life has been demonstrated by the sample data given. *Ans.*

11-18 Express Eq. (11-1) as

$$F_1^a L_1 = C_{10}^a L_{10} = K$$

For a ball bearing, $a = 3$ and for a 02-30 mm angular contact bearing, $C_{10} = 20.3$ kN.

$$K = (20.3)^3(10^6) = 8.365(10^9)$$

At a load of 18 kN, life L_1 is given by:

$$L_1 = \frac{K}{F_1^a} = \frac{8.365(10^9)}{18^3} = 1.434(10^6) \text{ rev}$$

For a load of 30 kN, life L_2 is:

$$L_2 = \frac{8.365(10^9)}{30^3} = 0.310(10^6) \text{ rev}$$

In this case, Eq. (7-57) – the Palmgren-Miner cycle ratio summation rule – can be expressed as

$$\frac{l_1}{L_1} + \frac{l_2}{L_2} = 1$$

Substituting,

$$\frac{200\,000}{1.434(10^6)} + \frac{l_2}{0.310(10^6)} = 1$$

$$l_2 = 0.267(10^6) \text{ rev} \quad \textit{Ans.}$$

11-19 *Total life in revolutions*

Let:

l = total turns
f_1 = fraction of turns at F_1
f_2 = fraction of turns at F_2

From the solution of Prob. 11-18, $L_1 = 1.434(10^6)$ rev and $L_2 = 0.310(10^6)$ rev.

Palmgren-Miner rule:

$$\frac{l_1}{L_1} + \frac{l_2}{L_2} = \frac{f_1 l}{L_1} + \frac{f_2 l}{L_2} = 1$$

from which

$$l = \frac{1}{f_1/L_1 + f_2/L_2}$$

$$l = \frac{1}{\{0.40/[1.434(10^6)]\} + \{0.60/[0.310(10^6)]\}}$$

$$= 451\,585 \text{ rev} \quad \textit{Ans.}$$

Total life in loading cycles

$$4 \text{ min at } 2000 \text{ rev/min} = 8000 \text{ rev}$$

$$\frac{6 \text{ min}}{10 \text{ min/cycle}} \text{ at } 2000 \text{ rev/min} = \frac{12\,000 \text{ rev}}{20\,000 \text{ rev/cycle}}$$

$$\frac{451\,585 \text{ rev}}{20\,000 \text{ rev/cycle}} = 22.58 \text{ cycles} \quad \textit{Ans.}$$

Total life in hours

$$\left(10\frac{\text{min}}{\text{cycle}}\right)\left(\frac{22.58 \text{ cycles}}{60 \text{ min/h}}\right) = 3.76 \text{ h} \quad \textit{Ans.}$$

11-20 While we made some use of the log *F*-log *x* plot in Probs. 11-15 and 11-17, the principal use of Fig. 11-5 is to understand equations (11-6) and (11-7) in the discovery of the catalog basic load rating for a case at hand.

Point D

$$F_D = 495.6 \text{ lbf}$$

$$\log F_D = \log 495.6 = 2.70$$

$$x_D = \frac{30\,000(300)(60)}{10^6} = 540$$

$$\log x_D = \log 540 = 2.73$$

$$K_D = F_D^3 x_D = (495.6)^3(540)$$

$$= 65.7(10^9) \text{ lbf}^3 \cdot \text{turns}$$

$$\log K_D = \log[65.7(10^9)] = 10.82$$

F_D has the following uses: F_{design}, F_{desired}, F_e when a thrust load is present. It can include application factor a_f, or not. It depends on context.

Point B

$$\begin{aligned} x_B &= 0.02 + 4.439[\ln(1/0.99)]^{1/1.483} \\ &= 0.220 \text{ turns} \\ \log x_B &= \log 0.220 = -0.658 \\ F_B &= F_D \left(\frac{x_D}{x_B}\right)^{1/3} = 495.6\left(\frac{540}{0.220}\right)^{1/3} = 6685 \text{ lbf} \end{aligned}$$

Note: Example 11-3 used Eq. (11-7). Whereas, here we basically used Eq. (11-6).

$$\begin{aligned} \log F_B &= \log(6685) = 3.825 \\ K_D &= 6685^3(0.220) = 65.7(10^9) \text{ lbf}^3 \cdot \text{turns} \qquad \text{(as it should)} \end{aligned}$$

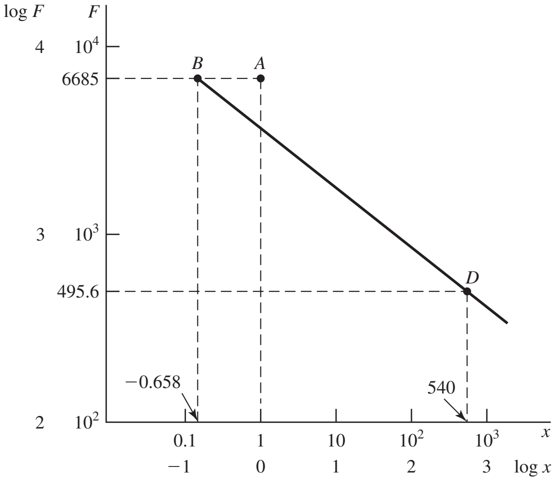
Point A

$$\begin{aligned} F_A &= F_B = C_{10} = 6685 \text{ lbf} \\ \log C_{10} &= \log(6685) = 3.825 \\ x_A &= 1 \\ \log x_A &= \log(1) = 0 \\ K_{10} &= F_A^3 x_A = C_{10}^3(1) = 6685^3 = 299(10^9) \text{ lbf}^3 \cdot \text{turns} \end{aligned}$$

Note that $K_D/K_{10} = 65.7(10^9)/[299(10^9)] = 0.220$, which is x_B. This is worth knowing since

$$K_{10} = \frac{K_D}{x_B}$$

$$\log K_{10} = \log[299(10^9)] = 11.48$$



Now $C_{10} = 6685$ lbf $= 29.748$ kN, which is required for a reliability goal of 0.99. If we select an angular contact 02-40 mm ball bearing, then $C_{10} = 31.9$ kN $= 7169$ lbf.

Chapter 12

12-1 Given $d_{\max} = 1.000$ in and $b_{\min} = 1.0015$ in, the minimum radial clearance is

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{1.0015 - 1.000}{2} = 0.000\,75 \text{ in}$$

Also $$l/d = 1$$

$$r \doteq 1.000/2 = 0.500$$

$$r/c = 0.500/0.000\,75 = 667$$

$$N = 1100/60 = 18.33 \text{ rev/s}$$

$$P = W/(ld) = 250/[(1)(1)] = 250 \text{ psi}$$

Eq. (12-7): $$S = (667^2)\left[\frac{8(10^{-6})(18.33)}{250}\right] = 0.261$$

Fig. 12-16: $$h_0/c = 0.595$$

Fig. 12-19: $$Q/(rcNl) = 3.98$$

Fig. 12-18: $$fr/c = 5.8$$

Fig. 12-20: $$Q_s/Q = 0.5$$

$$h_0 = 0.595(0.000\,75) = 0.000\,466 \text{ in} \quad Ans.$$

$$f = \frac{5.8}{r/c} = \frac{5.8}{667} = 0.0087$$

The power loss in Btu/s is

$$H = \frac{2\pi f W r N}{778(12)} = \frac{2\pi(0.0087)(250)(0.5)(18.33)}{778(12)}$$

$$= 0.0134 \text{ Btu/s} \quad Ans.$$

$$Q = 3.98rcNl = 3.98(0.5)(0.000\,75)(18.33)(1) = 0.0274 \text{ in}^3\text{/s}$$

$$Q_s = 0.5(0.0274) = 0.0137 \text{ in}^3\text{/s} \quad Ans.$$

12-2

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{1.252 - 1.250}{2} = 0.001 \text{ in}$$

$$r \doteq 1.25/2 = 0.625 \text{ in}$$

$$r/c = 0.625/0.001 = 625$$

$$N = 1150/60 = 19.167 \text{ rev/s}$$

$$P = \frac{400}{1.25(2.5)} = 128 \text{ psi}$$

$$l/d = 2.5/1.25 = 2$$

$$S = \frac{(625^2)(10)(10^{-6})(19.167)}{128} = 0.585$$

The interpolation formula of Eq. (12-16) will have to be used. From Figs. 12-16, 12-21, and 12-19

For $l/d = \infty, \quad h_o/c = 0.96, \quad P/p_{\max} = 0.84, \quad \dfrac{Q}{rcNl} = 3.09$

$$l/d = 1, \quad h_o/c = 0.77, \quad P/p_{\max} = 0.52, \quad \frac{Q}{rcNl} = 3.6$$

$$l/d = \frac{1}{2}, \quad h_o/c = 0.54, \quad P/p_{\max} = 0.42, \quad \frac{Q}{rcNl} = 4.4$$

$$l/d = \frac{1}{4}, \quad h_o/c = 0.31, \quad P/p_{\max} = 0.28, \quad \frac{Q}{rcNl} = 5.25$$

Equation (12-16) is easily programmed by code or by using a spreadsheet. The results are:

	l/d	y_∞	y_1	$y_{1/2}$	$y_{1/4}$	$y_{l/d}$
h_o/c	2	0.96	0.77	0.54	0.31	0.88
$P/p_{\max}$	2	0.84	0.52	0.42	0.28	0.64
$Q/rcNl$	2	3.09	3.60	4.40	5.25	3.28

$$\therefore\ h_o = 0.88(0.001) = 0.000\,88 \text{ in} \quad \textit{Ans.}$$

$$p_{\max} = \frac{128}{0.64} = 200 \text{ psi} \quad \textit{Ans.}$$

$$Q = 3.28(0.625)(0.001)(19.167)(2.5) = 0.098 \text{ in}^3\text{/s} \quad \textit{Ans.}$$

12-3

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{3.005 - 3.000}{2} = 0.0025 \text{ in}$$

$$r \doteq 3.000/2 = 1.500 \text{ in}$$

$$l/d = 1.5/3 = 0.5$$

$$r/c = 1.5/0.0025 = 600$$

$$N = 600/60 = 10 \text{ rev/s}$$

$$P = \frac{800}{1.5(3)} = 177.78 \text{ psi}$$

Fig. 12-12: SAE 10, $\mu' = 1.75\ \mu$reyn

$$S = (600^2)\left[\frac{1.75(10^{-6})(10)}{177.78}\right] = 0.0354$$

Figs. 12-16 and 12-21: $h_o/c = 0.11, \quad P/p_{\max} = 0.21$

$$h_o = 0.11(0.0025) = 0.000\,275 \text{ in} \quad \textit{Ans.}$$

$$p_{\max} = 177.78/0.21 = 847 \text{ psi} \quad \textit{Ans.}$$

Fig. 12-12: SAE 40, $\mu' = 4.5\ \mu\text{reyn}$

$$S = 0.0354\left(\frac{4.5}{1.75}\right) = 0.0910$$

$$h_o/c = 0.19, \quad P/p_{\max} = 0.275$$

$$h_o = 0.19(0.0025) = 0.000\,475\text{ in} \quad Ans.$$

$$p_{\max} = 177.78/0.275 = 646\text{ psi} \quad Ans.$$

12-4

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{3.006 - 3.000}{2} = 0.003$$

$$r \doteq 3.000/2 = 1.5\text{ in}$$

$$l/d = 1$$

$$r/c = 1.5/0.003 = 500$$

$$N = 750/60 = 12.5\text{ rev/s}$$

$$P = \frac{600}{3(3)} = 66.7\text{ psi}$$

Fig. 12-14: SAE 10W, $\mu' = 2.1\ \mu\text{reyn}$

$$S = (500^2)\left[\frac{2.1(10^{-6})(12.5)}{66.7}\right] = 0.0984$$

From Figs. 12-16 and 12-21:

$$h_o/c = 0.34, \quad P/p_{\max} = 0.395$$

$$h_o = 0.34(0.003) = 0.001\,020\text{ in} \quad Ans.$$

$$p_{\max} = \frac{66.7}{0.395} = 169\text{ psi} \quad Ans.$$

Fig. 12-14: SAE 20W-40, $\mu' = 5.05\ \mu\text{reyn}$

$$S = (500^2)\left[\frac{5.05(10^{-6})(12.5)}{66.7}\right] = 0.237$$

From Figs. 12-16 and 12-21:

$$h_o/c = 0.57, \quad P/p_{\max} = 0.47$$

$$h_o = 0.57(0.003) = 0.001\,71\text{ in} \quad Ans.$$

$$p_{\max} = \frac{66.7}{0.47} = 142\text{ psi} \quad Ans.$$

12-5

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{2.0024 - 2}{2} = 0.0012 \text{ in}$$

$$r \doteq \frac{d}{2} = \frac{2}{2} = 1 \text{ in}, \quad l/d = 1/2 = 0.50$$

$$r/c = 1/0.0012 = 833$$

$$N = 800/60 = 13.33 \text{ rev/s}$$

$$P = \frac{600}{2(1)} = 300 \text{ psi}$$

Fig. 12-12: SAE 20, $\mu' = 3.75\ \mu$reyn

$$S = (833^2)\left[\frac{3.75(10^{-6})(13.3)}{300}\right] = 0.115$$

From Figs. 12-16, 12-18 and 12-19:

$$h_o/c = 0.23, \quad rf/c = 3.8, \quad Q/(rcNl) = 5.3$$

$$h_o = 0.23(0.0012) = 0.000\,276 \text{ in} \quad Ans.$$

$$f = \frac{3.8}{833} = 0.004\,56$$

The power loss due to friction is

$$H = \frac{2\pi f W r N}{778(12)} = \frac{2\pi(0.004\,56)(600)(1)(13.33)}{778(12)}$$

$$= 0.0245 \text{ Btu/s} \quad Ans.$$

$$Q = 5.3 rcNl$$

$$= 5.3(1)(0.0012)(13.33)(1)$$

$$= 0.0848 \text{ in}^3\text{/s} \quad Ans.$$

12-6

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{25.04 - 25}{2} = 0.02 \text{ mm}$$

$$r \doteq d/2 = 25/2 = 12.5 \text{ mm}, \quad l/d = 1$$

$$r/c = 12.5/0.02 = 625$$

$$N = 1200/60 = 20 \text{ rev/s}$$

$$P = \frac{1250}{25^2} = 2 \text{ MPa}$$

For $\mu = 50$ mPa · s, $\quad S = (625^2)\left[\frac{50(10^{-3})(20)}{2(10^6)}\right] = 0.195$

From Figs. 12-16, 12-18 and 12-20:

$$h_o/c = 0.52, \quad fr/c = 4.5, \quad Q_s/Q = 0.57$$

$$h_o = 0.52(0.02) = 0.0104 \text{ mm} \quad Ans.$$

$$f = \frac{4.5}{625} = 0.0072$$

$$T = fWr = 0.0072(1.25)(12.5) = 0.1125 \text{ N} \cdot \text{m}$$

The power loss due to friction is

$$H = 2\pi T N = 2\pi(0.1125)(20) = 14.14 \text{ W} \quad Ans.$$

$$Q_s = 0.57Q \quad \text{The side flow is 57\% of } Q \quad Ans.$$

12-7

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{30.05 - 30.00}{2} = 0.025 \text{ mm}$$

$$r = \frac{d}{2} = \frac{30}{2} = 15 \text{ mm}$$

$$\frac{r}{c} = \frac{15}{0.025} = 600$$

$$N = \frac{1120}{60} = 18.67 \text{ rev/s}$$

$$P = \frac{2750}{30(50)} = 1.833 \text{ MPa}$$

$$S = (600^2)\left[\frac{60(10^{-3})(18.67)}{1.833(10^6)}\right] = 0.22$$

$$\frac{l}{d} = \frac{50}{30} = 1.67$$

This l/d requires use of the interpolation of Raimondi and Boyd, Eq. (12-16).

From Fig. 12-16, the h_o/c values are:

$$y_{1/4} = 0.18, \quad y_{1/2} = 0.34, \quad y_1 = 0.54, \quad y_\infty = 0.89$$

Substituting into Eq. (12-16), $\dfrac{h_o}{c} = 0.659$

From Fig. 12-18, the fr/c values are:

$$y_{1/4} = 7.4, \quad y_{1/2} = 6.0, \quad y_1 = 5.0, \quad y_\infty = 4.0$$

Substituting into Eq. (12-16), $\dfrac{fr}{c} = 4.59$

From Fig. 12-19, the $Q/(rcNl)$ values are:

$$y_{1/4} = 5.65, \quad y_{1/2} = 5.05, \quad y_1 = 4.05, \quad y_\infty = 2.95$$

Substituting into Eq. (12-16), $\dfrac{Q}{rcN\,l} = 3.605$

$$h_o = 0.659(0.025) = 0.0165 \text{ mm} \quad Ans.$$

$$f = 4.59/600 = 0.007\,65 \quad Ans.$$

$$Q = 3.605(15)(0.025)(18.67)(50) = 1263 \text{ mm}^3\text{/s} \quad Ans.$$

12-8

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{75.10 - 75}{2} = 0.05 \text{ mm}$$

$$l/d = 36/75 \doteq 0.5 \quad \text{(close enough)}$$

$$r = d/2 = 75/2 = 37.5 \text{ mm}$$

$$r/c = 37.5/0.05 = 750$$

$$N = 720/60 = 12 \text{ rev/s}$$

$$P = \frac{2000}{75(36)} = 0.741 \text{ MPa}$$

Fig. 12-13: SAE 20, $\mu = 18.5 \text{ mPa} \cdot \text{s}$

$$S = (750^2)\left[\frac{18.5(10^{-3})(12)}{0.741(10^6)}\right] = 0.169$$

From Figures 12-16, 12-18 and 12-21:

$$h_o/c = 0.29, \quad fr/c = 5.1, \quad P/p_{\max} = 0.315$$

$$h_o = 0.29(0.05) = 0.0145 \text{ mm} \quad Ans.$$

$$f = 5.1/750 = 0.0068$$

$$T = fWr = 0.0068(2)(37.5) = 0.51 \text{ N} \cdot \text{m}$$

The heat loss rate equals the rate of work on the film

$$H_{\text{loss}} = 2\pi TN = 2\pi(0.51)(12) = 38.5 \text{ W} \quad Ans.$$

$$p_{\max} = 0.741/0.315 = 2.35 \text{ MPa} \quad Ans.$$

Fig. 12-13: SAE 40, $\mu = 37 \text{ MPa} \cdot \text{s}$

$$S = 0.169(37)/18.5 = 0.338$$

From Figures 12-16, 12-18 and 12-21:

$$h_o/c = 0.42, \quad fr/c = 8.5, \quad P/p_{\max} = 0.38$$

$$h_o = 0.42(0.05) = 0.021 \text{ mm} \quad Ans.$$

$$f = 8.5/750 = 0.0113$$

$$T = fWr = 0.0113(2)(37.5) = 0.85 \text{ N} \cdot \text{m}$$

$$H_{\text{loss}} = 2\pi TN = 2\pi(0.85)(12) = 64 \text{ W} \quad Ans.$$

$$p_{\max} = 0.741/0.38 = 1.95 \text{ MPa} \quad Ans.$$

12-9

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{50.05 - 50}{2} = 0.025 \text{ mm}$$

$$r = d/2 = 50/2 = 25 \text{ mm}$$

$$r/c = 25/0.025 = 1000$$

$$l/d = 25/50 = 0.5, \quad N = 840/60 = 14 \text{ rev/s}$$

$$P = \frac{2000}{25(50)} = 1.6 \text{ MPa}$$

Fig. 12-13: SAE 30, $\mu = 34$ mPa · s

$$S = (1000^2)\left[\frac{34(10^{-3})(14)}{1.6(10^6)}\right] = 0.2975$$

From Figures 12-16, 12-18, 12-19 and 12-20:

$$h_o/c = 0.40, \quad fr/c = 7.8, \quad Q_s/Q = 0.74, \quad Q/(rcNl) = 4.9$$

$$h_o = 0.40(0.025) = 0.010 \text{ mm} \quad \textit{Ans.}$$

$$f = 7.8/1000 = 0.0078$$

$$T = fWr = 0.0078(2)(25) = 0.39 \text{ N} \cdot \text{m}$$

$$H = 2\pi TN = 2\pi(0.39)(14) = 34.3 \text{ W} \quad \textit{Ans.}$$

$$Q = 4.9rcNl = 4.9(25)(0.025)(14)(25) = 1072 \text{ mm}^2\text{/s}$$

$$Q_s = 0.74(1072) = 793 \text{ mm}^3\text{/s} \quad \textit{Ans.}$$

12-10 Consider the bearings as specified by

minimum f: $d^{+0}_{-t_d}, \quad b^{+t_b}_{-0}$

maximum W: $d'^{+0}_{-t_d}, \quad b^{+t_b}_{-0}$

and differing only in d and d'.

Preliminaries:

$$l/d = 1$$

$$P = 700/(1.25^2) = 448 \text{ psi}$$

$$N = 3600/60 = 60 \text{ rev/s}$$

Fig. 12-16:

minimum f: $S \doteq 0.08$

maximum W: $S \doteq 0.20$

Fig. 12-12: $\mu = 1.38(10^{-6})$ reyn

$$\mu N/P = 1.38(10^{-6})(60/448) = 0.185(10^{-6})$$

Eq. (12-7):

$$\frac{r}{c} = \sqrt{\frac{S}{\mu N/P}}$$

For minimum f:

$$\frac{r}{c} = \sqrt{\frac{0.08}{0.185(10^{-6})}} = 658$$

$$c = 0.625/658 = 0.000\,950 \doteq 0.001 \text{ in}$$

If this is $c_{\min}$,

$$b - d = 2(0.001) = 0.002 \text{ in}$$

The median clearance is

$$\bar{c} = c_{\min} + \frac{t_d + t_b}{2} = 0.001 + \frac{t_d + t_b}{2}$$

and the clearance range for *this* bearing is

$$\Delta c = \frac{t_d + t_b}{2}$$

which is a function only of the tolerances.

For maximum W:

$$\frac{r}{c} = \sqrt{\frac{0.2}{0.185(10^{-6})}} = 1040$$

$$c = 0.625/1040 = 0.000\,600 \doteq 0.0005 \text{ in}$$

If this is $c_{\min}$

$$b - d' = 2c_{\min} = 2(0.0005) = 0.001 \text{ in}$$

$$\bar{c} = c_{\min} + \frac{t_d + t_b}{2} = 0.0005 + \frac{t_d + t_b}{2}$$

$$\Delta c = \frac{t_d + t_b}{2}$$

The difference (mean) in clearance between the *two* clearance ranges, c_{range}, is

$$\begin{aligned} c_{\text{range}} &= 0.001 + \frac{t_d + t_b}{2} - \left(0.0005 + \frac{t_d + t_b}{2}\right) \\ &= 0.0005 \text{ in} \end{aligned}$$

For the minimum f bearing

$$b - d = 0.002 \text{ in}$$

or

$$d = b - 0.002 \text{ in}$$

For the maximum W bearing

$$d' = b - 0.001 \text{ in}$$

For the same b, t_b and t_d, we need to change the journal diameter by 0.001 in.

$$\begin{aligned} d' - d &= b - 0.001 - (b - 0.002) \\ &= 0.001 \text{ in} \end{aligned}$$

Increasing d of the minimum friction bearing by 0.001 in, defines d' of the maximum load bearing. Thus, the clearance range provides for bearing dimensions which are attainable in manufacturing. *Ans.*

12-11 Given: SAE 30, $N = 8$ rev/s, $T_s = 60°\text{C}$, $l/d = 1$, $d = 80$ mm, $b = 80.08$ mm, $W = 3000\,\text{N}$

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{80.08 - 80}{2} = 0.04 \text{ mm}$$

$$r = d/2 = 80/2 = 40 \text{ mm}$$

$$\frac{r}{c} = \frac{40}{0.04} = 1000$$

$$P = \frac{3000}{80(80)} = 0.469 \text{ MPa}$$

Trial #1: From Figure 12-13 for $T = 81°\text{C}$, $\mu = 12 \text{ mPa} \cdot \text{s}$

$$\Delta T = 2(81°\text{C} - 60°\text{C}) = 42°\text{C}$$

$$S = (1000^2)\left[\frac{12(10^{-3})(8)}{0.469(10^6)}\right] = 0.2047$$

From Fig. 12-24,

$$\frac{0.120\Delta T}{P} = 0.349 + 6.009(0.2047) + 0.0475(0.2047)^2 = 1.58$$

$$\Delta T = 1.58\left(\frac{0.469}{0.120}\right) = 6.2°\text{C}$$

Discrepancy = 42°C − 6.2°C = 35.8°C

Trial #2: From Figure 12-13 for $T = 68°\text{C}$, $\mu = 20 \text{ mPa} \cdot \text{s}$,

$$\Delta T = 2(68°\text{C} - 60°\text{C}) = 16°\text{C}$$

$$S = 0.2047\left(\frac{20}{12}\right) = 0.341$$

From Fig. 12-24,

$$\frac{0.120\Delta T}{P} = 0.349 + 6.009(0.341) + 0.0475(0.341)^2 = 2.4$$

$$\Delta T = 2.4\left(\frac{0.469}{0.120}\right) = 9.4°\text{C}$$

Discrepancy = 16°C − 9.4°C = 6.6°C

Trial #3: $\mu = 21 \text{ mPa} \cdot \text{s}$, $T = 65°\text{C}$

$$\Delta T = 2(65°\text{C} - 60°\text{C}) = 10°\text{C}$$

$$S = 0.2047\left(\frac{21}{12}\right) = 0.358$$

From Fig. 12-24,

$$\frac{0.120\Delta T}{P} = 0.349 + 6.009(0.358) + 0.0475(0.358)^2 = 2.5$$

$$\Delta T = 2.5\left(\frac{0.469}{0.120}\right) = 9.8°\text{C}$$

Discrepancy $= 10°\text{C} - 9.8°\text{C} = 0.2°\text{C}$ *O.K.*

$$T_{av} = 65°\text{C} \quad \textit{Ans.}$$

$$T_1 = T_{av} - \Delta T/2 = 65°\text{C} - (10°\text{C}/2) = 60°\text{C}$$

$$T_2 = T_{av} + \Delta T/2 = 65°\text{C} + (10°\text{C}/2) = 70°\text{C}$$

$$S = 0.358$$

From Figures 12-16, 12-18, 12-19 and 12-20:

$$\frac{h_o}{c} = 0.68, \quad fr/c = 7.5, \quad \frac{Q}{rcNl} = 3.8, \quad \frac{Q_s}{Q} = 0.44$$

$$h_o = 0.68(0.04) = 0.0272 \text{ mm} \quad \textit{Ans.}$$

$$f = \frac{7.5}{1000} = 0.0075$$

$$T = fWr = 0.0075(3)(40) = 0.9 \text{ N} \cdot \text{m}$$

$$H = 2\pi TN = 2\pi(0.9)(8) = 45.2 \text{ W} \quad \textit{Ans.}$$

$$Q = 3.8(40)(0.04)(8)(80) = 3891 \text{ mm}^3\text{/s}$$

$$Q_s = 0.44(3891) = 1712 \text{ mm}^3\text{/s} \quad \textit{Ans.}$$

12-12 Given: $d = 2.5$ in, $b = 2.504$ in, $c_{\min} = 0.002$ in, $W = 1200$ lbf, SAE $= 20$, $T_s = 110°$F, $N = 1120$ rev/min, and $l = 2.5$ in.

For a trial film temperature $T_f = 150°$F

T_f	μ'	S	ΔT (From Fig. 12-24)
150	2.421	0.0921	18.5

$$T_{av} = T_s + \frac{\Delta T}{2} = 110°\text{F} + \frac{18.5°\text{F}}{2} = 119.3°\text{F}$$

$$T_f - T_{av} = 150°\text{F} - 119.3°\text{F}$$

which is not 0.1 or less, therefore try averaging

$$(T_f)_{\text{new}} = \frac{150°\text{F} + 119.3°\text{F}}{2} = 134.6°\text{F}$$

Proceed with additional trials

Trial T_f	μ'	S	ΔT	T_{av}	New T_f
150.0	2.421	0.0921	18.5	119.3	134.6
134.6	3.453	0.1310	23.1	121.5	128.1
128.1	4.070	0.1550	25.8	122.9	125.5
125.5	4.255	0.1650	27.0	123.5	124.5
124.5	4.471	0.1700	27.5	123.8	124.1
124.1	4.515	0.1710	27.7	123.9	124.0
124.0	4.532	0.1720	27.8	123.7	123.9

Note that the convergence begins rapidly. There are ways to speed this, but at this point they would only add complexity. Depending where you stop, you can enter the analysis.

(a) $\mu = 4.541(10^{-6})$ reyn, $\quad S = 0.1724$

From Fig. 12-16: $\dfrac{h_o}{c} = 0.482, \quad h_o = 0.482(0.002) = 0.000\,964$ in

From Fig. 12-17: $\phi = 56^\circ$ *Ans.*

(b) $e = c - h_o = 0.002 - 0.000\,964 = 0.001\,04$ in *Ans.*

(c) From Fig. 12-18: $\dfrac{fr}{c} = 4.10, \quad f = 4.10(0.002/1.25) = 0.006\,56$ *Ans.*

(d) $T = fWr = 0.006\,56(1200)(1.25) = 9.84\ \text{lbf}\cdot\text{in}$

$$H = \frac{2\pi TN}{778(12)} = \frac{2\pi(9.84)(1120/60)}{778(12)} = 0.124\ \text{Btu/s} \quad \textit{Ans.}$$

(e) From Fig. 12-19: $\dfrac{Q}{rcNl} = 4.16, \quad Q = 4.16(1.25)(0.002)\left(\dfrac{1120}{60}\right)(2.5)$

$$= 0.485\ \text{in}^3/\text{s} \quad \textit{Ans.}$$

From Fig. 12-20: $\dfrac{Q_s}{Q} = 0.6, \quad Q_s = 0.6(0.485) = 0.291\ \text{in}^3/\text{s}$ *Ans.*

(f) From Fig. 12-21: $\dfrac{P}{p_{\max}} = 0.45, \quad p_{\max} = \dfrac{1200}{2.5^2(0.45)} = 427$ psi *Ans.*

$$\phi_{p_{\max}} = 16^\circ \quad \textit{Ans.}$$

(g) $\phi_{p_0} = 82^\circ$ *Ans.*

(h) $T_f = 123.9^\circ\text{F}$ *Ans.*

(i) $T_s + \Delta T = 110^\circ\text{F} + 27.8^\circ\text{F} = 137.8^\circ\text{F}$ *Ans.*

12-13 Given: $d = 1.250$ in, $t_d = 0.001$ in, $b = 1.252$ in, $t_b = 0.003$ in, $l = 1.25$ in, $W = 250$ lbf, $N = 1750$ rev/min, SAE 10 lubricant, sump temperature $T_s = 120°\text{F}$.

Below is a partial tabular summary for comparison purposes.

	$c_{\min}$ 0.001 in	$\bar{c}$ 0.002 in	$c_{\max}$ 0.003 in
T_f	132.2	125.8	124.0
ΔT	24.3	11.5	7.96
$T_{\max}$	144.3	131.5	128.0
μ'	2.587	3.014	3.150
S	0.184	0.053 7	0.024 9
ϵ	0.499	0.775 0	0.873
$\dfrac{fr}{c}$	4.317	1.881	1.243
$\dfrac{Q}{rcN_jl}$	4.129	4.572	4.691
$\dfrac{Q_s}{Q}$	0.582	0.824	0.903
$\dfrac{h_o}{c}$	0.501	0.225	0.127
f	0.006 9	0.006	0.005 9
Q	0.094 1	0.208	0.321
Q_s	0.054 8	0.172	0.290
h_o	0.000 501	0.000 495	0.000 382

Note the variations on each line. There is *not* a bearing, but an ensemble of many bearings, due to the random assembly of toleranced bushings and journals. Fortunately the distribution is bounded; the extreme cases, $c_{\min}$ and $c_{\max}$, coupled with $\bar{c}$ provide the charactistic description for the designer. All assemblies must be satisfactory.

The designer does not specify a journal-bushing bearing, but an ensemble of bearings.

12-14 Computer programs will vary—Fortran based, MATLAB, spreadsheet, etc.

12-15 In a step-by-step fashion, we are building a skill for natural circulation bearings.

- Given the average film temperature, establish the bearing properties.
- Given a sump temperature, find the average film temperature, then establish the bearing properties.
- Now we acknowledge the environmental temperature's role in establishing the sump temperature. Sec. 12-9 and Ex. 12-5 address this problem.

The task is to iteratively find the average film temperature, T_f, which makes H_{gen} and H_{loss} equal. The steps for determining $c_{\min}$ are provided within Trial #1 through Trial #3 on the following page.

Trial #1:

- Choose a value of T_f.
- Find the corresponding viscosity.
- Find the Sommerfeld number.
- Find fr/c, then

$$H_{\text{gen}} = \frac{2545}{1050} W N c \left(\frac{fr}{c}\right)$$

- Find $Q/(rcNl)$ and Q_s/Q. From Eq. (12-15)

$$\Delta T = \frac{0.103P(fr/c)}{(1 - 0.5Q_s/Q)[Q/(rcN_j l)]}$$

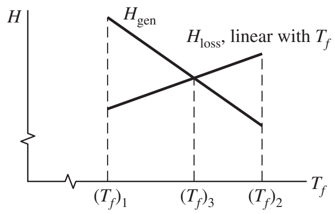
$$H_{\text{loss}} = \frac{\hbar_{\text{CR}} A(T_f - T_\infty)}{1 + \alpha}$$

- Display $T_f,\ S,\ H_{\text{gen}},\ H_{\text{loss}}$

Trial #2: Choose another T_f, repeating above drill.

Trial #3:

Plot the results of the first two trials.



Choose $(T_f)_3$ from plot. Repeat the drill. Plot the results of Trial #3 on the above graph. If you are not within 0.1°F, iterate again. Otherwise, stop, and find all the properties of the bearing for the first clearance, $c_{\min}$. See if Trumpler conditions are satisfied, and if so, analyze $\bar{c}$ and $c_{\max}$.

The bearing ensemble in the current problem statement meets Trumpler's criteria (for $n_d = 2$).

This adequacy assessment protocol can be used as a design tool by giving the students additional possible bushing sizes.

b (in)	t_b (in)
2.254	0.004
2.004	0.004
1.753	0.003

Otherwise, the design option includes reducing l/d to save on the cost of journal machining and vender-supplied bushings.

12-16 Continue to build a skill with pressure-fed bearings, that of finding the average temperature of the fluid film. First examine the case for $c = c_{\min}$

Trial #1:

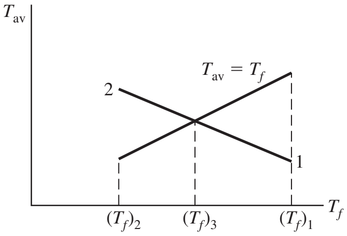
- Choose an initial T_f .
- Find the viscosity.
- Find the Sommerfeld number.
- Find fr/c, h_o/c, and ϵ.
- From Eq. (12-24), find ΔT .

$$T_{av} = T_s + \frac{\Delta T}{2}$$

- Display T_f, S, ΔT, and T_{av}.

Trial #2:

- Choose another T_f. Repeat the drill, and display the second set of values for T_f, S, ΔT, and T_{av}.
- Plot T_{av} vs T_f:



Trial #3:

Pick the third T_f from the plot and repeat the procedure. If $(T_f)_3$ and $(T_{av})_3$ differ by more than 0.1°F, plot the results for Trials #2 and #3 and try again. If they are within 0.1°F, determine the bearing parameters, check the Trumpler criteria, and compare H_{loss} with the lubricant's cooling capacity.

Repeat the entire procedure for $c = c_{\max}$ to assess the cooling capacity for the maximum radial clearance. Finally, examine $c = \bar{c}$ to characterize the ensemble of bearings.

12-17 An adequacy assessment associated with a design task is required. Trumpler's criteria will do.

$$d = 50.00^{+0.00}_{-0.05}\text{ mm}, \quad b = 50.084^{+0.010}_{-0.000}\text{ mm}$$

$$\text{SAE 30, } N = 2880\text{ rev/min or 48 rev/s}, \quad W = 10\text{ kN}$$

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{50.084 - 50}{2} = 0.042\text{ mm}$$

$$r = d/2 = 50/2 = 25\text{ mm}$$

$$r/c = 25/0.042 = 595$$

$$l' = \frac{1}{2}(55 - 5) = 25\text{ mm}$$

$$l'/d = 25/50 = 0.5$$

$$p = \frac{W}{4rl'} = \frac{10(10^6)}{4(0.25)(0.25)} = 4000\text{ kPa}$$

Trial #1: Choose $(T_f)_1 = 79°\text{C}$. From Fig. 12-13, $\mu = 13$ mPa · s.

$$S = (595^2)\left[\frac{13(10^{-3})(48)}{4000(10^3)}\right] = 0.055$$

From Figs. 12-18 and 12-16: $\dfrac{fr}{c} = 2.3, \quad \epsilon = 0.85.$

From Eq. (12-25), $\Delta T = \dfrac{978(10^6)}{1 + 1.5\epsilon^2}\,\dfrac{(fr/c)SW^2}{p_s r^4}$

$$= \frac{978(10^6)}{1 + 1.5(0.85)^2}\left[\frac{2.3(0.055)(10^2)}{200(25)^4}\right]$$

$$= 76.0°\text{C}$$

$$T_{av} = T_s + \Delta T/2 = 55°\text{C} + (76°\text{C}/2) = 93°\text{C}$$

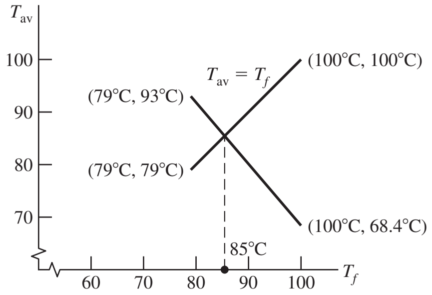
Trial #2: Choose $(T_f)_2 = 100°\text{C}$. From Fig. 12-13, $\mu = 7$ mPa · s.

$$S = 0.055\left(\frac{7}{13}\right) = 0.0296$$

From Figs. 12-18 and 12-16: $\dfrac{fr}{c} = 1.6, \quad \epsilon = 0.90$

$$\Delta T = \frac{978(10^6)}{1 + 1.5(0.9)^2}\left[\frac{1.6(0.0296)(10^2)}{200(25)^4}\right] = 26.8°\text{C}$$

$$T_{av} = 55°\text{C} + \frac{26.8°\text{C}}{2} = 68.4°\text{C}$$



Trial #3: Thus, the plot gives $(T_f)_3 = 85°\text{C}$. From Fig. 12-13, $\mu = 10.8$ mPa · s.

$$S = 0.055\left(\frac{10.8}{13}\right) = 0.0457$$

From Figs. 12-18 and 12-16: $\dfrac{fr}{c} = 2.2, \quad \epsilon = 0.875$

$$\Delta T = \frac{978(10^6)}{1 + 1.5(0.875^2)}\left[\frac{2.2(0.0457)(10^2)}{200(25)^4}\right] = 58.6°\text{C}$$

$$T_{av} = 55°\text{C} + \frac{58.6°\text{C}}{2} = 84.3°\text{C}$$

Result is close. Choose $\bar{T}_f = \dfrac{85°\text{C} + 84.3°\text{C}}{2} = 84.7°\text{C}$

Fig. 12-13: $\mu = 10.8\ \text{MPa}\cdot\text{s}$

$$S = 0.055\left(\frac{10.8}{13}\right) = 0.0457$$

$$\frac{fr}{c} = 2.23, \quad \epsilon = 0.874, \quad \frac{h_o}{c} = 0.13$$

$$\Delta T = \frac{978(10^6)}{1 + 1.5(0.874^2)}\left[\frac{2.23(0.0457)(10^2)}{200(25^4)}\right] = 59.5°\text{C}$$

$$T_{av} = 55°\text{C} + \frac{59.5°\text{C}}{2} = 84.7°\text{C} \qquad \textit{O.K.}$$

From Eq. (12-22)

$$\begin{aligned} Q_s &= (1 + 1.5\epsilon^2)\frac{\pi p_s r c^3}{3\mu l'} \\ &= [1 + 1.5(0.874^2)]\left[\frac{\pi(200)(0.042^3)(25)}{3(10)(10^{-6})(25)}\right] \\ &= 3334\ \text{mm}^3/\text{s} \end{aligned}$$

$$h_o = 0.13(0.042) = 0.005\,46\,\text{mm} \quad \text{or} \quad 0.000\,215\,\text{in}$$

Trumpler:

$$\begin{aligned} h_o &= 0.0002 + 0.000\,04(50/25.4) \\ &= 0.000\,279\,\text{in} \quad \textit{Not O.K.} \end{aligned}$$

$$T_{\max} = T_s + \Delta T = 55°\text{C} + 63.7°\text{C} = 118.7°\text{C} \quad \text{or} \quad 245.7°\text{F} \quad \textit{O.K.}$$

$$P_{st} = 4000\ \text{kPa} \quad \text{or} \quad 581\ \text{psi} \quad \textit{Not O.K.}$$

$$n = 1,\ \text{as done} \quad \textit{Not O.K.}$$

There is no point in proceeding further.

12-18 So far, we've performed elements of the design task. Now let's do it more completely. First, remember our viewpoint.

The values of the unilateral tolerances, t_b and t_d, reflect the routine capabilities of the bushing vendor and the in-house capabilities. While the designer has to live with these, his approach should not depend on them. They can be incorporated later.

First we shall find the minimum size of the journal which satisfies Trumpler's constraint of $P_{st} \le 300$ psi.

$$P_{st} = \frac{W}{2dl'} \le 300$$

$$\frac{W}{2d^2\, l'/d} \le 300 \quad \Rightarrow \quad d \ge \sqrt{\frac{W}{600(l'/d)}}$$

$$d_{\min} = \sqrt{\frac{900}{2(300)(0.5)}} = 1.73\ \text{in}$$

In this problem we will take journal diameter as the nominal value and the bushing bore as a variable. In the next problem, we will take the bushing bore as nominal and the journal diameter as free.

To determine where the constraints are, we will set $t_b = t_d = 0$, and thereby shrink the design window to a point.

We set
$$d = 2.000 \text{ in}$$
$$b = d + 2c_{\min} = d + 2c$$
$$n_d = 2 \quad \text{(This makes Trumpler's } n_d \le 2 \text{ tight)}$$

and construct a table.

c	b	d	$\bar{T}_f^*$	$T_{\max}$	h_o	P_{st}	$T_{\max}$	n	fom
0.0010	2.0020	2	215.50	312.0	×	✓	×	✓	−5.74
0.0011	2.0022	2	206.75	293.0	×	✓	✓	✓	−6.06
0.0012	2.0024	2	198.50	277.0	×	✓	✓	✓	−6.37
0.0013	2.0026	2	191.40	262.8	×	✓	✓	✓	−6.66
0.0014	2.0028	2	185.23	250.4	×	✓	✓	✓	−6.94
0.0015	2.0030	2	179.80	239.6	×	✓	✓	✓	−7.20
0.0016	2.0032	2	175.00	230.1	×	✓	✓	✓	−7.45
0.0017	2.0034	2	171.13	220.3	×	✓	✓	✓	−7.65
0.0018	2.0036	2	166.92	213.9	✓	✓	✓	✓	−7.91
0.0019	2.0038	2	163.50	206.9	✓	✓	✓	✓	−8.12
0.0020	2.0040	2	160.40	200.6	✓	✓	✓	✓	−8.32

*Sample calculation for the first entry of this column.

Iteration yields: $\bar{T}_f = 215.5°\text{F}$

With $\bar{T}_f = 215.5°\text{F}$, from Table 12-1

$$\mu = 0.0136(10^{-6})\exp[1271.6/(215.5 + 95)] = 0.817(10^{-6}) \text{ reyn}$$

$$N = 3000/60 = 50 \text{ rev/s}, \quad P = \frac{900}{4} = 225 \text{ psi}$$

$$S = \left(\frac{1}{0.001}\right)^2 \left[\frac{0.817(10^{-6})(50)}{225}\right] = 0.182$$

From Figs. 12-16 and 12-18: $\epsilon = 0.7, \quad fr/c = 5.5$

Eq. (12–24):
$$\Delta T_F = \frac{0.0123(5.5)(0.182)(900^2)}{[1 + 1.5(0.7^2)](30)(1^4)} = 191.6°\text{F}$$

$$T_{av} = 120°\text{F} + \frac{191.6°\text{F}}{2} = 215.8°\text{F} \doteq 215.5°\text{F}$$

For the nominal 2-in bearing, the various clearances show that we have been in contact with the recurving of $(h_o)_{\min}$. The figure of merit (the parasitic friction torque plus the pumping torque negated) is best at $c = 0.0018$ in. For the nominal 2-in bearing, we will place the top of the design window at $c_{\min} = 0.002$ in, and $b = d + 2(0.002) = 2.004$ in. At this point, add the b and d unilateral tolerances:

$$d = 2.000^{+0.000}_{-0.001} \text{ in}, \quad b = 2.004^{+0.003}_{-0.000} \text{ in}$$

Now we can check the performance at $c_{\min}$, $\bar{c}$, and $c_{\max}$. Of immediate interest is the fom of the median clearance assembly, -9.82, as compared to any other satisfactory bearing ensemble.

If a nominal 1.875 in bearing is possible, construct another table with $t_b = 0$ and $t_d = 0$.

c	b	d	$\bar{T}_f$	$T_{\max}$	h_o	P_{st}	$T_{\max}$	fos	fom
0.0020	1.879	1.875	157.2	194.30	×	✓	✓	✓	−7.36
0.0030	1.881	1.875	138.6	157.10	✓	✓	✓	✓	−8.64
0.0035	1.882	1.875	133.5	147.10	✓	✓	✓	✓	−9.05
0.0040	1.883	1.875	130.0	140.10	✓	✓	✓	✓	−9.32
0.0050	1.885	1.875	125.7	131.45	✓	✓	✓	✓	−9.59
0.0055	1.886	1.875	124.4	128.80	✓	✓	✓	✓	−9.63
0.0060	1.887	1.875	123.4	126.80	×	✓	✓	✓	−9.64

The range of clearance is $0.0030 < c < 0.0055$ in. That is enough room to fit in our design window.

$$d = 1.875^{+0.000}_{-0.001} \text{ in}, \quad b = 1.881^{+0.003}_{-0.000} \text{ in}$$

The ensemble median assembly has fom $= -9.31$.

We just had room to fit in a design window based upon the $(h_o)_{\min}$ constraint. Further reduction in nominal diameter will preclude any smaller bearings. A table constructed for a $d = 1.750$ in journal will prove this.

We choose the nominal 1.875-in bearing ensemble because it has the largest figure of merit. *Ans.*

12-19 This is the same as Prob. 12-18 but uses design variables of nominal bushing bore b and radial clearance c.

The approach is similar to that of Prob. 12-18 and the tables will change slightly. In the table for a nominal $b = 1.875$ in, note that at $c = 0.003$ the constraints are "loose." Set

$$b = 1.875 \text{ in}$$

$$d = 1.875 - 2(0.003) = 1.869 \text{ in}$$

For the ensemble

$$b = 1.875^{+0.003}_{-0.001}, \quad d = 1.869^{+0.000}_{-0.001}$$

Analyze at $c_{\min} = 0.003$, $\bar{c} = 0.004$ in and $c_{\max} = 0.005$ in

At $c_{\min} = 0.003$ in: $\bar{T}_f = 138.4°\text{F}$, $\mu' = 3.160$, $S = 0.0297$, $H_{\text{loss}} = 1035$ Btu/h and the Trumpler conditions are met.

At $\bar{c} = 0.004$ in: $\bar{T}_f = 130°\text{F}$, $\mu' = 3.872$, $S = 0.0205$, $H_{\text{loss}} = 1106$ Btu/h, fom $= -9.246$ and the Trumpler conditions are *O.K.*

At $c_{\max} = 0.005$ in: $\bar{T}_f = 125.68°\text{F}$, $\mu' = 4.325\ \mu\text{reyn}$, $S = 0.01466$, $H_{\text{loss}} = 1129$ Btu/h and the Trumpler conditions are *O.K.*

The ensemble figure of merit is slightly better; this bearing is *slightly* smaller. The lubricant cooler has sufficient capacity.

12-20 From Table 12-1, Seireg and Dandage, $\mu_0 = 0.0141(10^6)$ reyn and $b = 1360.0$

$$\mu(\mu\text{reyn}) = 0.0141\exp[1360/(T+95)] \qquad (T \text{ in °F})$$
$$= 0.0141\exp[1360/(1.8C+127)] \qquad (C \text{ in °C})$$
$$\mu(\text{mPa}\cdot\text{s}) = 6.89(0.0141)\exp[1360/(1.8C+127)] \qquad (C \text{ in °C})$$

For SAE 30 at 79°C

$$\mu = 6.89(0.0141)\exp\{1360/[1.8(79)+127]\}$$
$$= 15.2 \text{ mPa}\cdot\text{s} \quad \textit{Ans.}$$

12-21 Originally

$$d = 2.000^{+0.000}_{-0.001} \text{ in}, \quad b = 2.005^{+0.003}_{-0.000} \text{ in}$$

Doubled,

$$d = 4.000^{+0.000}_{-0.002} \text{ in}, \quad b = 4.010^{+0.006}_{-0.000}$$

The radial load quadrupled to 3600 lbf when the analyses for parts (a) and (b) were carried out. Some of the results are:

Part	$\bar{c}$	μ'	S	$\bar{T}_f$	fr/c	Q_s	h_o/c	ϵ	H_{loss}	h_o	Trumpler h_o	f
(a)	0.007	3.416	0.0310	135.1	0.1612	6.56	0.1032	0.897	9898	0.000 722	0.000 360	0.005 67
(b)	0.0035	3.416	0.0310	135.1	0.1612	0.870	0.1032	0.897	1237	0.000 361	0.000 280	0.005 67

The side flow Q_s differs because there is a c^3 term and consequently an 8-fold increase. H_{loss} is related by a 9898/1237 or an 8-fold increase. The existing h_o is related by a 2-fold increase. Trumpler's $(h_o)_{\min}$ is related by a 1.286-fold increase

$$\left.\begin{array}{ll} \text{fom} = -82.37 & \text{for double size} \\ \text{fom} = -10.297 & \text{for original size} \end{array}\right\} \text{ an 8-fold increase for double-size}$$

12-22 From Table 12-8: $K = 0.6(10^{-10}) \text{ in}^3\cdot\text{min/(lbf}\cdot\text{ft}\cdot\text{h)}$. $P = 500/[(1)(1)] = 500$ psi, $V = \pi DN/12 = \pi(1)(200)/12 = 52.4$ ft/min

Tables 12-10 and 12-11: $f_1 = 1.8, \quad f_2 = 1$

Table 12-12: $PV_{\max} = 46\,700 \text{ psi}\cdot\text{ft/min}, \quad P_{\max} = 3560 \text{ psi}, \quad V_{\max} = 100 \text{ ft/min}$

$$P_{\max} = \frac{4}{\pi}\frac{F}{DL} = \frac{4(500)}{\pi(1)(1)} = 637 \text{ psi} < 3560 \text{ psi} \quad O.K.$$

$$P = \frac{F}{DL} = 500 \text{ psi} \qquad V = 52.4 \text{ ft/min}$$

$$PV = 500(52.4) = 26\,200 \text{ psi}\cdot\text{ft/min} < 46\,700 \text{ psi}\cdot\text{ft/min} \quad O.K.$$

Solving Eq. (12-32) for t

$$t = \frac{\pi DLw}{4f_1 f_2\ KVF} = \frac{\pi(1)(1)(0.005)}{4(1.8)(1)(0.6)(10^{-10})(52.4)(500)} = 1388\ \text{h} = 83\,270\ \text{min}$$

Cycles $= Nt = 200(83\,270) = 16.7$ rev *Ans.*

12-23 Estimate bushing length with $f_1 = f_2 = 1$, and $K = 0.6(10^{-10})\ \text{in}^3 \cdot \text{min/(lbf} \cdot \text{ft} \cdot \text{h)}$

Eq. (12-32): $$L = \frac{1(1)(0.6)(10^{-10})(2)(100)(400)(1000)}{3(0.002)} = 0.80\ \text{in}$$

From Eq. (12-38), with $f_s = 0.03$ from Table 12-9 applying $n_d = 2$ to F and $\hbar_{\text{CR}} = 2.7\ \text{Btu/(h} \cdot \text{ft}^2 \cdot {}^\circ\text{F)}$

$$L \doteq \frac{720(0.03)(2)(100)(400)}{778(2.7)(300-70)} = 3.58\ \text{in}$$

$$0.80 \le L \le 3.58\ \text{in}$$

Trial 1: Let $L = 1$ in, $D = 1$ in

$$P_{\max} = \frac{4(2)(100)}{\pi(1)(1)} = 255\ \text{psi} < 3560\ \text{psi} \quad \textit{O.K.}$$

$$P = \frac{2(100)}{1(1)} = 200\ \text{psi}$$

$$V = \frac{\pi(1)(400)}{12} = 104.7\ \text{ft/min} > 100\ \text{ft/min} \quad \textit{Not O.K.}$$

Trial 2: Try $D = 7/8$ in, $L = 1$ in

$$P_{\max} = \frac{4(2)(100)}{\pi(7/8)(1)} = 291\ \text{psi} < 3560\ \text{psi} \quad \textit{O.K.}$$

$$P = \frac{2(100)}{7/8(1)} = 229\ \text{psi}$$

$$V = \frac{\pi(7/8)(400)}{12} = 91.6\ \text{ft/min} < 100\ \text{ft/min} \quad \textit{O.K.}$$

$PV = 229(91.6) = 20\,976\ \text{psi} \cdot \text{ft/min} < 46\,700\ \text{psi} \cdot \text{ft/min}$ *O.K.*

V	f_1
33	1.3
91.6	f_1
100	1.8

$$\Rightarrow \quad f_1 = 1.3 + (1.8 - 1.3)\left(\frac{91.6-33}{100-33}\right) = 1.74$$

$$L = 0.80(1.74) = 1.39\ \text{in}$$

Trial 3: Try $D = 7/8$ in, $L = 1.5$ in

$$P_{\text{max}} = \frac{4(2)(100)}{\pi(7/8)(1.5)} = 194 \text{ psi} < 3560 \text{ psi} \quad O.K.$$

$$P = \frac{2(100)}{7/8(1.5)} = 152 \text{ psi}, \quad V = 91.6 \text{ ft/min}$$

$$PV = 152(91.6) = 13\,923 \text{ psi} \cdot \text{ft/min} < 46\,700 \text{ psi} \cdot \text{ft/min} \quad O.K.$$

$$D = 7/8 \text{ in}, \quad L = 1.5 \text{ in} \qquad \text{is acceptable} \quad Ans.$$

Suggestion: Try smaller sizes.

Chapter 14

14-1

$$d = \frac{N}{P} = \frac{22}{6} = 3.667 \text{ in}$$

Table 14-2: $Y = 0.331$

$$V = \frac{\pi dn}{12} = \frac{\pi(3.667)(1200)}{12} = 1152 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 1152}{1200} = 1.96$$

$$W^t = \frac{T}{d/2} = \frac{63\,025H}{nd/2} = \frac{63\,025(15)}{1200(3.667/2)} = 429.7 \text{ lbf}$$

Eq. (14-7):

$$\sigma = \frac{K_v W^t P}{FY} = \frac{1.96(429.7)(6)}{2(0.331)} = 7633 \text{ psi} = 7.63 \text{ kpsi} \quad \textit{Ans.}$$

14-2

$$d = \frac{16}{12} = 1.333 \text{ in}, \quad Y = 0.296$$

$$V = \frac{\pi(1.333)(700)}{12} = 244.3 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 244.3}{1200} = 1.204$$

$$W^t = \frac{63\,025H}{nd/2} = \frac{63\,025(1.5)}{700(1.333/2)} = 202.6 \text{ lbf}$$

Eq. (14-7):

$$\sigma = \frac{K_v W^t P}{FY} = \frac{1.204(202.6)(12)}{0.75(0.296)} = 13\,185 \text{ psi} = 13.2 \text{ kpsi} \quad \textit{Ans.}$$

14-3

$$d = mN = 1.25(18) = 22.5 \text{ mm}, \quad Y = 0.309$$

$$V = \frac{\pi(22.5)(10^{-3})(1800)}{60} = 2.121 \text{ m/s}$$

Eq. (14-6*b*): $$K_v = \frac{6.1 + 2.121}{6.1} = 1.348$$

$$W^t = \frac{60H}{\pi dn} = \frac{60(0.5)(10^3)}{\pi(22.5)(10^{-3})(1800)} = 235.8 \text{ N}$$

Eq. (14-8): $$\sigma = \frac{K_v W^t}{FmY} = \frac{1.348(235.8)}{12(1.25)(0.309)} = 68.6 \text{ MPa} \quad \textit{Ans.}$$

14-4

$$d = 5(15) = 75 \text{ mm}, \quad Y = 0.290$$

$$V = \frac{\pi(75)(10^{-3})(200)}{60} = 0.7854 \text{ m/s}$$

Assume steel and apply Eq. (14-6*b*):

$$K_v = \frac{6.1 + 0.7854}{6.1} = 1.129$$

$$W^t = \frac{60H}{\pi dn} = \frac{60(5)(10^3)}{\pi(75)(10^{-3})(200)} = 6366 \text{ N}$$

Eq. (14-8): $$\sigma = \frac{K_v W^t}{FmY} = \frac{1.129(6366)}{60(5)(0.290)} = 82.6 \text{ MPa} \quad \textit{Ans.}$$

14-5

$$d = 1(16) = 16 \text{ mm}, \quad Y = 0.296$$

$$V = \frac{\pi(16)(10^{-3})(400)}{60} = 0.335 \text{ m/s}$$

Assume steel and apply Eq. (14-6*b*):

$$K_v = \frac{6.1 + 0.335}{6.1} = 1.055$$

$$W^t = \frac{60H}{\pi dn} = \frac{60(0.15)(10^3)}{\pi(16)(10^{-3})(400)} = 447.6 \text{ N}$$

Eq. (14-8): $$F = \frac{K_v W^t}{\sigma mY} = \frac{1.055(447.6)}{150(1)(0.296)} = 10.6 \text{ mm}$$

From Table A-17, use $F = 11$ mm *Ans.*

14-6

$$d = 1.5(17) = 25.5 \text{ mm}, \quad Y = 0.303$$

$$V = \frac{\pi(25.5)(10^{-3})(400)}{60} = 0.534 \text{ m/s}$$

Eq. (14-6*b*): $$K_v = \frac{6.1 + 0.534}{6.1} = 1.088$$

$$W^t = \frac{60H}{\pi dn} = \frac{60(0.25)(10^3)}{\pi(25.5)(10^{-3})(400)} = 468 \text{ N}$$

Eq. (14-8): $$F = \frac{K_v W^t}{\sigma mY} = \frac{1.088(468)}{75(1.5)(0.303)} = 14.9 \text{ mm}$$

Use $F = 15$ mm *Ans.*

14-7

$$d = \frac{24}{5} = 4.8 \text{ in}, \quad Y = 0.337$$

$$V = \frac{\pi(4.8)(50)}{12} = 62.83 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 62.83}{1200} = 1.052$$

$$W^t = \frac{63\,025H}{nd/2} = \frac{63\,025(6)}{50(4.8/2)} = 3151 \text{ lbf}$$

Eq. (14-7): $$F = \frac{K_v W^t P}{\sigma Y} = \frac{1.052(3151)(5)}{20(10^3)(0.337)} = 2.46 \text{ in}$$

Use $F = 2.5$ in *Ans.*

14-8

$$d = \frac{16}{5} = 3.2 \text{ in}, \quad Y = 0.296$$

$$V = \frac{\pi(3.2)(600)}{12} = 502.7 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 502.7}{1200} = 1.419$$

$$W^t = \frac{63\,025(15)}{600(3.2/2)} = 984.8 \text{ lbf}$$

Eq. (14-7): $$F = \frac{K_v W^t P}{\sigma Y} = \frac{1.419(984.8)(5)}{10(10^3)(0.296)} = 2.38 \text{ in}$$

Use $F = 2.5$ in *Ans.*

14-9 Try $P = 8$ which gives $d = 18/8 = 2.25$ in and $Y = 0.309$.

$$V = \frac{\pi(2.25)(600)}{12} = 353.4 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 353.4}{1200} = 1.295$$

$$W^t = \frac{63\,025(2.5)}{600(2.25/2)} = 233.4 \text{ lbf}$$

Eq. (14-7): $$F = \frac{K_v W^t P}{\sigma Y} = \frac{1.295(233.4)(8)}{10(10^3)(0.309)} = 0.783 \text{ in}$$

Using coarse integer pitches from Table 13-2, the following table is formed.

P	d	V	K_v	W^t	F
2	9.000	1413.717	2.178	58.356	0.082
3	6.000	942.478	1.785	87.535	0.152
4	4.500	706.858	1.589	116.713	0.240
6	3.000	471.239	1.393	175.069	0.473
8	2.250	353.429	1.295	233.426	0.782
10	1.800	282.743	1.236	291.782	1.167
12	1.500	235.619	1.196	350.139	1.627
16	1.125	176.715	1.147	466.852	2.773

Other considerations may dictate the selection. Good candidates are $P = 8$ ($F = 7/8$ in) and $P = 10$ ($F = 1.25$ in). *Ans.*

14-10 Try $m = 2$ mm which gives $d = 2(18) = 36$ mm and $Y = 0.309$.

$$V = \frac{\pi(36)(10^{-3})(900)}{60} = 1.696 \text{ m/s}$$

Eq. (14-6*b*):
$$K_v = \frac{6.1 + 1.696}{6.1} = 1.278$$

$$W^t = \frac{60(1.5)(10^3)}{\pi(36)(10^{-3})(900)} = 884 \text{ N}$$

Eq. (14-8):
$$F = \frac{1.278(884)}{75(2)(0.309)} = 24.4 \text{ mm}$$

Using the preferred module sizes from Table 13-2:

m	d	V	K_v	W^t	F
1.00	18.0	0.848	1.139	1768.388	86.917
1.25	22.5	1.060	1.174	1414.711	57.324
1.50	27.0	1.272	1.209	1178.926	40.987
2.00	36.0	1.696	1.278	884.194	24.382
3.00	54.0	2.545	1.417	589.463	12.015
4.00	72.0	3.393	1.556	442.097	7.422
5.00	90.0	4.241	1.695	353.678	5.174
6.00	108.0	5.089	1.834	294.731	3.888
8.00	144.0	6.786	2.112	221.049	2.519
10.00	180.0	8.482	2.391	176.839	1.824
12.00	216.0	10.179	2.669	147.366	1.414
16.00	288.0	13.572	3.225	110.524	0.961
20.00	360.0	16.965	3.781	88.419	0.721
25.00	450.0	21.206	4.476	70.736	0.547
32.00	576.0	27.143	5.450	55.262	0.406
40.00	720.0	33.929	6.562	44.210	0.313
50.00	900.0	42.412	7.953	35.368	0.243

Other design considerations may dictate the size selection. For the present design, $m = 2$ mm ($F = 25$ mm) is a good selection. *Ans.*

14-11

$$d_P = \frac{22}{6} = 3.667 \text{ in}, \quad d_G = \frac{60}{6} = 10 \text{ in}$$

$$V = \frac{\pi(3.667)(1200)}{12} = 1152 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 1152}{1200} = 1.96$$

$$W^t = \frac{63\,025(15)}{1200(3.667/2)} = 429.7 \text{ lbf}$$

Table 14-8: $C_p = 2100\sqrt{\text{psi}}$ [Note: using Eq. (14-13) can result in wide variation in C_p due to wide variation in cast iron properties]

Eq. (14-12): $r_1 = \dfrac{3.667 \sin 20^\circ}{2} = 0.627$ in, $r_2 = \dfrac{10 \sin 20^\circ}{2} = 1.710$ in

Eq. (14-14): $$\sigma_C = -C_p \left[\frac{K_v W^t}{F \cos\phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2}$$

$$= -2100 \left[\frac{1.96(429.7)}{2 \cos 20^\circ} \left(\frac{1}{0.627} + \frac{1}{1.710} \right) \right]^{1/2}$$

$$= -65.6(10^3) \text{ psi} = -65.6 \text{ kpsi} \quad \textit{Ans.}$$

14-12

$$d_P = \frac{16}{12} = 1.333 \text{ in}, \quad d_G = \frac{48}{12} = 4 \text{ in}$$

$$V = \frac{\pi(1.333)(700)}{12} = 244.3 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 244.3}{1200} = 1.204$$

$$W^t = \frac{63\,025(1.5)}{700(1.333/2)} = 202.6 \text{ lbf}$$

Table 14-8: $C_p = 2100\sqrt{\text{psi}}$ (see note in Prob. 14-11 solution)

Eq. (14-12): $r_1 = \dfrac{1.333 \sin 20^\circ}{2} = 0.228$ in, $r_2 = \dfrac{4 \sin 20^\circ}{2} = 0.684$ in

Eq. (14-14):

$$\sigma_C = -2100 \left[\frac{1.202(202.6)}{F \cos 20^\circ} \left(\frac{1}{0.228} + \frac{1}{0.684} \right) \right]^{1/2} = -100(10^3)$$

$$F = \left(\frac{2100}{100(10^3)} \right)^2 \left[\frac{1.202(202.6)}{\cos 20^\circ} \right] \left(\frac{1}{0.228} + \frac{1}{0.684} \right) = 0.668 \text{ in}$$

Use $F = 0.75$ in *Ans.*

14-13

$$d_P = \frac{24}{5} = 4.8 \text{ in}, \quad d_G = \frac{48}{5} = 9.6 \text{ in}$$

$$V = \frac{\pi(4.8)(50)}{12} = 62.83 \text{ ft/min}$$

Eq. (14-4*a*): $$K_v = \frac{600 + 62.83}{600} = 1.105$$

$$W^t = \frac{63\,025H}{50(4.8/2)} = 525.2H$$

Table 14-8: $C_p = 1960\sqrt{\text{psi}}$ (see note in Prob. 14-11 solution)

Eq. (14-12): $$r_1 = \frac{4.8 \sin 20^\circ}{2} = 0.821 \text{ in}, \quad r_2 = 2r_1 = 1.642 \text{ in}$$

Eq. (14-14): $$-100(10^3) = -1960\left[\frac{1.105(525.2H)}{2.5\cos 20^\circ}\left(\frac{1}{0.821} + \frac{1}{1.642}\right)\right]^{1/2}$$

$$H = 5.77 \text{ hp} \quad \textit{Ans.}$$

14-14

$$d_P = 4(20) = 80 \text{ mm}, \quad d_G = 4(32) = 128 \text{ mm}$$

$$V = \frac{\pi(80)(10^{-3})(1000)}{60} = 4.189 \text{ m/s}$$

Eq. (14-6*a*): $$K_v = \frac{3.05 + 4.189}{3.05} = 2.373$$

$$W^t = \frac{60(10)(10^3)}{\pi(80)(10^{-3})(1000)} = 2387 \text{ N}$$

Table 14-8: $C_p = 163\sqrt{\text{MPa}}$ (see note in Prob. 14-11 solution)

Eq. (14-12): $$r_1 = \frac{80 \sin 20^\circ}{2} = 13.68 \text{ mm}, \quad r_2 = \frac{128 \sin 20^\circ}{2} = 21.89 \text{ mm}$$

Eq. (14-14): $$\sigma_C = -163\left[\frac{2.373(2387)}{50\cos 20^\circ}\left(\frac{1}{13.68} + \frac{1}{21.89}\right)\right]^{1/2} = -617 \text{ MPa} \quad \textit{Ans.}$$

14-15 The pinion controls the design.

Bending $Y_P = 0.303, \quad Y_G = 0.359$

$$d_P = \frac{17}{12} = 1.417 \text{ in}, \quad d_G = \frac{30}{12} = 2.500 \text{ in}$$

$$V = \frac{\pi d_P n}{12} = \frac{\pi(1.417)(525)}{12} = 194.8 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 194.8}{1200} = 1.162$$

Eq. (6-8): $S'_e = 0.5(76) = 38$ kpsi

Eq. (6-19): $k_a = 2.70(76)^{-0.265} = 0.857$

$$l = \frac{2.25}{P_d} = \frac{2.25}{12} = 0.1875 \text{ in}$$

Eq. (14-3): $$x = \frac{3Y_P}{2P} = \frac{3(0.303)}{2(12)} = 0.0379 \text{ in}$$

Eq. (*b*), p. 717: $$t = \sqrt{4(0.1875)(0.0379)} = 0.1686 \text{ in}$$

Eq. (6-25): $$d_e = 0.808\sqrt{0.875(0.1686)} = 0.310 \text{ in}$$

Eq. (6-20): $$k_b = \left(\frac{0.310}{0.30}\right)^{-0.107} = 0.996$$

$$k_c = k_d = k_e = 1, \quad k_{f_1} = 1.66 \quad \text{(see Ex. 14-2)}$$

$$r_f = \frac{0.300}{12} = 0.025 \text{ in} \quad \text{(see Ex. 14-2)}$$

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.025}{0.1686} = 0.148$$

Approximate $D/d = \infty$ with $D/d = 3$; from Fig. A-15-6, $K_t = 1.68$.

From Fig. 6-20, with $S_{ut} = 76$ kpsi and $r = 0.025$ in, $q = 0.62$. From Eq. (6-32)

$$K_f = 1 + 0.62(1.68 - 1) = 1.42$$

Miscellaneous-Effects Factor:

$$k_f = k_{f1}k_{f2} = 1.65\left(\frac{1}{1.323}\right) = 1.247$$

Eq. (7-17): $$S_e = 0.857(0.996)(1)(1)(1)(1.247)(38\,000)$$
$$= 40\,450 \text{ psi}$$

$$\sigma_{\text{all}} = \frac{40\,770}{2.25} = 18\,120 \text{ psi}$$

$$W^t = \frac{FY_P\sigma_{\text{all}}}{K_v P_d} = \frac{0.875(0.303)(18\,120)}{1.162(12)}$$
$$= 345 \text{ lbf}$$

$$H = \frac{345(194.8)}{33\,000} = 2.04 \text{ hp} \quad \textit{Ans.}$$

Wear

$$\nu_1 = \nu_2 = 0.292, \quad E_1 = E_2 = 30(10^6) \text{ psi}$$

Eq. (14-13): $$C_p = \left[\frac{1}{2\pi\left(\frac{1-0.292^2}{30(10^6)}\right)}\right] = 2285\sqrt{\text{psi}}$$

Eq. (14-12): $$r_1 = \frac{d_P}{2}\sin\phi = \frac{1.417}{2}\sin 20^\circ = 0.242 \text{ in}$$

$$r_2 = \frac{d_G}{2}\sin\phi = \frac{2.500}{2}\sin 20^\circ = 0.428$$

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{0.242} + \frac{1}{0.428} = 6.469 \text{ in}^{-1}$$

From Eq. (6-68),

$$(S_C)_{10^8} = 0.4H_B - 10 \text{ kpsi}$$

$$= [0.4(149) - 10](10^3) = 49\,600 \text{ psi}$$

$$\sigma_{C,\text{all}} = -\frac{(S_C)_{10^8}}{\sqrt{n}} = \frac{-49\,600}{\sqrt{2.25}} = -33\,067 \text{ psi}$$

Eq. (14-14): $$W^t = \left(\frac{-33\,067}{2285}\right)^2 \left[\frac{0.875\cos 20^\circ}{1.162(6.469)}\right] = 22.6 \text{ lbf}$$

$$H = \frac{22.6(194.8)}{33\,000} = 0.133 \text{ hp} \quad \textit{Ans.}$$

Rating power (pinion controls):

$$H_1 = 2.04 \text{ hp}$$

$$H_2 = 0.133 \text{ hp}$$

$H_{\text{all}} = (\min 2.04, 0.133) = 0.133$ hp *Ans.*

14-16 See Prob. 14-15 solution for equation numbers.

Pinion controls: $Y_P = 0.322, \quad Y_G = 0.447$

Bending $$d_P = 20/3 = 6.667 \text{ in}, \quad d_G = 33.333 \text{ in}$$

$$V = \pi d_P n/12 = \pi(6.667)(870)/12 = 1519 \text{ ft/min}$$

$$K_v = (1200 + 1519)/1200 = 2.266$$

$$S'_e = 0.5(113) = 56.5 \text{ kpsi}$$

$$k_a = 2.70(113)^{-0.265} = 0.771$$

$$l = 2.25/P_d = 2.25/3 = 0.75 \text{ in}$$

$$x = 3(0.322)/[2(3)] = 0.161 \text{ in}$$

$$t = \sqrt{4(0.75)(0.161)} = 0.695 \text{ in}$$

$$d_e = 0.808\sqrt{2.5(0.695)} = 1.065 \text{ in}$$

$$k_b = (1.065/0.30)^{-0.107} = 0.873$$

$$k_c = k_d = k_e = 1$$

$$r_f = 0.300/3 = 0.100 \text{ in}$$

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.100}{0.695} = 0.144$$

From Table A-15-6, $K_t = 1.75$; Fig. 6-20, $q = 0.85$; Eq. (6-32), $K_f = 1.64$

$$k_{f2} = 1/1.597, \quad k_f = k_{f1}k_{f2} = 1.66/1.597 = 1.039$$

$$S_e = 0.771(0.873)(1)(1)(1)(1.039)(56\,500) = 39\,500 \text{ psi}$$

$$\sigma_{\text{all}} = S_e/n = 39\,500/1.5 = 26\,330 \text{ psi}$$

$$W^t = \frac{FY_P\sigma_{\text{all}}}{K_v P_d} = \frac{2.5(0.322)(26\,330)}{2.266(3)} = 3118 \text{ lbf}$$

$$H = W^t V/33\,000 = 3118(1519)/33\,000 = 144 \text{ hp} \quad \textit{Ans.}$$

Wear

Eq. (14-13): $$C_p = 2285\sqrt{\text{psi}}$$

Eq. (14-12): $$r_1 = (6.667/2)\sin 20° = 1.140 \text{ in}$$

$$r_2 = (33.333/2)\sin 20° = 5.700 \text{ in}$$

Eq. (6-68): $$S_C = [0.4(262) - 10](10^3) = 94\,800 \text{ psi}$$

$$\sigma_{C,\text{all}} = -S_C/\sqrt{n_d} = -94\,800/\sqrt{1.5} = -77\,404 \text{ psi}$$

$$W^t = \left(\frac{\sigma_{C,\text{all}}}{C_p}\right)^2 \frac{F\cos\phi}{K_v}\frac{1}{1/r_1 + 1/r_2}$$

$$= \left(\frac{-77\,404}{2300}\right)^2 \left(\frac{2.5\cos 20°}{2.266}\right)\left(\frac{1}{1/1.140 + 1/5.700}\right)$$

$$= 1115 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{1115(1519)}{33\,000} = 51.3\,\text{hp} \quad \textit{Ans.}$$

For 10^8 cycles (revolutions of the pinion), the power based on wear is 51.3 hp.

Rating power–pinion controls

$$H_1 = 144 \text{ hp}$$

$$H_2 = 51.3 \text{ hp}$$

$$H_{\text{rated}} = \min(144, 51.3) = 51.3 \text{ hp} \quad \textit{Ans.}$$

14-17 Given: $\phi = 20°$, $n = 1145$ rev/min, $m = 6$ mm, $F = 75$ mm, $N_P = 16$ milled teeth, $N_G = 30T$, $S_{ut} = 900$ MPa, $H_B = 260$, $n_d = 3$, $Y_P = 0.296$, and $Y_G = 0.359$.

Pinion bending

$$d_P = mN_P = 6(16) = 96 \text{ mm}$$

$$d_G = 6(30) = 180 \text{ mm}$$

$$V = \frac{\pi d_P n}{12} = \frac{\pi(96)(1145)(10^{-3})(12)}{(12)(60)} = 5.76 \text{ m/s}$$

Eq. (14-6*b*): $$K_v = \frac{6.1 + 5.76}{6.1} = 1.944$$

$$S'_e = 0.5(900) = 450 \text{ MPa}$$

$$a = 4.45, \quad b = -0.265$$

$$k_a = 4.51(900)^{-0.265} = 0.744$$

$$l = 2.25m = 2.25(6) = 13.5 \text{ mm}$$

$$x = 3Ym/2 = 3(0.296)6/2 = 2.664 \text{ mm}$$

$$t = \sqrt{4lx} = \sqrt{4(13.5)(2.664)} = 12.0 \text{ mm}$$

$$d_e = 0.808\sqrt{75(12.0)} = 24.23 \text{ mm}$$

$$k_b = \left(\frac{24.23}{7.62}\right)^{-0.107} = 0.884$$

$$k_c = k_d = k_e = 1$$

$$r_f = 0.300m = 0.300(6) = 1.8\text{ mm}$$

From Fig. A-15-6 for $r/d = r_f/t = 1.8/12 = 0.15$, $K_t = 1.68$.
Figure 6-20, $q = 0.86$; Eq. (6-32),

$$K_f = 1 + 0.86(1.68 - 1) = 1.58$$

$$k_{f1} = 1.66 \quad \text{(Gerber failure criterion)}$$

$$k_{f2} = 1/K_f = 1/1.537 = 0.651$$

$$k_f = k_{f1}k_{f2} = 1.66(0.651) = 1.08$$

$$S_e = 0.744(0.884)(1)(1)(1)(1.08)(450) = 319.6\text{ MPa}$$

$$\sigma_{\text{all}} = \frac{S_e}{n_d} = \frac{319.6}{1.3} = 245.8\text{ MPa}$$

Eq. (14-8): $$W^t = \frac{FYm\sigma_{\text{all}}}{K_v} = \frac{75(0.296)(6)(245.8)}{1.944} = 16\,840\text{ N}$$

$$H = \frac{Tn}{9.55} = \frac{16\,840(96/2)(1145)}{9.55(10^6)} = 96.9\text{ kW} \quad Ans.$$

Wear: Pinion and gear

Eq. (14-12): $$r_1 = (96/2)\sin 20^\circ = 16.42\text{ mm}$$

$$r_2 = (180/2)\sin 20^\circ = 30.78\text{ mm}$$

Eq. (14-13), with $E = 207(10^3)$ MPa and $\nu = 0.292$, gives

$$C_p = \left[\frac{1}{2\pi(1 - 0.292^2)/(207 \times 10^3)}\right] = 190\sqrt{\text{MPa}}$$

Eq. (6-68): $$S_C = 6.89[0.4(260) - 10] = 647.7\text{ MPa}$$

$$\sigma_{C,\text{all}} = -\frac{S_C}{\sqrt{n}} = -\frac{647.7}{\sqrt{1.3}} = -568\text{ MPa}$$

Eq. (14-14): $$W^t = \left(\frac{\sigma_{C,\text{all}}}{C_p}\right)^2 \frac{F\cos\phi}{K_v}\,\frac{1}{1/r_1 + 1/r_2}$$

$$= \left(\frac{-568}{191}\right)^2 \left(\frac{75\cos 20^\circ}{1.944}\right)\left(\frac{1}{1/16.42 + 1/30.78}\right)$$

$$= 3433\text{ N}$$

$$T = \frac{W^t d_P}{2} = \frac{3433(96)}{2} = 164\,784\text{ N}\cdot\text{mm} = 164.8\text{ N}\cdot\text{m}$$

$$H = \frac{Tn}{9.55} = \frac{164.8(1145)}{9.55} = 19\,758.7\text{ W} = 19.8\text{ kW} \quad Ans.$$

Thus, wear controls the gearset power rating; $H = 19.8$ kW. *Ans.*

14-18 Preliminaries: $N_P = 17, \quad N_G = 51$

$$d_P = \frac{N}{P_d} = \frac{17}{6} = 2.833 \text{ in}$$

$$d_G = \frac{51}{6} = 8.500 \text{ in}$$

$$V = \pi d_P n/12 = \pi(2.833)(1120)/12 = 830.7 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = (1200 + 830.7)/1200 = 1.692$$

$$\sigma_{\text{all}} = \frac{S_y}{n_d} = \frac{90\,000}{2} = 45\,000 \text{ psi}$$

Table 14-2: $$Y_P = 0.303, \quad Y_G = 0.410$$

Eq. (14-7): $$W^t = \frac{FY_P\sigma_{\text{all}}}{K_v P_d} = \frac{2(0.303)(45\,000)}{1.692(6)} = 2686 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{2686(830.7)}{33\,000} = 67.6 \text{ hp}$$

Based on yielding in bending, the power is 67.6 hp.

(a) Pinion fatigue

Bending

Eq. (2-17): $S_{ut} \doteq 0.5H_B = 0.5(232) = 116$ kpsi

Eq. (6-8): $S'_e = 0.5S_{ut} = 0.5(116) = 58$ kpsi

Eq. (6-19): $a = 2.70, \quad b = -0.265, \quad k_a = 2.70(116)^{-0.265} = 0.766$

Table 13-1: $$l = \frac{1}{P_d} + \frac{1.25}{P_d} = \frac{2.25}{P_d} = \frac{2.25}{6} = 0.375 \text{ in}$$

Eq. (14-3): $$x = \frac{3Y_P}{2P_d} = \frac{3(0.303)}{2(6)} = 0.0758$$

Eq. (*b*), p. 717: $$t = \sqrt{4lx} = \sqrt{4(0.375)(0.0758)} = 0.337 \text{ in}$$

Eq. (6-25): $$d_e = 0.808\sqrt{Ft} = 0.808\sqrt{2(0.337)} = 0.663 \text{ in}$$

Eq. (6-20): $$k_b = \left(\frac{0.663}{0.30}\right)^{-0.107} = 0.919$$

$k_c = k_d = k_e = 1$. Assess two components contributing to k_f. First, based upon one-way bending and the Gerber failure criterion, $k_{f1} = 1.66$ (see Ex. 14-2). Second, due to stress-concentration,

$$r_f = \frac{0.300}{P_d} = \frac{0.300}{6} = 0.050 \text{ in} \quad \text{(see Ex. 14-2)}$$

Fig. A-15-6: $$\frac{r}{d} = \frac{r_f}{t} = \frac{0.05}{0.338} = 0.148$$

Estimate $D/d = \infty$ by setting $D/d = 3$, $K_t = 1.68$. From Fig. 6-20, $q = 0.86$, and Eq. (6-32)

$$K_f = 1 + 0.86(1.68 - 1) = 1.58$$

$$k_{f2} = \frac{1}{K_f} = \frac{1}{1.58} = 0.633$$

$$k_f = k_{f1}k_{f2} = 1.66(0.633) = 1.051$$

$$S_e = 0.766(0.919)(1)(1)(1)(1.051)(58) = 42.9 \text{ kpsi}$$

$$\sigma_{\text{all}} = \frac{S_e}{n_d} = \frac{42.9}{2} = 21.5 \text{ kpsi}$$

$$W^t = \frac{FY_P\sigma_{\text{all}}}{K_v P_d} = \frac{2(0.303)(21\,500)}{1.692(6)} = 1283 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{1283(830.7)}{33\,000} = 32.3 \text{ hp} \quad \textit{Ans.}$$

(b) Pinion fatigue

Wear

From Table A-5 for steel: $\nu = 0.292, \quad E = 30(10^6)$ psi

Eq. (14-13) or Table 14-8:

$$C_p = \left\{\frac{1}{2\pi[(1 - 0.292^2)/30(10^6)]}\right\}^{1/2} = 2285\sqrt{\text{psi}}$$

In preparation for Eq. (14-14):

Eq. (14-12): $$r_1 = \frac{d_P}{2}\sin\phi = \frac{2.833}{2}\sin 20^\circ = 0.485 \text{ in}$$

$$r_2 = \frac{d_G}{2}\sin\phi = \frac{8.500}{2}\sin 20^\circ = 1.454 \text{ in}$$

$$\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{1}{0.485} + \frac{1}{1.454} = 2.750 \text{ in}$$

Eq. (6-68): $$(S_C)_{10^8} = 0.4H_B - 10 \text{ kpsi}$$

In terms of gear notation

$$\sigma_C = [0.4(232) - 10]10^3 = 82\,800 \text{ psi}$$

We will introduce the design factor of $n_d = 2$ and because it is a contact stress apply it to the load W^t by dividing by $\sqrt{2}$.

$$\sigma_{C,\text{all}} = -\frac{\sigma_c}{\sqrt{2}} = -\frac{82\,800}{\sqrt{2}} = -58\,548 \text{ psi}$$

Solve Eq. (14-14) for W^t:

$$W^t = \left(\frac{-58\,548}{2285}\right)^2 \left[\frac{2\cos 20^\circ}{1.692(2.750)}\right] = 265 \text{ lbf}$$

$$H_{\text{all}} = \frac{265(830.7)}{33\,000} = 6.67 \text{ hp} \quad \textit{Ans.}$$

For 10^8 cycles (turns of pinion), the allowable power is 6.67 hp.

(c) Gear fatigue due to bending and wear

Bending

Eq. (14-3): $x = \dfrac{3Y_G}{2P_d} = \dfrac{3(0.4103)}{2(6)} = 0.1026$ in

Eq. (b), p. 717: $t = \sqrt{4(0.375)(0.1026)} = 0.392$ in

Eq. (6-25): $d_e = 0.808\sqrt{2(0.392)} = 0.715$ in

Eq. (6-20): $k_b = \left(\dfrac{0.715}{0.30}\right)^{-0.107} = 0.911$

$$k_c = k_d = k_e = 1$$

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.050}{0.392} = 0.128$$

Approximate $D/d = \infty$ by setting $D/d = 3$ for Fig. A-15-6; $K_t = 1.80$. Use $K_f = 1.80$.

$$k_{f2} = \frac{1}{1.80} = 0.556, \quad k_f = 1.66(0.556) = 0.923$$

$$S_e = 0.766(0.911)(1)(1)(1)(0.923)(58) = 37.36 \text{ kpsi}$$

$$\sigma_{\text{all}} = \frac{S_e}{n_d} = \frac{37.36}{2} = 18.68 \text{ kpsi}$$

$$W^t = \frac{FY_G\sigma_{\text{all}}}{K_v - P_d} = \frac{2(0.4103)(18\,680)}{1.692(6)} = 1510 \text{ lbf}$$

$$H_{\text{all}} = \frac{1510(830.7)}{33\,000} = 38.0 \text{ hp} \quad \textit{Ans.}$$

The gear is thus stronger than the pinion in bending.

Wear Since the material of the pinion and the gear are the same, and the contact stresses are the same, the allowable power transmission of both is the same. Thus, $H_{\text{all}} = 6.67$ hp for 10^8 revolutions of each. As yet, we have no way to establish S_C for $10^8/3$ revolutions.

(d) Pinion bending: $H_1 = 32.3$ hp

Pinion wear: $H_2 = 6.67$ hp

Gear bending: $H_3 = 38.0$ hp

Gear wear: $H_4 = 6.67$ hp

Power rating of the gear set is thus

$$H_{\text{rated}} = \min(32.3, 6.67, 38.0, 6.67) = 6.67 \text{ hp} \quad \textit{Ans.}$$

14-19 $d_P = 16/6 = 2.667$ in, $\quad d_G = 48/6 = 8$ in

$$V = \frac{\pi(2.667)(300)}{12} = 209.4 \text{ ft/min}$$

$$W^t = \frac{33\,000(5)}{209.4} = 787.8 \text{ lbf}$$

Assuming uniform loading, $K_o = 1$. From Eq. (14-28),

$$Q_v = 6, \quad B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (14-27):

$$K_v = \left(\frac{59.77 + \sqrt{209.4}}{59.77}\right)^{0.8255} = 1.196$$

From Table 14-2,

$$N_P = 16T, \quad Y_P = 0.296$$

$$N_G = 48T, \quad Y_G = 0.4056$$

From Eq. (*a*), Sec. 14-10 with $F = 2$ in

$$(K_s)_P = 1.192\left(\frac{2\sqrt{0.296}}{6}\right)^{0.0535} = 1.088$$

$$(K_s)_G = 1.192\left(\frac{2\sqrt{0.4056}}{6}\right)^{0.0535} = 1.097$$

From Eq. (14-30) with $C_{mc} = 1$

$$C_{pf} = \frac{2}{10(2.667)} - 0.0375 + 0.0125(2) = 0.0625$$

$$C_{pm} = 1, \quad C_{ma} = 0.093 \quad \text{(Fig. 14-11)}, \qquad C_e = 1$$

$$K_m = 1 + 1[0.0625(1) + 0.093(1)] = 1.156$$

Assuming constant thickness of the gears → $K_B = 1$

$$m_G = N_G/N_P = 48/16 = 3$$

With N (pinion) $= 10^8$ cycles and N (gear) $= 10^8/3$, Fig. 14-14 provides the relations:

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8/3)^{-0.0178} = 0.996$$

Fig. 14-6: $J_P = 0.27, \quad J_G \doteq 0.38$

From Table 14-10 for $R = 0.9,\ K_R = 0.85$

$$K_T = C_f = 1$$

Eq. (14-23) with $m_N = 1$ $\quad I = \dfrac{\cos 20^\circ \sin 20^\circ}{2}\left(\dfrac{3}{3+1}\right) = 0.1205$

Table 14-8: $C_p = 2300\sqrt{\text{psi}}$

Strength: Grade 1 steel with $H_{BP} = H_{BG} = 200$

Fig. 14-2: $(S_t)_P = (S_t)_G = 77.3(200) + 12\,800 = 28\,260$ psi

Fig. 14-5: $(S_c)_P = (S_c)_G = 322(200) + 29\,100 = 93\,500$ psi

Fig. 14-15: $(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$

$$(Z_N)_G = 1.4488(10^8/3)^{-0.023} = 0.973$$

Fig. 14-12: $H_{BP}/H_{BG} = 1 \quad \therefore\ C_H = 1$

Pinion tooth bending

Eq. (14-15):

$$(\sigma)_P = W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J} = 787.8(1)(1.196)(1.088)\left(\frac{6}{2}\right)\left[\frac{(1.156)(1)}{0.27}\right]$$
$$= 13\,167\ \text{psi} \quad \textit{Ans.}$$

Factor of safety from Eq. (14-41)

$$(S_F)_P = \left[\frac{S_t Y_N/(K_T K_R)}{\sigma}\right] = \frac{28\,260(0.977)/[(1)(0.85)]}{13\,167} = 2.47 \quad \textit{Ans.}$$

Gear tooth bending

$$(\sigma)_G = 787.8(1)(1.196)(1.097)\left(\frac{6}{2}\right)\left[\frac{(1.156)(1)}{0.38}\right] = 9433\ \text{psi} \quad \textit{Ans.}$$

$$(S_F)_G = \frac{28\,260(0.996)/[(1)(0.85)]}{9433} = 3.51 \quad \textit{Ans.}$$

Pinion tooth wear

Eq. (14-16): $\quad (\sigma_c)_P = C_p\left(W^t K_o K_v K_s \dfrac{K_m}{d_P F}\dfrac{C_f}{I}\right)_P^{1/2}$

$$= 2300\left[787.8(1)(1.196)(1.088)\left(\frac{1.156}{2.667(2)}\right)\left(\frac{1}{0.1205}\right)\right]^{1/2}$$
$$= 98\,760\ \text{psi} \quad \textit{Ans.}$$

Eq. (14-42):

$$(S_H)_P = \left[\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right]_P = \left\{\frac{93\,500(0.948)/[(1)(0.85)]}{98\,760}\right\} = 1.06 \quad \textit{Ans.}$$

Gear tooth wear

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2} (\sigma_c)_P = \left(\frac{1.097}{1.088}\right)^{1/2} (98\,760) = 99\,170 \text{ psi} \quad \textit{Ans.}$$

$$(S_H)_G = \frac{93\,500(0.973)(1)/[(1)(0.85)]}{99\,170} = 1.08 \quad \textit{Ans.}$$

The hardness of the pinion and the gear should be increased.

14-20 $d_P = 2.5(20) = 50$ mm, $d_G = 2.5(36) = 90$ mm

$$V = \frac{\pi d_P n_P}{60} = \frac{\pi(50)(10^{-3})(100)}{60} = 0.2618 \text{ m/s}$$

$$W^t = \frac{60(120)}{\pi(50)(10^{-3})(100)} = 458.4 \text{ N}$$

Eq. (14-28): $K_o = 1, \quad Q_v = 6, \quad B = 0.25(12-6)^{2/3} = 0.8255$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (14-27): $K_v = \left[\dfrac{59.77 + \sqrt{200(0.2618)}}{59.77}\right]^{0.8255} = 1.099$

Table 14-2: $Y_P = 0.322, \quad Y_G = 0.3775$

Similar to Eq. (*a*) of Sec. 14-10 but for SI units:

$$K_s = \frac{1}{k_b} = 0.8433\left(mF\sqrt{Y}\right)^{0.0535}$$

$$(K_s)_P = 0.8433\left[2.5(18)\sqrt{0.322}\right]^{0.0535} = 1.003 \quad \text{use } 1$$

$$(K_s)_G = 0.8433\left[2.5(18)\sqrt{0.3775}\right]^{0.0535} > 1 \quad \text{use } 1$$

$$C_{mc} = 1, \quad F = 18/25.4 = 0.709 \text{ in}, \quad C_{pf} = \frac{18}{10(50)} - 0.025 = 0.011$$

$$C_{pm} = 1, \quad C_{ma} = 0.247 + 0.0167(0.709) - 0.765(10^{-4})(0.709^2) = 0.259$$

$$C_e = 1$$

$$K_H = 1 + 1[0.011(1) + 0.259(1)] = 1.27$$

Eq. (14-40): $K_B = 1, \quad m_G = N_G/N_P = 36/20 = 1.8$

Fig. 14-14: $(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$

$$(Y_N)_G = 1.3558(10^8/1.8)^{-0.0178} = 0.987$$

Fig. 14-6: $(Y_J)_P = 0.33, \quad (Y_J)_G = 0.38$

Eq. (14-38): $Y_Z = 0.658 - 0.0759 \ln(1 - 0.95) = 0.885$

Sec. 14-15: $Y_\theta = Z_R = 1$

Eq. (14-23) with $m_N = 1$:

$$Z_I = \frac{\cos 20^\circ \sin 20^\circ}{2}\left(\frac{1.8}{1.8+1}\right) = 0.103$$

Table 14-8: $Z_E = 191\sqrt{\text{MPa}}$

Strength Grade 1 steel, given $H_{BP} = H_{BG} = 200$

Fig. 14-2: $(\sigma_{FP})_P = (\sigma_{FP})_G = 0.533(200) + 88.3 = 194.9$ MPa

Fig. 14-5: $(\sigma_{HP})_P = (\sigma_{HP})_G = 2.22(200) + 200 = 644$ MPa

Fig. 14-15: $(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$

$(Z_N)_G = 1.4488(10^8/1.8)^{-0.023} = 0.961$

Fig. 14-12: $H_{BP}/H_{BG} = 1 \qquad \therefore Z_W = 1$

Pinion tooth bending

$$(\sigma)_P = \left(W^t K_o K_v K_s \frac{1}{bm_t}\frac{K_H K_B}{Y_J}\right)_P$$

$$= 458.4(1)(1.099)(1)\left[\frac{1}{18(2.5)}\right]\left[\frac{1.27(1)}{0.33}\right] = 43.08 \text{ MPa} \quad \textit{Ans.}$$

Eq. (14-41): $$(S_F)_P = \left(\frac{\sigma_{FP}}{\sigma}\frac{Y_N}{Y_\theta Y_Z}\right)_P = \frac{194.9}{43.08}\left[\frac{0.977}{1(0.885)}\right] = 4.99 \quad \textit{Ans.}$$

Gear tooth bending

Eq. (14-15): $$(\sigma)_G = 458.4(1)(1.099)(1)\left[\frac{1}{18(2.5)}\right]\left[\frac{1.27(1)}{0.38}\right] = 37.42 \text{ MPa} \quad \textit{Ans.}$$

$$(S_F)_G = \frac{194.9}{37.42}\left[\frac{0.987}{1(0.885)}\right] = 5.81 \quad \textit{Ans.}$$

Pinion tooth wear

Eq. (14-16): $$(\sigma_c)_P = \left(Z_E\sqrt{W^t K_o K_v K_s \frac{K_H}{d_{w1} b}\frac{Z_R}{Z_I}}\right)_P$$

$$= 191\sqrt{458.4(1)(1.099)(1)\left[\frac{1.27}{50(18)}\right]\left[\frac{1}{0.103}\right]} = 501.8 \text{ MPa} \quad \textit{Ans.}$$

Eq. (14-42): $$(S_H)_P = \left(\frac{\sigma_{HP}}{\sigma_c}\frac{Z_N Z_W}{Y_\theta Y_Z}\right)_P = \frac{644}{501.8}\left[\frac{0.948(1)}{1(0.885)}\right] = 1.37 \quad \textit{Ans.}$$

Gear tooth wear

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1}{1}\right)^{1/2}(501.8) = 501.8 \text{ MPa} \quad \textit{Ans.}$$

$$(S_H)_G = \frac{644}{501.8}\left[\frac{0.961(1)}{1(0.885)}\right] = 1.39 \quad \textit{Ans.}$$

14-21

$$P_t = P_n \cos\psi = 6\cos 30^\circ = 5.196 \text{ teeth/in}$$

$$d_P = \frac{16}{5.196} = 3.079 \text{ in}, \quad d_G = \frac{48}{16}(3.079) = 9.238 \text{ in}$$

$$V = \frac{\pi(3.079)(300)}{12} = 241.8 \text{ ft/min}$$

$$W^t = \frac{33\,000(5)}{241.8} = 682.3 \text{ lbf}, \quad K_v = \left(\frac{59.77 + \sqrt{241.8}}{59.77}\right)^{0.8255} = 1.210$$

From Prob. 14-19:

$$Y_P = 0.296, \quad Y_G = 0.4056$$

$$(K_s)_P = 1.088, \quad (K_s)_G = 1.097, \quad K_B = 1$$

$$m_G = 3, \quad (Y_N)_P = 0.977, \quad (Y_N)_G = 0.996, \quad K_R = 0.85$$

$$(S_t)_P = (S_t)_G = 28\,260 \text{ psi}, \quad C_H = 1, \quad (S_c)_P = (S_c)_G = 93\,500 \text{ psi}$$

$$(Z_N)_P = 0.948, \quad (Z_N)_G = 0.973, \quad C_p = 2300\sqrt{\text{psi}}$$

The pressure angle is:

Eq. (13-19): $\phi_t = \tan^{-1}\left(\dfrac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.80^\circ$

$$(r_b)_P = \frac{3.079}{2}\cos 22.8^\circ = 1.419 \text{ in}, \quad (r_b)_G = 3(r_b)_P = 4.258 \text{ in}$$

$$a = 1/P_n = 1/6 = 0.167 \text{ in}$$

Eq. (14-25):

$$Z = \left[\left(\frac{3.079}{2} + 0.167\right)^2 - 1.419^2\right]^{1/2} + \left[\left(\frac{9.238}{2} + 0.167\right)^2 - 4.258^2\right]^{1/2} - \left(\frac{3.079}{2} + \frac{9.238}{2}\right)\sin 22.8^\circ$$

$$= 0.9479 + 2.1852 - 2.3865 = 0.7466 \qquad \text{Conditions } O.K. \text{ for use}$$

$$p_N = p_n \cos\phi_n = \frac{\pi}{6}\cos 20^\circ = 0.4920 \text{ in}$$

Eq. (14-21): $m_N = \dfrac{p_N}{0.95Z} = \dfrac{0.492}{0.95(0.7466)} = 0.6937$

Eq. (14-23): $I = \left[\dfrac{\sin 22.8^\circ \cos 22.8^\circ}{2(0.6937)}\right]\left(\dfrac{3}{3+1}\right) = 0.193$

Fig. 14-7: $J'_P \doteq 0.45, \quad J'_G \doteq 0.54$

Fig. 14-8: Corrections are 0.94 and 0.98

$$J_P = 0.45(0.94) = 0.423, \quad J_G = 0.54(0.98) = 0.529$$

$$C_{mc} = 1, \quad C_{pf} = \frac{2}{10(3.079)} - 0.0375 + 0.0125(2) = 0.0525$$

$$C_{pm} = 1, \quad C_{ma} = 0.093, \quad C_e = 1$$

$$K_m = 1 + (1)[0.0525(1) + 0.093(1)] = 1.146$$

Pinion tooth bending

$$(\sigma)_P = 682.3(1)(1.21)(1.088)\left(\frac{5.196}{2}\right)\left[\frac{1.146(1)}{0.423}\right] = 6323 \text{ psi} \quad \textit{Ans.}$$

$$(S_F)_P = \frac{28\,260(0.977)/[1(0.85)]}{6323} = 5.14 \quad \textit{Ans.}$$

Gear tooth bending

$$(\sigma)_G = 682.3(1)(1.21)(1.097)\left(\frac{5.196}{2}\right)\left[\frac{1.146(1)}{0.529}\right] = 5097 \text{ psi} \quad \textit{Ans.}$$

$$(S_F)_G = \frac{28\,260(0.996)/[1(0.85)]}{5097} = 6.50 \quad \textit{Ans.}$$

Pinion tooth wear

$$(\sigma_c)_P = 2300\left\{682.3(1)(1.21)(1.088)\left[\frac{1.146}{3.078(2)}\right]\left(\frac{1}{0.193}\right)\right\}^{1/2} = 67\,700 \text{ psi} \quad \textit{Ans.}$$

$$(S_H)_P = \frac{93\,500(0.948)/[(1)(0.85)]}{67\,700} = 1.54 \quad \textit{Ans.}$$

Gear tooth wear

$$(\sigma_c)_G = \left[\frac{1.097}{1.088}\right]^{1/2}(67\,700) = 67\,980 \text{ psi} \quad \textit{Ans.}$$

$$(S_H)_G = \frac{93\,500(0.973)/[(1)(0.85)]}{67\,980} = 1.57 \quad \textit{Ans.}$$

14-22 Given: $N_P = 17T$, $N_G = 51T$, $R = 0.99$ at 10^8 cycles, $H_B = 232$ through-hardening Grade 1, core and case, both gears.

Table 14-2: $Y_P = 0.303, Y_G = 0.4103$

Fig. 14-6: $J_P = 0.292, J_G = 0.396$

$$d_P = N_P/P = 17/6 = 2.833 \text{ in}, d_G = 51/6 = 8.5 \text{ in}$$

Pinion bending

From Fig. 14-2:

$$_{0.99}(S_t)_{10^7} = 77.3H_B + 12\,800$$

$$= 77.3(232) + 12\,800 = 30\,734 \text{ psi}$$

Fig. 14-14: $Y_N = 1.6831(10^8)^{-0.0323} = 0.928$

$$V = \pi d_P n/12 = \pi(2.833)(1120/12) = 830.7 \text{ ft/min}$$

$$K_T = K_R = 1, \quad S_F = 2, \quad S_H = \sqrt{2}$$

$$\sigma_{\text{all}} = \frac{30\,734(0.928)}{2(1)(1)} = 14\,261 \text{ psi}$$

$$Q_v = 5, \quad B = 0.25(12-5)^{2/3} = 0.9148$$

$$A = 50 + 56(1 - 0.9148) = 54.77$$

$$K_v = \left(\frac{54.77 + \sqrt{830.7}}{54.77}\right)^{0.9148} = 1.472$$

$$K_s = 1.192\left(\frac{2\sqrt{0.303}}{6}\right)^{0.0535} = 1.089 \quad \Rightarrow \quad \text{use } 1$$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{mc} = 1$$

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.0125F$$

$$= \frac{2}{10(2.833)} - 0.0375 + 0.0125(2)$$

$$= 0.0581$$

$$C_{pm} = 1$$

$$C_{ma} = 0.127 + 0.0158(2) - 0.093(10^{-4})(2^2) = 0.1586$$

$$C_e = 1$$

$$K_m = 1 + 1[0.0581(1) + 0.1586(1)] = 1.2167$$

$$K_\beta = 1$$

Eq. (14-15): $W^t = \dfrac{F J_P \sigma_{\text{all}}}{K_o K_v K_s P_d K_m K_B}$

$$= \frac{2(0.292)(14\,261)}{1(1.472)(1)(6)(1.2167)(1)} = 775 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{775(830.7)}{33\,000} = 19.5 \text{ hp}$$

Pinion wear

Fig. 14-15: $Z_N = 2.466N^{-0.056} = 2.466(10^8)^{-0.056} = 0.879$

$$M_G = 51/17 = 3$$

$$I = \frac{\sin 20^\circ \cos 20^\circ}{2}\left(\frac{3}{3+1}\right) = 1.205, \quad C_H = 1$$

Fig. 14-5: $${}_{0.99}(S_c)_{10^7} = 322H_B + 29\,100$$

$$= 322(232) + 29\,100 = 103\,804 \text{ psi}$$

$$\sigma_{c,\text{all}} = \frac{103\,804(0.879)}{\sqrt{2}(1)(1)} = 64\,519 \text{ psi}$$

Eq. (14-16): $$W^t = \left(\frac{\sigma_{c,\text{all}}}{C_p}\right)^2 \frac{F d_P I}{K_o K_v K_s K_m C_f}$$

$$= \left(\frac{64\,519}{2300}\right)^2 \left[\frac{2(2.833)(0.1205)}{1(1.472)(1)(1.2167)(1)}\right]$$

$$= 300 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{300(830.7)}{33\,000} = 7.55 \text{ hp}$$

The pinion controls therefore $H_{\text{rated}} = 7.55$ hp *Ans.*

14-23

$$l = 2.25/P_d, \quad x = \frac{3Y}{2P_d}$$

$$t = \sqrt{4lx} = \sqrt{4\left(\frac{2.25}{P_d}\right)\left(\frac{3Y}{2P_d}\right)} = \frac{3.674}{P_d}\sqrt{Y}$$

$$d_e = 0.808\sqrt{Ft} = 0.808\sqrt{F\left(\frac{3.674}{P_d}\right)\sqrt{Y}} = 1.5487\sqrt{\frac{F\sqrt{Y}}{P_d}}$$

$$k_b = \left(\frac{1.5487\sqrt{F\sqrt{Y}/P_d}}{0.30}\right)^{-0.107} = 0.8389\left(\frac{F\sqrt{Y}}{P_d}\right)^{-0.0535}$$

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P_d}\right)^{0.0535} \qquad Ans.$$

14-24 $Y_P = 0.331$, $Y_G = 0.422$, $J_P = 0.345$, $J_G = 0.410$, $K_o = 1.25$. The service conditions are adequately described by K_o. Set $S_F = S_H = 1$.

$$d_P = 22/4 = 5.500 \text{ in}$$

$$d_G = 60/4 = 15.000 \text{ in}$$

$$V = \frac{\pi(5.5)(1145)}{12} = 1649 \text{ ft/min}$$

Pinion bending

$$ {}_{0.99}(S_t)_{10^7} = 77.3H_B + 12\,800 = 77.3(250) + 12\,800 = 32\,125 \text{ psi}$$

$$Y_N = 1.6831[3(10^9)]^{-0.0323} = 0.832$$

Eq. (14-17): $$(\sigma_{\text{all}})_P = \frac{32\,125(0.832)}{1(1)(1)} = 26\,728 \text{ psi}$$

$$B = 0.25(12-6)^{2/3} = 0.8255$$

$$A = 50 + 56(1-0.8255) = 59.77$$

$$K_v = \left(\frac{59.77+\sqrt{1649}}{59.77}\right)^{0.8255} = 1.534$$

$$K_s = 1, \quad C_m = 1$$

$$C_{mc} = \frac{F}{10d} - 0.0375 + 0.0125F$$

$$= \frac{3.25}{10(5.5)} - 0.0375 + 0.0125(3.25) = 0.0622$$

$$C_{ma} = 0.127 + 0.0158(3.25) - 0.093(10^{-4})(3.25^2) = 0.178$$

$$C_e = 1$$

$$K_m = C_{mf} = 1 + (1)[0.0622(1) + 0.178(1)] = 1.240$$

$$K_B = 1, \quad K_T = 1$$

Eq. (14-15): $$W_1^t = \frac{26\,728(3.25)(0.345)}{1.25(1.534)(1)(4)(1.240)} = 3151 \text{ lbf}$$

$$H_1 = \frac{3151(1649)}{33\,000} = 157.5 \text{ hp}$$

Gear bending By similar reasoning, $W_2^t = 3861$ lbf and $H_2 = 192.9$ hp

Pinion wear

$$m_G = 60/22 = 2.727$$

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2}\left(\frac{2.727}{1+2.727}\right) = 0.1176$$

$$_{0.99}(S_c)_{10^7} = 322(250) + 29\,100 = 109\,600 \text{ psi}$$

$$(Z_N)_P = 2.466[3(10^9)]^{-0.056} = 0.727$$

$$(Z_N)_G = 2.466[3(10^9)/2.727]^{-0.056} = 0.769$$

$$(\sigma_{c,\text{all}})_P = \frac{109\,600(0.727)}{1(1)(1)} = 79\,679 \text{ psi}$$

$$W_3^t = \left(\frac{\sigma_{c,\text{all}}}{C_p}\right)^2 \frac{F d_P I}{K_o K_v K_s K_m C_f}$$

$$= \left(\frac{79\,679}{2300}\right)^2 \left[\frac{3.25(5.5)(0.1176)}{1.25(1.534)(1)(1.24)(1)}\right] = 1061 \text{ lbf}$$

$$H_3 = \frac{1061(1649)}{33\,000} = 53.0 \text{ hp}$$

Gear wear

Similarly, $W_4^t = 1182\,\text{lbf}, \quad H_4 = 59.0\,\text{hp}$

Rating

$$\begin{aligned} H_{\text{rated}} &= \min(H_1, H_2, H_3, H_4) \\ &= \min(157.5, 192.9, 53, 59) = 53\text{ hp} \quad \textit{Ans.} \end{aligned}$$

Note differing capacities. Can these be equalized?

14-25 From Prob. 14-24:

$$\begin{aligned} &W_1^t = 3151\text{ lbf}, \quad W_2^t = 3861\text{ lbf}, \\ &W_3^t = 1061\text{ lbf}, \quad W_4^t = 1182\text{ lbf} \\ &W^t = \frac{33\,000 K_o H}{V} = \frac{33\,000(1.25)(40)}{1649} = 1000\text{ lbf} \end{aligned}$$

Pinion bending: The factor of safety, based on load and stress, is

$$(S_F)_P = \frac{W_1^t}{1000} = \frac{3151}{1000} = 3.15$$

Gear bending based on load and stress

$$(S_F)_G = \frac{W_2^t}{1000} = \frac{3861}{1000} = 3.86$$

Pinion wear

based on load: $n_3 = \dfrac{W_3^t}{1000} = \dfrac{1061}{1000} = 1.06$

based on stress: $(S_H)_P = \sqrt{1.06} = 1.03$

Gear wear

based on load: $n_4 = \dfrac{W_4^t}{1000} = \dfrac{1182}{1000} = 1.18$

based on stress: $(S_H)_G = \sqrt{1.18} = 1.09$

Factors of safety are used to assess the relative threat of loss of function 3.15, 3.86, 1.06, 1.18 where the threat is from pinion wear. By comparison, the AGMA safety factors

$$(S_F)_P, \ (S_F)_G, \ (S_H)_P, \ (S_H)_G$$

are

$$3.15,\ 3.86,\ 1.03,\ 1.09 \quad \text{or} \quad 3.15,\ 3.86,\ 1.06^{1/2},\ 1.18^{1/2}$$

and the threat is again from pinion wear. Depending on the magnitude of the numbers, using S_F and S_H as defined by AGMA, does not *necessarily* lead to the same conclusion concerning threat. Therefore be cautious.

14-26 Solution summary from Prob. 14-24: $n = 1145$ rev/min, $K_o = 1.25$, Grade 1 materials, $N_P = 22T$, $N_G = 60T$, $m_G = 2.727$, $Y_P = 0.331$, $Y_G = 0.422$, $J_P = 0.345$, $J_G = 0.410$, $P_d = 4T/\text{in}$, $F = 3.25$ in, $Q_v = 6$, $(N_c)_P = 3(10^9)$, $R = 0.99$

Pinion H_B: 250 core, 390 case

Gear H_B: 250 core, 390 case

$K_m = 1.240$, $K_T = 1$, $K_B = 1$, $d_P = 5.500$ in, $d_G = 15.000$ in,
$V = 1649$ ft/min, $K_v = 1.534$, $(K_s)_P = (K_s)_G = 1$, $(Y_N)_P = 0.832$,
$(Y_N)_G = 0.859$, $K_R = 1$

Bending

$$(\sigma_{\text{all}})_P = 26\,728 \text{ psi} \qquad (S_t)_P = 32\,125 \text{ psi}$$
$$(\sigma_{\text{all}})_G = 27\,546 \text{ psi} \qquad (S_t)_G = 32\,125 \text{ psi}$$
$$W_1^t = 3151 \text{ lbf}, \qquad H_1 = 157.5 \text{ hp}$$
$$W_2^t = 3861 \text{ lbf}, \qquad H_2 = 192.9 \text{ hp}$$

Wear

$$\phi = 20^\circ, \quad I = 0.1176, \quad (Z_N)_P = 0.727,$$
$$(Z_N)_G = 0.769, \quad C_P = 2300\sqrt{\text{psi}}$$
$$(S_c)_P = S_c = 322(390) + 29\,100 = 154\,680 \text{ psi}$$
$$(\sigma_{c,\text{all}})_P = \frac{154\,680(0.727)}{1(1)(1)} = 112\,450 \text{ psi}$$
$$(\sigma_{c,\text{all}})_G = \frac{154\,680(0.769)}{1(1)(1)} = 118\,950 \text{ psi}$$
$$W_3^t = \left(\frac{112\,450}{79\,679}\right)^2(1061) = 2113 \text{ lbf}, \qquad H_3 = \frac{2113(1649)}{33\,000} = 105.6 \text{ hp}$$
$$W_4^t = \left(\frac{118\,950}{109\,600(0.769)}\right)^2(1182) = 2354 \text{ lbf}, \qquad H_4 = \frac{2354(1649)}{33\,000} = 117.6 \text{ hp}$$

Rated power

$$H_{\text{rated}} = \min(157.5, 192.9, 105.6, 117.6) = 105.6 \text{ hp} \quad \textit{Ans.}$$

Prob. 14-24

$$H_{\text{rated}} = \min(157.5, 192.9, 53.0, 59.0) = 53 \text{ hp}$$

The rated power approximately doubled.

14-27 The gear and the pinion are 9310 grade 1, carburized and case-hardened to obtain Brinell 285 core and Brinell 580–600 case.

Table 14-3:

$$_{0.99}(S_t)_{10^7} = 55\,000 \text{ psi}$$

Modification of S_t by $(Y_N)_P = 0.832$ produces

$$(\sigma_{\text{all}})_P = 45\,657 \text{ psi},$$

Similarly for $(Y_N)_G = 0.859$

$$(\sigma_{\text{all}})_G = 47\,161 \text{ psi}, \quad \text{and}$$

$$W_1^t = 4569 \text{ lbf}, \quad H_1 = 228 \text{ hp}$$

$$W_2^t = 5668 \text{ lbf}, \quad H_2 = 283 \text{ hp}$$

From Table 14-8, $C_p = 2300\sqrt{\text{psi}}$. Also, from Table 14-6:

$$_{0.99}(S_c)_{10^7} = 180\,000 \text{ psi}$$

Modification of S_c by (Y_N) produces

$$(\sigma_{c,\text{all}})_P = 130\,525 \text{ psi}$$

$$(\sigma_{c,\text{all}})_G = 138\,069 \text{ psi}$$

and

$$W_3^t = 2489 \text{ lbf}, \quad H_3 = 124.3 \text{ hp}$$

$$W_4^t = 2767 \text{ lbf}, \quad H_4 = 138.2 \text{ hp}$$

Rating

$$H_{\text{rated}} = \min(228, 283, 124, 138) = 124 \text{ hp} \quad \textit{Ans.}$$

14-28 Grade 2 9310 carburized and case-hardened to 285 core and 580 case in Prob. 14-27.

Summary:

Table 14-3: $$_{0.99}(S_t)_{10^7} = 65\,000 \text{ psi}$$

$$(\sigma_{\text{all}})_P = 53\,959 \text{ psi}$$

$$(\sigma_{\text{all}})_G = 55\,736 \text{ psi}$$

and it follows that

$$W_1^t = 5399.5 \text{ lbf}, \quad H_1 = 270 \text{ hp}$$

$$W_2^t = 6699 \text{ lbf}, \quad H_2 = 335 \text{ hp}$$

From Table 14-8, $C_p = 2300\sqrt{\text{psi}}$. Also, from Table 14-6:

$$S_c = 225\,000 \text{ psi}$$

$$(\sigma_{c,\text{all}})_P = 181\,285 \text{ psi}$$

$$(\sigma_{c,\text{all}})_G = 191\,762 \text{ psi}$$

Consequently,

$$W_3^t = 4801 \text{ lbf}, \quad H_3 = 240 \text{ hp}$$

$$W_4^t = 5337 \text{ lbf}, \quad H_4 = 267 \text{ hp}$$

Rating

$$H_{\text{rated}} = \min(270, 335, 240, 267) = 240 \text{ hp.} \quad \textit{Ans.}$$

14-29 $n = 1145$ rev/min, $K_o = 1.25$, $N_P = 22T$, $N_G = 60T$, $m_G = 2.727$, $d_P = 2.75$ in, $d_G = 7.5$ in, $Y_P = 0.331$, $Y_G = 0.422$, $J_P = 0.335$, $J_G = 0.405$, $P = 8T$/in, $F = 1.625$ in, $H_B = 250$, case and core, both gears. $C_m = 1$, $F/d_P = 0.0591$, $C_f = 0.0419$, $C_{pm} = 1$, $C_{ma} = 0.152$, $C_e = 1$, $K_m = 1.1942$, $K_T = 1$, $K_\beta = 1$, $K_s = 1$, $V = 824$ ft/min, $(Y_N)_P = 0.8318$, $(Y_N)_G = 0.859$, $K_R = 1$, $I = 0.117\,58$

$$_{0.99}(S_t)_{10^7} = 32\,125 \text{ psi}$$
$$(\sigma_{\text{all}})_P = 26\,668 \text{ psi}$$
$$(\sigma_{\text{all}})_G = 27\,546 \text{ psi}$$

and it follows that

$$W_1^t = 879.3 \text{ lbf}, \quad H_1 = 21.97 \text{ hp}$$
$$W_2^t = 1098 \text{ lbf}, \quad H_2 = 27.4 \text{ hp}$$

For wear

$$W_3^t = 304 \text{ lbf}, \quad H_3 = 7.59 \text{ hp}$$
$$W_4^t = 340 \text{ lbf}, \quad H_4 = 8.50 \text{ hp}$$

Rating

$$H_{\text{rated}} = \min(21.97, 27.4, 7.59, 8.50) = 7.59 \text{ hp}$$

In Prob. 14-24, $H_{\text{rated}} = 53$ hp

Thus

$$\frac{7.59}{53.0} = 0.1432 = \frac{1}{6.98}, \quad \text{not } \frac{1}{8} \quad Ans.$$

The transmitted load rating is

$$W_{\text{rated}}^t = \min(879.3, 1098, 304, 340) = 304 \text{ lbf}$$

In Prob. 14-24

$$W_{\text{rated}}^t = 1061 \text{ lbf}$$

Thus

$$\frac{304}{1061} = 0.2865 = \frac{1}{3.49}, \quad \text{not } \frac{1}{4}, \quad Ans.$$

14-30 $S_P = S_H = 1$, $P_d = 4$, $J_P = 0.345$, $J_G = 0.410$, $K_o = 1.25$

Bending

Table 14-4: $_{0.99}(S_t)_{10^7} = 13\,000$ psi

$$(\sigma_{\text{all}})_P = (\sigma_{\text{all}})_G = \frac{13\,000(1)}{1(1)(1)} = 13\,000 \text{ psi}$$
$$W_1^t = \frac{\sigma_{\text{all}} F J_P}{K_o K_v K_s P_d K_m K_B} = \frac{13\,000(3.25)(0.345)}{1.25(1.534)(1)(4)(1.24)(1)} = 1533 \text{ lbf}$$
$$H_1 = \frac{1533(1649)}{33\,000} = 76.6 \text{ hp}$$
$$W_2^t = W_1^t J_G/J_P = 1533(0.410)/0.345 = 1822 \text{ lbf}$$
$$H_2 = H_1 J_G/J_P = 76.6(0.410)/0.345 = 91.0 \text{ hp}$$

Wear

Table 14-8: $C_p = 1960\sqrt{\text{psi}}$

Table 14-7: ${}_{0.99}(S_c)_{10^7} = 75\,000 \text{ psi} = (\sigma_{c,\text{all}})_P = (\sigma_{c,\text{all}})_G$

$$W_3^t = \left(\frac{(\sigma_{c,\text{all}})_P}{C_p}\right)^2 \frac{FdpI}{K_o K_v K_s K_m C_f}$$

$$W_3^t = \left(\frac{75\,000}{1960}\right)^2 \frac{3.25(5.5)(0.1176)}{1.25(1.534)(1)(1.24)(1)} = 1295 \text{ lbf}$$

$$W_4^t = W_3^t = 1295 \text{ lbf}$$

$$H_4 = H_3 = \frac{1295(1649)}{33\,000} = 64.7 \text{ hp}$$

Rating

$$H_{\text{rated}} = \min(76.7, 94.4, 64.7, 64.7) = 64.7 \text{ hp} \quad Ans.$$

Notice that the balance between bending and wear power is improved due to CI's more favorable S_c/S_t ratio. Also note that the life is 10^7 pinion revolutions which is $(1/300)$ of $3(10^9)$. Longer life goals require power derating.

14-31 From Table A-24*a*, $E_{av} = 11.8(10^6)$

For $\phi = 14.5^\circ$ and $H_B = 156$

$$S_C = \sqrt{\frac{1.4(81)}{2\sin 14.5^\circ/[11.8(10^6)]}} = 51\,693 \text{ psi}$$

For $\phi = 20^\circ$

$$S_C = \sqrt{\frac{1.4(112)}{2\sin 20^\circ/[11.8(10^6)]}} = 52\,008 \text{ psi}$$

$$S_C = 0.32(156) = 49.9 \text{ kpsi}$$

14-32 Programs will vary.

14-33

$$(Y_N)_P = 0.977, \quad (Y_N)_G = 0.996$$

$$(S_t)_P = (S_t)_G = 82.3(250) + 12\,150 = 32\,725 \text{ psi}$$

$$(\sigma_{\text{all}})_P = \frac{32\,725(0.977)}{1(0.85)} = 37\,615 \text{ psi}$$

$$W_1^t = \frac{37\,615(1.5)(0.423)}{1(1.404)(1.043)(8.66)(1.208)(1)} = 1558 \text{ lbf}$$

$$H_1 = \frac{1558(925)}{33\,000} = 43.7 \text{ hp}$$

$$(\sigma_{\text{all}})_G = \frac{32\,725(0.996)}{1(0.85)} = 38\,346 \text{ psi}$$

$$W_2^t = \frac{38\,346(1.5)(0.5346)}{1(1.404)(1.043)(8.66)(1.208)(1)} = 2007 \text{ lbf}$$

$$H_2 = \frac{2007(925)}{33\,000} = 56.3 \text{ hp}$$

$$(Z_N)_P = 0.948, \quad (Z_N)_G = 0.973$$

Table 14-6: $_{0.99}(S_c)_{10^7} = 150\,000$ psi

$$(\sigma_{c,\text{allow}})_P = 150\,000\left[\frac{0.948(1)}{1(0.85)}\right] = 167\,294 \text{ psi}$$

$$W_3^t = \left(\frac{167\,294}{2300}\right)^2\left[\frac{1.963(1.5)(0.195)}{1(1.404)(1.043)}\right] = 2074 \text{ lbf}$$

$$H_3 = \frac{2074(925)}{33\,000} = 58.1 \text{ hp}$$

$$(\sigma_{c,\text{allow}})_G = \frac{0.973}{0.948}(167\,294) = 171\,706 \text{ psi}$$

$$W_4^t = \left(\frac{171\,706}{2300}\right)^2\left[\frac{1.963(1.5)(0.195)}{1(1.404)(1.052)}\right] = 2167 \text{ lbf}$$

$$H_4 = \frac{2167(925)}{33\,000} = 60.7 \text{ hp}$$

$$H_{\text{rated}} = \min(43.7, 56.3, 58.1, 60.7) = 43.7 \text{ hp} \quad Ans.$$

Pinion bending controlling

14-34

$$(Y_N)_P = 1.6831(10^8)^{-0.0323} = 0.928$$
$$(Y_N)_G = 1.6831(10^8/3.059)^{-0.0323} = 0.962$$

Table 14-3: $S_t = 55\,000$ psi

$$(\sigma_{\text{all}})_P = \frac{55\,000(0.928)}{1(0.85)} = 60\,047 \text{ psi}$$

$$W_1^t = \frac{60\,047(1.5)(0.423)}{1(1.404)(1.043)(8.66)(1.208)(1)} = 2487 \text{ lbf}$$

$$H_1 = \frac{2487(925)}{33\,000} = 69.7 \text{ hp}$$

$$(\sigma_{\text{all}})_G = \frac{0.962}{0.928}(60\,047) = 62\,247 \text{ psi}$$

$$W_2^t = \frac{62\,247}{60\,047}\left(\frac{0.5346}{0.423}\right)(2487) = 3258 \text{ lbf}$$

$$H_2 = \frac{3258}{2487}(69.7) = 91.3 \text{ hp}$$

Table 14-6: $S_c = 180\,000$ psi

$$(Z_N)_P = 2.466(10^8)^{-0.056} = 0.8790$$

$$(Z_N)_G = 2.466(10^8/3.059)^{-0.056} = 0.9358$$

$$(\sigma_{c,\text{all}})_P = \frac{180\,000(0.8790)}{1(0.85)} = 186\,141 \text{ psi}$$

$$W_3^t = \left(\frac{186\,141}{2300}\right)^2 \left[\frac{1.963(1.5)(0.195)}{1(1.404)(1.043)}\right] = 2568 \text{ lbf}$$

$$H_3 = \frac{2568(925)}{33\,000} = 72.0 \text{ hp}$$

$$(\sigma_{c,\text{all}})_G = \frac{0.9358}{0.8790}(186\,141) = 198\,169 \text{ psi}$$

$$W_4^t = \left(\frac{198\,169}{186\,141}\right)^2 \left(\frac{1.043}{1.052}\right)(2568) = 2886 \text{ lbf}$$

$$H_4 = \frac{2886(925)}{33\,000} = 80.9 \text{ hp}$$

$$H_{\text{rated}} = \min(69.7, 91.3, 72, 80.9) = 69.7 \text{ hp} \quad Ans.$$

Pinion bending controlling

14-35 $(Y_N)_P = 0.928$, $(Y_N)_G = 0.962$ (See Prob. 14-34)

Table 14-3: $S_t = 65\,000$ psi

$$(\sigma_{\text{all}})_P = \frac{65\,000(0.928)}{1(0.85)} = 70\,965 \text{ psi}$$

$$W_1^t = \frac{70\,965(1.5)(0.423)}{1(1.404)(1.043)(8.66)(1.208)} = 2939 \text{ lbf}$$

$$H_1 = \frac{2939(925)}{33\,000} = 82.4 \text{ hp}$$

$$(\sigma_{\text{all}})_G = \frac{65\,000(0.962)}{1(0.85)} = 73\,565 \text{ psi}$$

$$W_2^t = \frac{73\,565}{70\,965}\left(\frac{0.5346}{0.423}\right)(2939) = 3850 \text{ lbf}$$

$$H_2 = \frac{3850}{2939}(82.4) = 108 \text{ hp}$$

Table 14-6: $S_c = 225\,000$ psi

$$(Z_N)_P = 0.8790, \quad (Z_N)_G = 0.9358$$

$$(\sigma_{c,\text{all}})_P = \frac{225\,000(0.879)}{1(0.85)} = 232\,676 \text{ psi}$$

$$W_3^t = \left(\frac{232\,676}{2300}\right)^2 \left[\frac{1.963(1.5)(0.195)}{1(1.404)(1.043)}\right] = 4013 \text{ lbf}$$

$$H_3 = \frac{4013(925)}{33\,000} = 112.5 \text{ hp}$$

$$(\sigma_{c,\text{all}})_G = \frac{0.9358}{0.8790}(232\,676) = 247\,711 \text{ psi}$$

$$W_4^t = \left(\frac{247\,711}{232\,676}\right)^2 \left(\frac{1.043}{1.052}\right)(4013) = 4509 \text{ lbf}$$

$$H_4 = \frac{4509(925)}{33\,000} = 126 \text{ hp}$$

$$H_{\text{rated}} = \min(82.4,\ 108,\ 112.5,\ 126) = 82.4 \text{ hp} \quad Ans.$$

The bending of the pinion is the controlling factor.

Chapter 15

15-1 Given: Uncrowned, through-hardened 300 Brinell core and case, Grade 1, $N_C = 10^9$ rev of pinion at $R = 0.999$, $N_P = 20$ teeth, $N_G = 60$ teeth, $Q_v = 6$, $P_d = 6$ teeth/in, shaft angle 90°, $n_p = 900$ rev/min, $J_P = 0.249$ and $J_G = 0.216$ (Fig. 15-7), $F = 1.25$ in, $S_F = S_H = 1$, $K_o = 1$.

Mesh $$d_P = 20/6 = 3.333 \text{ in}$$

$$d_G = 60/6 = 10.000 \text{ in}$$

Eq. (15-7): $$v_t = \pi(3.333)(900/12) = 785.3 \text{ ft/min}$$

Eq. (15-6): $$B = 0.25(12-6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (15-5): $$K_v = \left(\frac{59.77 + \sqrt{785.3}}{59.77}\right)^{0.8255} = 1.374$$

Eq. (15-8): $$v_{t,\max} = [59.77 + (6-3)]^2 = 3940 \text{ ft/min}$$

Since $785.3 < 3904$, $K_v = 1.374$ is valid. The size factor for bending is:

Eq. (15-10): $$K_s = 0.4867 + 0.2132/6 = 0.5222$$

For one gear straddle-mounted, the load-distribution factor is:

Eq. (15-11): $$K_m = 1.10 + 0.0036(1.25)^2 = 1.106$$

Eq. (15-15): $$(K_L)_P = 1.6831(10^9)^{-0.0323} = 0.862$$

$$(K_L)_G = 1.6831(10^9/3)^{-0.0323} = 0.893$$

Eq. (15-14): $$(C_L)_P = 3.4822(10^9)^{-0.0602} = 1$$

$$(C_L)_G = 3.4822(10^9/3)^{-0.0602} = 1.069$$

Eq. (15-19): $$K_R = 0.50 - 0.25\log(1 - 0.999) = 1.25 \quad \text{(or Table 15-3)}$$

$$C_R = \sqrt{K_R} = \sqrt{1.25} = 1.118$$

Bending

Fig. 15-13: $${}_{0.99}S_t = s_{at} = 44(300) + 2100 = 15\,300 \text{ psi}$$

Eq. (15-4): $$(\sigma_{\text{all}})_P = s_{wt} = \frac{s_{at}K_L}{S_F K_T K_R} = \frac{15\,300(0.862)}{1(1)(1.25)} = 10\,551 \text{ psi}$$

Eq. (15-3): $$W_P^t = \frac{(\sigma_{\text{all}})_P F K_x J_P}{P_d K_o K_v K_s K_m}$$

$$= \frac{10\,551(1.25)(1)(0.249)}{6(1)(1.374)(0.5222)(1.106)} = 690 \text{ lbf}$$

$$H_1 = \frac{690(785.3)}{33\,000} = 16.4 \text{ hp}$$

Eq. (15-4): $$(\sigma_{\text{all}})_G = \frac{15\,300(0.893)}{1(1)(1.25)} = 10\,930 \text{ psi}$$

$$W_G^t = \frac{10\,930(1.25)(1)(0.216)}{6(1)(1.374)(0.5222)(1.106)} = 620 \text{ lbf}$$

$$H_2 = \frac{620(785.3)}{33\,000} = 14.8 \text{ hp} \quad \textit{Ans.}$$

The gear controls the bending rating.

15-2 Refer to Prob. 15-1 for the gearset specifications.

Wear

Fig. 15-12: $s_{ac} = 341(300) + 23\,620 = 125\,920$ psi

For the pinion, $C_H = 1$. From Prob. 15-1, $C_R = 1.118$. Thus, from Eq. (15-2):

$$(\sigma_{c,\text{all}})_P = \frac{s_{ac}(C_L)_P C_H}{S_H K_T C_R}$$

$$(\sigma_{c,\text{all}})_P = \frac{125\,920(1)(1)}{1(1)(1.118)} = 112\,630 \text{ psi}$$

For the gear, from Eq. (15-16),

$$B_1 = 0.008\,98(300/300) - 0.008\,29 = 0.000\,69$$
$$C_H = 1 + 0.000\,69(3-1) = 1.001\,38$$

And Prob. 15-1, $(C_L)_G = 1.0685$. Equation (15-2) thus gives

$$(\sigma_{c,\text{all}})_G = \frac{s_{ac}(C_L)_G C_H}{S_H K_T C_R}$$

$$(\sigma_{c,\text{all}})_G = \frac{125\,920(1.0685)(1.001\,38)}{1(1)(1.118)} = 120\,511 \text{ psi}$$

For steel: $C_p = 2290\sqrt{\text{psi}}$

Eq. (15-9): $C_s = 0.125(1.25) + 0.4375 = 0.593\,75$

Fig. 15-6: $I = 0.083$

Eq. (15-12): $C_{xc} = 2$

Eq. (15-1):
$$W_P^t = \left(\frac{(\sigma_{c,\text{all}})_P}{C_p}\right)^2 \frac{F d_P I}{K_o K_v K_m C_s C_{xc}}$$

$$= \left(\frac{112\,630}{2290}\right)^2 \left[\frac{1.25(3.333)(0.083)}{1(1.374)(1.106)(0.5937)(2)}\right]$$

$$= 464 \text{ lbf}$$

$$H_3 = \frac{464(785.3)}{33\,000} = 11.0 \text{ hp}$$

$$W_G^t = \left(\frac{120\,511}{2290}\right)^2 \left[\frac{1.25(3.333)(0.083)}{1(1.374)(1.106)(0.593\,75)(2)}\right]$$

$$= 531 \text{ lbf}$$

$$H_4 = \frac{531(785.3)}{33\,000} = 12.6 \text{ hp}$$

The pinion controls wear: $H = 11.0$ hp *Ans.*

The power rating of the mesh, considering the power ratings found in Prob. 15-1, is

$$H = \min(16.4, 14.8, 11.0, 12.6) = 11.0 \text{ hp} \quad \textit{Ans.}$$

15-3 AGMA 2003-B97 does not fully address cast iron gears, however, approximate comparisons can be useful. This problem is similar to Prob. 15-1, but not identical. We will organize the method. A follow-up could consist of completing Probs. 15-1 and 15-2 with identical pinions, and cast iron gears.

Given: Uncrowned, straight teeth, $P_d = 6$ teeth/in, $N_P = 30$ teeth, $N_G = 60$ teeth, ASTM 30 cast iron, material Grade 1, shaft angle 90°, $F = 1.25$, $n_P = 900$ rev/min, $\phi_n = 20°$, one gear straddle-mounted, $K_o = 1, J_P = 0.268, J_G = 0.228, S_F = 2, S_H = \sqrt{2}$.

Mesh

$$d_P = 30/6 = 5.000 \text{ in}$$

$$d_G = 60/6 = 10.000 \text{ in}$$

$$v_t = \pi(5)(900/12) = 1178 \text{ ft/min}$$

Set $N_L = 10^7$ cycles for the pinion. For $R = 0.99$,

Table 15-7: $s_{at} = 4500$ psi

Table 15-5: $s_{ac} = 50\,000$ psi

Eq. (15-4): $s_{wt} = \dfrac{s_{at}K_L}{S_F K_T K_R} = \dfrac{4500(1)}{2(1)(1)} = 2250$ psi

The velocity factor K_v represents stress augmentation due to mislocation of tooth profiles along the pitch surface and the resulting "falling" of teeth into engagement. Equation (5-67) shows that the induced bending moment in a cantilever (tooth) varies directly with $\sqrt{E}$ of the tooth material. If only the material varies (cast iron vs. steel) in the same geometry, I is the same. From the Lewis equation of Section 14-1,

$$\sigma = \frac{M}{I/c} = \frac{K_v W^t P}{FY}$$

We expect the ratio $\sigma_{\text{CI}}/\sigma_{\text{steel}}$ to be

$$\frac{\sigma_{\text{CI}}}{\sigma_{\text{steel}}} = \frac{(K_v)_{\text{CI}}}{(K_v)_{\text{steel}}} = \sqrt{\frac{E_{\text{CI}}}{E_{\text{steel}}}}$$

In the case of ASTM class 30, from Table A-24(*a*)

$$(E_{\text{CI}})_{av} = (13 + 16.2)/2 = 14.7 \text{ kpsi}$$

Then

$$(K_v)_{\text{CI}} = \sqrt{\frac{14.7}{30}}(K_v)_{\text{steel}} = 0.7(K_v)_{\text{steel}}$$

Our modeling is rough, but it convinces us that $(K_v)_{\text{CI}} < (K_v)_{\text{steel}}$, but we are not sure of the value of $(K_v)_{\text{CI}}$. We will use K_v for steel as a basis for a conservative rating.

Eq. (15-6): $$B = 0.25(12-6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (15-5): $$K_v = \left(\frac{59.77 + \sqrt{1178}}{59.77}\right)^{0.8255} = 1.454$$

Pinion bending $(\sigma_{\text{all}})_P = s_{wt} = 2250$ psi

From Prob. 15-1, $K_x = 1, \quad K_m = 1.106, \quad K_s = 0.5222$

Eq. (15-3): $$W_P^t = \frac{(\sigma_{\text{all}})_P F K_x J_P}{P_d K_o K_v K_s K_m}$$

$$= \frac{2250(1.25)(1)(0.268)}{6(1)(1.454)(0.5222)(1.106)} = 149.6 \text{ lbf}$$

$$H_1 = \frac{149.6(1178)}{33\,000} = 5.34 \text{ hp}$$

Gear bending

$$W_G^t = W_P^t \frac{J_G}{J_P} = 149.6\left(\frac{0.228}{0.268}\right) = 127.3 \text{ lbf}$$

$$H_2 = \frac{127.3(1178)}{33\,000} = 4.54 \text{ hp}$$

The gear controls in bending fatigue.

$$H = 4.54 \text{ hp} \quad \textit{Ans.}$$

15-4 Continuing Prob. 15-3,

Table 15-5: $$s_{ac} = 50\,000 \text{ psi}$$

$$s_{wt} = \sigma_{c,\text{all}} = \frac{50\,000}{\sqrt{2}} = 35\,355 \text{ psi}$$

Eq. (15-1): $$W^t = \left(\frac{\sigma_{c,\text{all}}}{C_p}\right)^2 \frac{F d_P I}{K_o K_v K_m C_s C_{xc}}$$

Fig. 15-6: $I = 0.86$

Eq. (15-9) $C_s = 0.125(1.25) + 0.4375 = 0.593\,75$

Eq. (15-10) $K_s = 0.4867 + 0.2132/6 = 0.5222$

Eq. (15-11) $K_m = 1.10 + 0.0036(1.25)^2 = 1.106$

Eq. (15-12) $C_{xc} = 2$

From Table 14-8: $C_p = 1960\sqrt{\text{psi}}$

Thus, $$W^t = \left(\frac{35\,355}{1960}\right)^2 \left[\frac{1.25(5.000)(0.086)}{1(1.454)(1.106)(0.59375)(2)}\right] = 91.6 \text{ lbf}$$

$$H_3 = H_4 = \frac{91.6(1178)}{33\,000} = 3.27 \text{ hp} \quad \textit{Ans.}$$

Rating Based on results of Probs. 15-3 and 15-4,

$$H = \min(5.34, 4.54, 3.27, 3.27) = 3.27 \text{ hp} \quad \textit{Ans.}$$

The mesh is weakest in wear fatigue.

15-5 Uncrowned, through-hardened to 180 Brinell (core and case), Grade 1, 10^9 rev of pinion at $R = 0.999$, $N_P = z_1 = 22$ teeth, $N_G = z_2 = 24$ teeth, $Q_v = 5$, $m_{et} = 4$ mm, shaft angle 90°, $n_1 = 1800$ rev/min, $S_F = 1$, $S_H = \sqrt{S_F} = \sqrt{1}$, $J_P = Y_{J1} = 0.23$, $J_G = Y_{J2} = 0.205$, $F = b = 25$ mm, $K_o = K_A = K_T = K_\theta = 1$ and $C_p = 190\sqrt{\text{MPa}}$.

Mesh $$d_P = d_{e1} = mz_1 = 4(22) = 88 \text{ mm}$$

$$d_G = m_{et} z_2 = 4(24) = 96 \text{ mm}$$

Eq. (15-7): $$v_{et} = 5.236(10^{-5})(88)(1800) = 8.29 \text{ m/s}$$

Eq. (15-6): $$B = 0.25(12 - 5)^{2/3} = 0.9148$$

$$A = 50 + 56(1 - 0.9148) = 54.77$$

Eq. (15-5): $$K_v = \left(\frac{54.77 + \sqrt{200(8.29)}}{54.77}\right)^{0.9148} = 1.663$$

Eq. (15-10): $$K_s = Y_x = 0.4867 + 0.008\,339(4) = 0.520$$

Eq. (15-11) with $K_{mb} = 1$ (both straddle-mounted),

$$K_m = K_{H\beta} = 1 + 5.6(10^{-6})(25^2) = 1.0035$$

From Fig. 15-8,

$$(C_L)_P = (Z_{NT})_P = 3.4822(10^9)^{-0.0602} = 1.00$$

$$(C_L)_G = (Z_{NT})_G = 3.4822[10^9(22/24)]^{-0.0602} = 1.0054$$

Eq. (15-12): $$C_{xc} = Z_{xc} = 2 \quad \text{(uncrowned)}$$

Eq. (15-19): $$K_R = Y_Z = 0.50 - 0.25 \log(1 - 0.999) = 1.25$$

$$C_R = Z_Z = \sqrt{Y_Z} = \sqrt{1.25} = 1.118$$

From Fig. 15-10, $$C_H = Z_w = 1$$

Eq. (15-9): $$Z_x = 0.004\,92(25) + 0.4375 = 0.560$$

Wear of Pinion

Fig. 15-12: $$\sigma_{H\,\lim} = 2.35 H_B + 162.89$$

$$= 2.35(180) + 162.89 = 585.9 \text{ MPa}$$

Fig. 15-6: $$I = Z_I = 0.066$$

Eq. (15-2): $$(\sigma_H)_P = \frac{(\sigma_{H\,\lim})_P (Z_{NT})_P Z_W}{S_H K_\theta Z_Z}$$

$$= \frac{585.9(1)(1)}{\sqrt{1}(1)(1.118)} = 524.1 \text{ MPa}$$

Eq. (15-1): $$W_P^t = \left(\frac{\sigma_H}{C_p}\right)^2 \frac{b d_{e1} Z_I}{1000 K_A K_v K_{H\beta} Z_x Z_{xc}}$$

The constant 1000 expresses W^t in kN

$$W_P^t = \left(\frac{524.1}{190}\right)^2 \left[\frac{25(88)(0.066)}{1000(1)(1.663)(1.0035)(0.56)(2)}\right] = 0.591 \text{ kN}$$

Eq. (13-36): $$H_3 = \frac{\pi dn\, W^t}{60\,000} = \frac{\pi(88)1800(0.591)}{60\,000} = 4.90 \text{ kW}$$

Wear of Gear $\sigma_{H\,\text{lim}} = 585.9$ MPa

$$(\sigma_H)_G = \frac{585.9(1.0054)}{\sqrt{1}(1)(1.118)} = 526.9 \text{ MPa}$$

$$W_G^t = W_P^t \frac{(\sigma_H)_G}{(\sigma_H)_P} = 0.591\left(\frac{526.9}{524.1}\right) = 0.594 \text{ kN}$$

$$H_4 = \frac{\pi(88)1800(0.594)}{60\,000} = 4.93 \text{ kW}$$

Thus in wear, the pinion controls the power rating; $H = 4.90$ kW *Ans.*

We will rate the gear set after solving Prob. 15-6.

15-6 Refer to Prob. 15-5 for terms not defined below.

Bending of Pinion

Eq. (15-15): $$(K_L)_P = (Y_{NT})_P = 1.6831(10^9)^{-0.0323} = 0.862$$

$$(K_L)_G = (Y_{NT})_G = 1.6831[10^9(22/24)]^{-0.0323} = 0.864$$

Fig. 15-13: $$\sigma_{F\,\text{lim}} = 0.30H_B + 14.48$$

$$= 0.30(180) + 14.48 = 68.5 \text{ MPa}$$

Eq. (15-13): $$K_x = Y_\beta = 1$$

From Prob. 15-5: $Y_Z = 1.25$, $v_{et} = 8.29$ m/s

$$K_A = 1,\quad K_v = 1.663,\quad K_\theta = 1,\quad Y_x = 0.52,\quad K_{H\beta} = 1.0035,\quad Y_{J1} = 0.23$$

Eq. (5-4): $$(\sigma_F)_P = \frac{\sigma_{F\,\text{lim}} Y_{NT}}{S_F K_\theta Y_Z} = \frac{68.5(0.862)}{1(1)(1.25)} = 47.2 \text{ MPa}$$

Eq. (5-3): $$W_p^t = \frac{(\sigma_F)_P b m_{et} Y_\beta Y_{J1}}{1000 K_A K_v Y_x K_{H\beta}}$$

$$= \frac{47.2(25)(4)(1)(0.23)}{1000(1)(1.663)(0.52)(1.0035)} = 1.25 \text{ kN}$$

$$H_1 = \frac{\pi(88)1800(1.25)}{60\,000} = 10.37 \text{ kW}$$

Bending of Gear

$$\sigma_{F\,\text{lim}} = 68.5 \text{ MPa}$$

$$(\sigma_F)_G = \frac{68.5(0.864)}{1(1)(1.25)} = 47.3 \text{ MPa}$$

$$W_G^t = \frac{47.3(25)(4)(1)(0.205)}{1000(1)(1.663)(0.52)(1.0035)} = 1.12 \text{ kN}$$

$$H_2 = \frac{\pi(88)1800(1.12)}{60\,000} = 9.29 \text{ kW}$$

Rating of mesh is

$$H_{\text{rating}} = \min(10.37, 9.29, 4.90, 4.93) = 4.90\ \text{kW} \quad Ans.$$

with pinion wear controlling.

15-7

(a)

$$(S_F)_P = \left(\frac{\sigma_{\text{all}}}{\sigma}\right)_P = (S_F)_G = \left(\frac{\sigma_{\text{all}}}{\sigma}\right)_G$$

$$\frac{(s_{at}K_L/K_TK_R)_P}{(W^tP_dK_oK_vK_sK_m/FK_xJ)_P} = \frac{(s_{at}K_L/K_TK_R)_G}{(W^tP_dK_oK_vK_sK_m/FK_xJ)_G}$$

All terms cancel except for s_{at}, K_L, and J,

$$(s_{at})_P(K_L)_PJ_P = (s_{at})_G(K_L)_GJ_G$$

From which

$$(s_{at})_G = \frac{(s_{at})_P(K_L)_PJ_P}{(K_L)_GJ_G} = (s_{at})_P\frac{J_P}{J_G}m_G^{\beta}$$

Where $\beta = -0.0178$ or $\beta = -0.0323$ as appropriate. This equation is the same as Eq. (14-44). *Ans.*

(b) In bending

$$W^t = \left(\frac{\sigma_{\text{all}}}{S_F}\frac{FK_xJ}{P_dK_oK_vK_sK_m}\right)_{11} = \left(\frac{s_{at}}{S_F}\frac{K_L}{K_TK_R}\frac{FK_xJ}{P_dK_oK_vK_sK_m}\right)_{11} \tag{1}$$

In wear

$$\left(\frac{s_{ac}C_LC_U}{S_HK_TC_R}\right)_{22} = C_p\left(\frac{W^tK_oK_vK_mC_sC_{xc}}{Fd_PI}\right)_{22}^{1/2}$$

Squaring and solving for W^t gives

$$W^t = \left(\frac{s_{ac}^2C_L^2C_H^2}{S_H^2K_T^2C_R^2C_P^2}\right)_{22}\left(\frac{Fd_PI}{K_oK_vK_mC_sC_{xc}}\right)_{22} \tag{2}$$

Equating the right-hand sides of Eqs. (1) and (2) and canceling terms, and recognizing that $C_R = \sqrt{K_R}$ and $P_dd_P = N_P$,

we obtain

$$(s_{ac})_{22} = \frac{C_p}{(C_L)_{22}}\sqrt{\frac{S_H^2}{S_F}\frac{(s_{at})_{11}(K_L)_{11}K_xJ_{11}K_TC_sC_{xc}}{C_H^2N_PK_sI}}$$

For equal W^t in bending and wear

$$\frac{S_H^2}{S_F} = \frac{\left(\sqrt{S_F}\right)^2}{S_F} = 1$$

So we get

$$(s_{ac})_G = \frac{C_p}{(C_L)_GC_H}\sqrt{\frac{(s_{at})_P(K_L)_PJ_PK_xK_TC_sC_{xc}}{N_PIK_s}} \quad Ans.$$

(c)

$$(S_H)_P = (S_H)_G = \left(\frac{\sigma_{c,\text{all}}}{\sigma_c}\right)_P = \left(\frac{\sigma_{c,\text{all}}}{\sigma_c}\right)_G$$

Substituting in the right-hand equality gives

$$\frac{[s_{ac}C_L/(C_R K_T)]_P}{\left[C_p\sqrt{W^t K_o K_v K_m C_s C_{xc}/(F d_P I)}\right]_P} = \frac{[s_{ac}C_L C_H/(C_R K_T)]_G}{\left[C_p\sqrt{W^t K_o K_v K_m C_s C_{xc}/(F d_P I)}\right]_G}$$

Denominators cancel leaving

$$(s_{ac})_P(C_L)_P = (s_{ac})_G(C_L)_G C_H$$

Solving for $(s_{ac})_P$ gives,

$$(s_{ac})_P = (s_{ac})_G \frac{(C_L)_G}{(C_L)_P} C_H \tag{1}$$

From Eq. (15-14), $(C_L)_P = 3.4822 N_L^{-0.0602}$, $(C_L)_G = 3.4822(N_L/m_G)^{-0.0602}$. Thus,

$$(s_{ac})_P = (s_{ac})_G(1/m_G)^{-0.0602}C_H = (s_{ac})_G m_G^{0.0602} C_H \quad \textit{Ans.}$$

This equation is the transpose of Eq. (14-45).

15-8

	Core	Case
Pinion	$(H_B)_{11}$	$(H_B)_{12}$
Gear	$(H_B)_{21}$	$(H_B)_{22}$

Given $(H_B)_{11} = 300$ Brinell

Eq. (15-23): $(s_{at})_P = 44(300) + 2100 = 15\,300$ psi

From Prob. 15-7,

$$(s_{at})_G = (s_{at})_P \frac{J_P}{J_G} m_G^{-0.0323} = 15\,300\left(\frac{0.249}{0.216}\right)\left(3^{-0.0323}\right) = 17\,023 \text{ psi}$$

$$(H_B)_{21} = \frac{17\,023 - 2100}{44} = 339 \text{ Brinell} \quad \textit{Ans.}$$

$$(s_{ac})_G = \frac{2290}{1.0685(1)}\sqrt{\frac{15\,300(0.862)(0.249)(1)(0.593\,25)(2)}{20(0.086)(0.5222)}} = 141\,160 \text{ psi}$$

$$(H_B)_{22} = \frac{141\,160 - 23\,600}{341} = 345 \text{ Brinell} \quad \textit{Ans.}$$

$$(s_{ac})_P = (s_{ac})_G m_G^{0.0602} C_H \doteq 141\,160(3^{0.0602})(1) = 150\,811 \text{ psi}$$

$$(H_B)_{12} = \frac{150\,811 - 23\,600}{341} = 373 \text{ Brinell} \quad \textit{Ans.}$$

	Care	Case
Pinion	300	373
Gear	339	345

Ans.

15-9 *Pinion core*

$$(s_{at})_P = 44(300) + 2100 = 15\,300 \text{ psi}$$

$$(\sigma_{\text{all}})_P = \frac{15\,300(0.862)}{1(1)(1.25)} = 10\,551 \text{ psi}$$

$$W^t = \frac{10\,551(1.25)(0.249)}{6(1)(1.374)(0.5222)(1.106)} = 689.7 \text{ lbf}$$

Gear core

$$(s_{at})_G = 44(352) + 2100 = 17\,588 \text{ psi}$$

$$(\sigma_{\text{all}})_G = \frac{17\,588(0.893)}{1(1)(1.25)} = 12\,565 \text{ psi}$$

$$W^t = \frac{12\,565(1.25)(0.216)}{6(1)(1.374)(0.5222)(1.106)} = 712.5 \text{ lbf}$$

Pinion case

$$(s_{ac})_P = 341(372) + 23\,620 = 150\,472 \text{ psi}$$

$$(\sigma_{c,\text{all}})_P = \frac{150\,472(1)}{1(1)(1.118)} = 134\,590 \text{ psi}$$

$$W^t = \left(\frac{134\,590}{2290}\right)^2 \left[\frac{1.25(3.333)(0.086)}{1(1.374)(1.106)(0.593\,75)(2)}\right] = 685.8 \text{ lbf}$$

Gear case

$$(s_{ac})_G = 341(344) + 23\,620 = 140\,924 \text{ psi}$$

$$(\sigma_{c,\text{all}})_G = \frac{140\,924(1.0685)(1)}{1(1)(1.118)} = 134\,685 \text{ psi}$$

$$W^t = \left(\frac{134\,685}{2290}\right)^2 \left[\frac{1.25(3.333)(0.086)}{1(1.374)(1.106)(0.593\,75)(2)}\right] = 686.8 \text{ lbf}$$

The rating load would be

$$W^t_{\text{rated}} = \min(689.7, 712.5, 685.8, 686.8) = 685.8 \text{ lbf}$$

which is slightly less than intended.

Pinion core

$$(s_{at})_P = 15\,300 \text{ psi} \qquad \text{(as before)}$$

$$(\sigma_{\text{all}})_P = 10\,551 \qquad \text{(as before)}$$

$$W^t = 689.7 \qquad \text{(as before)}$$

Gear core

$$(s_{at})_G = 44(339) + 2100 = 17\,016 \text{ psi}$$

$$(\sigma_{\text{all}})_G = \frac{17\,016(0.893)}{1(1)(1.25)} = 12\,156 \text{ psi}$$

$$W^t = \frac{12\,156(1.25)(0.216)}{6(1)(1.374)(0.5222)(1.106)} = 689.3 \text{ lbf}$$

Pinion case

$$(s_{ac})_P = 341(373) + 23\,620 = 150\,813 \text{ psi}$$

$$(\sigma_{c,\text{all}})_P = \frac{150\,813(1)}{1(1)(1.118)} = 134\,895 \text{ psi}$$

$$W^t = \left(\frac{134\,895}{2290}\right)^2 \left[\frac{1.25(3.333)(0.086)}{1(1.374)(1.106)(0.593\,75)(2)}\right] = 689.0 \text{ lbf}$$

Gear case

$$(s_{ac})_G = 341(345) + 23\,620 = 141\,265 \text{ psi}$$

$$(\sigma_{c,\text{all}})_G = \frac{141\,265(1.0685)(1)}{1(1)(1.118)} = 135\,010 \text{ psi}$$

$$W^t = \left(\frac{135\,010}{2290}\right)^2 \left[\frac{1.25(3.333)(0.086)}{1(1.1374)(1.106)(0.593\,75)(2)}\right] = 690.1 \text{ lbf}$$

The equations developed within Prob. 15-7 are effective.

15-10 The catalog rating is 5.2 hp at 1200 rev/min for a straight bevel gearset. Also given: $N_P = 20$ teeth, $N_G = 40$ teeth, $\phi_n = 20^\circ$, $F = 0.71$ in, $J_P = 0.241$, $J_G = 0.201$, $P_d = 10$ teeth/in, through-hardened to 300 Brinell-General Industrial Service, and $Q_v = 5$ uncrowned.

Mesh

$$d_P = 20/10 = 2.000 \text{ in}, \quad d_G = 40/10 = 4.000 \text{ in}$$

$$v_t = \frac{\pi d_P n_P}{12} = \frac{\pi(2)(1200)}{12} = 628.3 \text{ ft/min}$$

$$K_o = 1, \quad S_F = 1, \quad S_H = 1$$

Eq. (15-6): $B = 0.25(12 - 5)^{2/3} = 0.9148$

$A = 50 + 56(1 - 0.9148) = 54.77$

Eq. (15-5): $$K_v = \left(\frac{54.77 + \sqrt{628.3}}{54.77}\right)^{0.9148} = 1.412$$

Eq. (15-10): $K_s = 0.4867 + 0.2132/10 = 0.508$

Eq. (15-11): $K_m = 1.25 + 0.0036(0.71)^2 = 1.252$

where $K_{mb} = 1.25$

Eq. (15-15): $(K_L)_P = 1.6831(10^9)^{-0.0323} = 0.862$

$(K_L)_G = 1.6831(10^9/2)^{-0.0323} = 0.881$

Eq. (15-14): $(C_L)_P = 3.4822(10^9)^{-0.0602} = 1.000$

$(C_L)_G = 3.4822(10^9/2)^{-0.0602} = 1.043$

Analyze for 10^9 pinion cycles at 0.999 reliability

Eq. (15-19):
$$K_R = 0.50 - 0.25\log(1 - 0.999) = 1.25$$
$$C_R = \sqrt{K_R} = \sqrt{1.25} = 1.118$$

Bending
Pinion:

Eq. (15-23):
$$(s_{at})_P = 44(300) + 2100 = 15\,300 \text{ psi}$$

Eq. (15-4):
$$(s_{wt})_P = \frac{15\,300(0.862)}{1(1)(1.25)} = 10\,551 \text{ psi}$$

Eq. (15-3):
$$W^t = \frac{(s_{wt})_P F K_x J_P}{P_d K_o K_v K_s K_m}$$
$$= \frac{10\,551(0.71)(1)(0.241)}{10(1)(1.412)(0.508)(1.252)} = 201 \text{ lbf}$$
$$H_1 = \frac{201(628.3)}{33\,000} = 3.8 \text{ hp}$$

Gear:
$$(s_{at})_G = 15\,300 \text{ psi}$$

Eq. (15-4):
$$(s_{wt})_G = \frac{15\,300(0.881)}{1(1)(1.25)} = 10\,783 \text{ psi}$$

Eq. (15-3):
$$W^t = \frac{10\,783(0.71)(1)(0.201)}{10(1)(1.412)(0.508)(1.252)} = 171.4 \text{ lbf}$$
$$H_2 = \frac{171.4(628.3)}{33\,000} = 3.3 \text{ hp}$$

Wear
Pinion:
$$(C_H)_G = 1, \quad I = 0.078, \quad C_p = 2290\sqrt{\text{psi}}, \quad C_{xc} = 2$$
$$C_s = 0.125(0.71) + 0.4375 = 0.526\,25$$

Eq. (15-22):
$$(s_{ac})_P = 341(300) + 23\,620 = 125\,920 \text{ psi}$$
$$(\sigma_{c,\text{all}})_P = \frac{125\,920(1)(1)}{1(1)(1.118)} = 112\,630 \text{ psi}$$

Eq. (15-1):
$$W^t = \left[\frac{(\sigma_{c,\text{all}})_P}{C_p}\right]^2 \frac{F d_P I}{K_o K_v K_m C_s C_{xc}}$$
$$= \left(\frac{112\,630}{2290}\right)^2 \left[\frac{0.71(2.000)(0.078)}{1(1.412)(1.252)(0.526\,25)(2)}\right]$$
$$= 144.0 \text{ lbf}$$
$$H_3 = \frac{144(628.3)}{33\,000} = 2.7 \text{ hp}$$

Gear:

$$(s_{ac})_G = 125\,920 \text{ psi}$$

$$(\sigma_{c,\text{all}}) = \frac{125\,920(1.043)(1)}{1(1)(1.118)} = 117\,473 \text{ psi}$$

$$W^t = \left(\frac{117\,473}{2290}\right)^2 \left[\frac{0.71(2.000)(0.078)}{1(1.412)(1.252)(0.526\,25)(2)}\right] = 156.6 \text{ lbf}$$

$$H_4 = \frac{156.6(628.3)}{33\,000} = 3.0 \text{ hp}$$

Rating:

$$H = \min(3.8, 3.3, 2.7, 3.0) = 2.7 \text{ hp}$$

Pinion wear controls the power rating. While the basis of the catalog rating is unknown, it is overly optimistic (by a factor of 1.9).

15-11 From Ex. 15-1, the core hardness of both the pinion and gear is 180 Brinell. So $(H_B)_{11}$ and $(H_B)_{21}$ are 180 Brinell and the bending stress numbers are:

$$(s_{at})_P = 44(180) + 2100 = 10\,020 \text{ psi}$$

$$(s_{at})_G = 10\,020 \text{ psi}$$

The contact strength of the gear case, based upon the equation derived in Prob. 15-7, is

$$(s_{ac})_G = \frac{C_p}{(C_L)_G C_H}\sqrt{\frac{S_H^2}{S_F}\left(\frac{(s_{at})_P (K_L)_P K_x J_P K_T C_s C_{xc}}{N_P I K_s}\right)}$$

Substituting $(s_{at})_P$ from above and the values of the remaining terms from Ex. 15-1,

$$\frac{2290}{1.32(1)}\sqrt{\frac{1.5^2}{1.5}\left(\frac{10\,020(1)(1)(0.216)(1)(0.575)(2)}{25(0.065)(0.529)}\right)} = 114\,331 \text{ psi}$$

$$(H_B)_{22} = \frac{114\,331 - 23\,620}{341} = 266 \text{ Brinell}$$

The pinion contact strength is found using the relation from Prob. 15-7:

$$(s_{ac})_P = (s_{ac})_G m_G^{0.0602} C_H = 114\,331(1)^{0.0602}(1) = 114\,331 \text{ psi}$$

$$(H_B)_{12} = \frac{114\,331 - 23\,600}{341} = 266 \text{ Brinell}$$

	Core	Case
Pinion	180	266
Gear	180	266

Realization of hardnesses

The response of students to this part of the question would be a function of the extent to which heat-treatment procedures were covered in their materials and manufacturing prerequisites, and how quantitative it was. The most important thing is to have the student think about it.

The instructor can comment in class when students curiosity is heightened. Options that will surface may include:

- Select a through-hardening steel which will meet or exceed core hardness in the hot-rolled condition, then heat-treating to gain the additional 86 points of Brinell hardness by bath-quenching, then tempering, then generating the teeth in the blank.
- Flame or induction hardening are possibilities.
- The hardness goal for the case is sufficiently modest that carburizing and case hardening may be too costly. In this case the material selection will be different.
- The initial step in a nitriding process brings the core hardness to 33–38 Rockwell C-scale (about 300–350 Brinell) which is too much.

Emphasize that development procedures are necessary in order to tune the "Black Art" to the occasion. Manufacturing personnel know what to do and the direction of adjustments, but how much is obtained by asking the gear (or gear blank). Refer your students to D. W. Dudley, *Gear Handbook,* library reference section, for descriptions of heat-treating processes.

15-12 Computer programs will vary.

15-13 A design program would ask the user to make the a priori decisions, as indicated in Sec. 15-5, p. 786, SMED8. The decision set can be organized as follows:

A priori decisions

- Function: H, K_o, rpm, m_G, temp., N_L, R
- Design factor: n_d $(S_F = n_d,\ S_H = \sqrt{n_d})$
- Tooth system: Involute, Straight Teeth, Crowning, ϕ_n
- Straddling: K_{mb}
- Tooth count: $N_P(N_G = m_G N_P)$

Design decisions

- Pitch and Face: P_d, F
- Quality number: Q_v
- Pinion hardness: $(H_B)_1$, $(H_B)_3$
- Gear hardness: $(H_B)_2$, $(H_B)_4$

First gather all of the equations one needs, then arrange them before coding. Find the required hardnesses, express the consequences of the chosen hardnesses, and allow for revisions as appropriate.

	Pinion Bending	Gear Bending	Pinion Wear	Gear Wear
Load-induced stress (Allowable stress)	$s_t = \dfrac{W^t P K_o K_v K_m K_s}{F K_x J_P} = s_{11}$	$s_t = \dfrac{W^t P K_o K_v K_m K_s}{F K_x J_G} = s_{21}$	$\sigma_c = C_p \left(\dfrac{W^t K_o K_v C_s C_{xc}}{F d_P I} \right)^{1/2} = s_{12}$	$s_{22} = s_{12}$
Tabulated strength	$(s_{at})_P = \dfrac{s_{11} S_F K_T K_R}{(K_L)_P}$	$(s_{at})_G = \dfrac{s_{21} S_F K_T K_R}{(K_L)_G}$	$(s_{ac})_P = \dfrac{s_{12} S_H K_T C_R}{(C_L)_P (C_H)_P}$	$(s_{ac})_G = \dfrac{s_{22} S_H K_T C_R}{(C_L)_G (C_H)_G}$
Associated hardness	$\text{Bhn} = \begin{cases} \dfrac{(s_{at})_P - 2100}{44} \\ \dfrac{(s_{at})_P - 5980}{48} \end{cases}$	$\text{Bhn} = \begin{cases} \dfrac{(s_{at})_G - 2100}{44} \\ \dfrac{(s_{at})_G - 5980}{48} \end{cases}$	$\text{Bhn} = \begin{cases} \dfrac{(s_{ac})_P - 23\,620}{341} \\ \dfrac{(s_{ac})_P - 29\,560}{363.6} \end{cases}$	$\text{Bhn} = \begin{cases} \dfrac{(s_{ac})_G - 23\,620}{341} \\ \dfrac{(s_{ac})_G - 29\,560}{363.6} \end{cases}$
Chosen hardness	$(H_B)_{11}$	$(H_B)_{21}$	$(H_B)_{12}$	$(H_B)_{22}$
New tabulated strength	$(s_{at1})_P = \begin{cases} 44(H_B)_{11} + 2100 \\ 48(H_B)_{11} + 5980 \end{cases}$	$(s_{at1})_G = \begin{cases} 44(H_B)_{21} + 2100 \\ 48(H_B)_{21} + 5980 \end{cases}$	$(s_{ac1})_P = \begin{cases} 341(H_B)_{12} + 23\,620 \\ 363.6(H_B)_{12} + 29\,560 \end{cases}$	$(s_{ac1})_G = \begin{cases} 341(H_B)_{22} + 23\,620 \\ 363.6(H_B)_{22} + 29\,560 \end{cases}$
Factor of safety	$n_{11} = \dfrac{\sigma_{\text{all}}}{\sigma} = \dfrac{(s_{at1})_P (K_L)_P}{s_{11} K_T K_R}$	$n_{21} = \dfrac{(s_{at1})_G (K_L)_G}{s_{21} K_T K_R}$	$n_{12} = \left[\dfrac{(s_{ac1})_P (C_L)_P (C_H)_P}{s_{12} K_T C_R} \right]^2$	$n_{22} = \left[\dfrac{(s_{ac1})_G (C_L)_G (C_H)_G}{s_{22} K_T C_R} \right]^2$

Note: $S_F = n_d,\ S_H = \sqrt{S_F}$

15-14 $N_W = 1,\ N_G = 56,\ P_t = 8$ teeth/in, $d = 1.5$ in, $H_o = 1$hp, $\phi_n = 20^\circ$, $t_a = 70^\circ$F, $K_a = 1.25, \quad n_d = 1, \quad F_e = 2$ in, $\quad A = 850$ in^2

(a)

$$m_G = N_G/N_W = 56, \quad D = N_G/P_t = 56/8 = 7.0 \text{ in}$$

$$p_x = \pi/8 = 0.3927 \text{ in}, \quad C = 1.5 + 7 = 8.5 \text{ in}$$

Eq. (15-39): $a = p_x/\pi = 0.3927/\pi = 0.125$ in

Eq. (15-40): $b = 0.3683 p_x = 0.1446$ in

Eq. (15-41): $h_t = 0.6866 p_x = 0.2696$ in

Eq. (15-42): $d_o = 1.5 + 2(0.125) = 1.75$ in

Eq. (15-43): $d_r = 3 - 2(0.1446) = 2.711$ in

Eq. (15-44): $D_t = 7 + 2(0.125) = 7.25$ in

Eq. (15-45): $D_r = 7 - 2(0.1446) = 6.711$ in

Eq. (15-46): $c = 0.1446 - 0.125 = 0.0196$ in

Eq. (15-47): $(F_W)_{\max} = 2\sqrt{2(7)0.125} = 2.646$ in

$$V_W = \pi(1.5)(1725/12) = 677.4 \text{ ft/min}$$

$$V_G = \frac{\pi(7)(1725/56)}{12} = 56.45 \text{ ft/min}$$

Eq. (13-28): $L = p_x N_W = 0.3927$ in, $\quad \lambda = \tan^{-1}\left(\dfrac{0.3927}{\pi(1.5)}\right) = 4.764^\circ$

$$P_n = \frac{P_t}{\cos\lambda} = \frac{8}{\cos 4.764^\circ} = 8.028$$

$$p_n = \frac{\pi}{P_n} = 0.3913 \text{ in}$$

Eq. (15-62): $V_s = \dfrac{\pi(1.5)(1725)}{12\cos 4.764^\circ} = 679.8$ ft/min

(b) Eq. (15-38): $f = 0.103 \exp\left[-0.110(679.8)^{0.450}\right] + 0.012 = 0.0250$

Eq. (15-54): The efficiency is,

$$e = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda} = \frac{\cos 20^\circ - 0.0250\tan 4.764^\circ}{\cos 20^\circ + 0.0250\cot 4.764^\circ} = 0.7563 \quad \textit{Ans.}$$

Eq. (15-58): $W_G^t = \dfrac{33\,000\, n_d H_o K_a}{V_G e} = \dfrac{33\,000(1)(1)(1.25)}{56.45(0.7563)} = 966$ lbf $\quad$ *Ans.*

Eq. (15-57): $W_W^t = W_G^t\left(\dfrac{\cos\phi_n\sin\lambda + f\cos\lambda}{\cos\phi_n\cos\lambda - f\sin\lambda}\right)$

$$= 966\left(\frac{\cos 20^\circ \sin 4.764^\circ + 0.025\cos 4.764^\circ}{\cos 20^\circ\cos 4.764^\circ - 0.025\sin 4.764^\circ}\right) = 106.4 \text{ lbf} \quad \textit{Ans.}$$

(c) Eq. (15-33): $C_s = 1190 - 477 \log 7.0 = 787$

Eq. (15-36): $C_m = 0.0107\sqrt{-56^2 + 56(56) + 5145} = 0.767$

Eq. (15-37): $C_v = 0.659 \exp[-0.0011(679.8)] = 0.312$

Eq. (15-38): $(W^t)_{\text{all}} = 787(7)^{0.8}(2)(0.767)(0.312) = 1787 \text{ lbf}$

Since $W_G^t < (W^t)_{\text{all}}$, the mesh will survive at least 25 000 h.

Eq. (15-61): $W_f = \dfrac{0.025(966)}{0.025 \sin 4.764° - \cos 20° \cos 4.764°} = -29.5 \text{ lbf}$

Eq. (15-63): $H_f = \dfrac{29.5(679.8)}{33\,000} = 0.608 \text{ hp}$

$$H_W = \frac{106.4(677.4)}{33\,000} = 2.18 \text{ hp}$$

$$H_G = \frac{966(56.45)}{33\,000} = 1.65 \text{ hp}$$

The mesh is sufficient *Ans.*

$$P_n = P_t/\cos\lambda = 8/\cos 4.764° = 8.028$$

$$p_n = \pi/8.028 = 0.3913 \text{ in}$$

$$\sigma_G = \frac{966}{0.3913(0.5)(0.125)} = 39\,500 \text{ psi}$$

The stress is high. At the rated horsepower,

$$\sigma_G = \frac{1}{1.65} 39\,500 = 23\,940 \text{ psi} \quad \text{acceptable}$$

(d) Eq. (15-52): $A_{\min} = 43.2(8.5)^{1.7} = 1642 \text{ in}^2 < 1700 \text{ in}^2$

Eq. (15-49): $H_{\text{loss}} = 33\,000(1 - 0.7563)(2.18) = 17\,530 \text{ ft} \cdot \text{lbf/min}$

Assuming a fan exists on the worm shaft,

Eq. (15-50): $\hbar_{CR} = \dfrac{1725}{3939} + 0.13 = 0.568 \text{ ft} \cdot \text{lbf/(min} \cdot \text{in}^2 \cdot °\text{F)}$

Eq. (15-51): $t_s = 70 + \dfrac{17\,530}{0.568(1700)} = 88.2°\text{F}$ *Ans.*

15-15 to 15-22

Problem statement values of 25 hp, 1125 rev/min, $m_G = 10$, $K_a = 1.25$, $n_d = 1.1$, $\phi_n = 20°$, $t_a = 70°\text{F}$ are not referenced in the table.

Parameters Selected		15-15	15-16	15-17	15-18	15-19	15-20	15-21	15-22
#1	p_x	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75
#2	d_W	3.60	3.60	3.60	3.60	3.60	4.10	3.60	3.60
#3	F_G	2.40	1.68	1.43	1.69	2.40	2.25	2.4	2.4
#4	A	2000	2000	2000	2000	2000	2000	2500	2600
								FAN	FAN
	H_W	38.2	38.2	38.2	38.2	38.2	38.0	41.2	41.2
	H_G	36.2	36.2	36.2	36.2	36.2	36.1	37.7	37.7
	H_f	1.87	1.47	1.97	1.97	1.97	1.85	3.59	3.59
	N_W	3	3	3	3	3	3	3	3
	N_G	30	30	30	30	30	30	30	30
	K_W				125	80	50	115	185
	C_s	607	854	1000					
	C_m	0.759	0.759	0.759					
	C_v	0.236	0.236	0.236					
	V_G	492	492	492	492	492	563	492	492
	W_G^t	2430	2430	2430	2430	2430	2120	2524	2524
	W_W^t	1189	1189	1189	1189	1189	1038	1284	1284
	f	0.0193	0.0193	0.0193	0.0193	0.0193	0.0183	0.034	0.034
	e	0.948	0.948	0.948	0.948	0.948	0.951	0.913	0.913
	$(P_t)_G$	1.795	1.795	1.795	1.795	1.795	1.571	1.795	1.795
	P_n	1.979	1.979	1.979	1.979	1.979	1.732	1.979	1.979
	C-to-C	10.156	10.156	10.156	10.156	10.156	11.6	10.156	10.156
	t_s	177	177	177	177	177	171	179.6	179.6
	L	5.25	5.25	5.25	5.25	5.25	6.0	5.25	5.25
	λ	24.9	24.9	24.9	24.9	24.9	24.98	24.9	24.9
	σ_G	5103	7290	8565	7247	5103	4158	5301	5301
	d_G	16.71	16.71	16.71	16.71	16.71	19.099	16.7	16.71

Chapter 16

16-1

(a) $\theta_1 = 0^\circ, \quad \theta_2 = 120^\circ, \quad \theta_a = 90^\circ, \quad \sin\theta_a = 1, \quad a = 5 \text{ in}$

$$\text{Eq. (16-2):} \quad M_f = \frac{0.28 p_a (1.5)(6)}{1} \int_{0^\circ}^{120^\circ} \sin\theta(6 - 5\cos\theta)\, d\theta$$

$$= 17.96 p_a \text{ lbf} \cdot \text{in}$$

$$\text{Eq. (16-3):} \quad M_N = \frac{p_a(1.5)(6)(5)}{1} \int_{0^\circ}^{120^\circ} \sin^2\theta\, d\theta = 56.87 p_a \text{ lbf} \cdot \text{in}$$

$$c = 2(5\cos 30^\circ) = 8.66 \text{ in}$$

$$\text{Eq. (16-4):} \quad F = \frac{56.87 p_a - 17.96 p_a}{8.66} = 4.49 p_a$$

$$p_a = F/4.49 = 500/4.49 = 111.4 \text{ psi} \quad \text{for cw rotation}$$

$$\text{Eq. (16-7):} \quad 500 = \frac{56.87 p_a + 17.96 p_a}{8.66}$$

$$p_a = 57.9 \text{ psi for ccw rotation}$$

A maximum pressure of 111.4 psioccurs on the RH shoe for cw rotation. *Ans.*

(b) *RH shoe*:

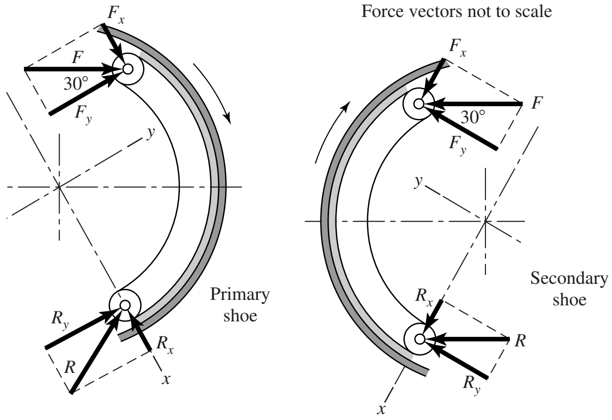
$$\text{Eq. (16-6):} \quad T_R = \frac{0.28(111.4)(1.5)(6)^2(\cos 0^\circ - \cos 120^\circ)}{1} = 2530 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

LH shoe:

$$\text{Eq. (16-6):} \quad T_L = \frac{0.28(57.9)(1.5)(6)^2(\cos 0^\circ - \cos 120^\circ)}{1} = 1310 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$T_{\text{total}} = 2530 + 1310 = 3840 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

(c)



RH shoe: $F_x = 500\sin 30° = 250\text{ lbf},\quad F_y = 500\cos 30° = 433\text{ lbf}$

Eqs. (16-8): $$A = \left(\frac{1}{2}\sin^2\theta\right)_{0°}^{120°} = 0.375,\quad B = \left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)_0^{2\pi/3\text{ rad}} = 1.264$$

Eqs. (16-9): $$R_x = \frac{111.4(1.5)(6)}{1}[0.375 - 0.28(1.264)] - 250 = -229\text{ lbf}$$

$$R_y = \frac{111.4(1.5)(6)}{1}[1.264 + 0.28(0.375)] - 433 = 940\text{ lbf}$$

$$R = [(-229)^2 + (940)^2]^{1/2} = 967\text{ lbf}\quad \textit{Ans.}$$

LH shoe: $F_x = 250\text{ lbf},\quad F_y = 433\text{ lbf}$

Eqs. (16-10): $$R_x = \frac{57.9(1.5)(6)}{1}[0.375 + 0.28(1.264)] - 250 = 130\text{ lbf}$$

$$R_y = \frac{57.9(1.5)(6)}{1}[1.264 - 0.28(0.375)] - 433 = 171\text{ lbf}$$

$$R = [(130)^2 + (171)^2]^{1/2} = 215\text{ lbf}\quad \textit{Ans.}$$

16-2 $\theta_1 = 15°,\quad \theta_2 = 105°,\quad \theta_a = 90°,\quad \sin\theta_a = 1,\quad a = 5\text{ in}$

Eq. (16-2): $$M_f = \frac{0.28p_a(1.5)(6)}{1}\int_{15°}^{105°}\sin\theta(6 - 5\cos\theta)\,d\theta = 13.06p_a$$

Eq. (16-3): $$M_N = \frac{p_a(1.5)(6)(5)}{1}\int_{15°}^{105°}\sin^2\theta\,d\theta = 46.59p_a$$

$$c = 2(5\cos 30°) = 8.66\text{ in}$$

Eq. (16-4): $$F = \frac{46.59p_a - 13.06p_a}{8.66} = 3.872p_a$$

RH shoe:

$$p_a = 500/3.872 = 129.1\text{ psi}\quad\text{on RH shoe for cw rotation}\quad \textit{Ans.}$$

Eq. (16-6): $$T_R = \frac{0.28(129.1)(1.5)(6^2)(\cos 15° - \cos 105°)}{1} = 2391\text{ lbf}\cdot\text{in}$$

LH shoe:

$$500 = \frac{46.59p_a + 13.06p_a}{8.66}\quad \Rightarrow\quad p_a = 72.59\text{ psi on LH shoe for ccw rotation}\quad \textit{Ans.}$$

$$T_L = \frac{0.28(72.59)(1.5)(6^2)(\cos 15° - \cos 105°)}{1} = 1344\text{ lbf}\cdot\text{in}$$

$$T_{\text{total}} = 2391 + 1344 = 3735\text{ lbf}\cdot\text{in}\quad \textit{Ans.}$$

Comparing this result with that of Prob. 16-1, a 2.7% reduction in torque is achieved by using 25% less braking material.

16-3 Given: $\theta_1 = 0°$, $\theta_2 = 120°$, $\theta_a = 90°$, $\sin\theta_a = 1$, $a = R = 90$ mm, $f = 0.30$, $F = 1000$ N $= 1$ kN, $r = 280/2 = 140$ mm, counter-clockwise rotation.

LH shoe:

$$M_f = \frac{f p_a b r}{\sin\theta_a}\left[r(1-\cos\theta_2) - \frac{a}{2}\sin^2\theta_2\right]$$

$$= \frac{0.30 p_a(0.030)(0.140)}{1}\left[0.140(1-\cos 120°) - \frac{0.090}{2}\sin^2 120°\right]$$

$$= 0.000\,222 p_a \text{ N} \cdot \text{m}$$

$$M_N = \frac{p_a b r a}{\sin\theta_a}\left[\frac{\theta_2}{2} - \frac{1}{4}\sin 2\theta_2\right]$$

$$= \frac{p_a(0.030)(0.140)(0.090)}{1}\left[\frac{120°}{2}\left(\frac{\pi}{180}\right) - \frac{1}{4}\sin 2(120°)\right]$$

$$= 4.777(10^{-4}) p_a \text{ N} \cdot \text{m}$$

$$c = 2r\cos\left(\frac{180° - \theta_2}{2}\right) = 2(0.090)\cos 30° = 0.155\,88 \text{ m}$$

$$F = 1 = p_a\left[\frac{4.777(10^{-4}) - 2.22(10^{-4})}{0.155\,88}\right] = 1.64(10^{-3}) p_a$$

$$p_a = 1/1.64(10^{-3}) = 610 \text{ kPa}$$

$$T_L = \frac{f p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

$$= \frac{0.30(610)(10^3)(0.030)(0.140^2)}{1}[1-(-0.5)]$$

$$= 161.4 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

RH shoe:

$$M_f = 2.22(10^{-4}) p_a \text{ N} \cdot \text{m}$$

$$M_N = 4.77(10^{-4}) p_a \text{ N} \cdot \text{m}$$

$$c = 0.155\,88 \text{ m}$$

$$F = 1 = p_a\left[\frac{4.77(10^{-4}) + 2.22(10^{-4})}{0.155\,88}\right] = 4.49(10^{-3}) p_a$$

$$p_a = \frac{1}{4.49(10^{-3})} = 222.8 \text{ kPa} \quad \textit{Ans.}$$

$$T_R = (222.8/610)(161.4) = 59.0 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

16-4

(a) Given: $\theta_1 = 10°$, $\theta_2 = 75°$, $\theta_a = 75°$, $p_a = 10^6$ Pa, $f = 0.24$, $b = 0.075$ m (shoe width), a = 0.150 m, r = 0.200 m, d = 0.050 m, c = 0.165 m.

Some of the terms needed are evaluated as:

$$A = \left[r\int_{\theta_1}^{\theta_2} \sin\theta\, d\theta - a\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\, d\theta\right] = r\big[-\cos\theta\big]_{\theta_1}^{\theta_2} - a\left[\frac{1}{2}\sin^2\theta\right]_{\theta_1}^{\theta_2}$$

$$= 200\big[-\cos\theta\big]_{10°}^{75°} - 150\left[\frac{1}{2}\sin^2\theta\right]_{10°}^{75°} = 77.5 \text{ mm}$$

$$B = \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta = \left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_{10\pi/180 \text{ rad}}^{75\pi/180 \text{ rad}} = 0.528$$

$$C = \int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\, d\theta = 0.4514$$

Now converting to pascals and meters, we have from Eq. (16-2),

$$M_f = \frac{f p_a b r}{\sin\theta_a} A = \frac{0.24[(10)^6](0.075)(0.200)}{\sin 75°}(0.0775) = 289 \text{ N}\cdot\text{m}$$

From Eq. (16-3),

$$M_N = \frac{p_a b r a}{\sin\theta_a} B = \frac{[(10)^6](0.075)(0.200)(0.150)}{\sin 75°}(0.528) = 1230 \text{ N}\cdot\text{m}$$

Finally, using Eq. (16-4), we have

$$F = \frac{M_N - M_f}{c} = \frac{1230 - 289}{165} = 5.70 \text{ kN} \quad \textit{Ans.}$$

(b) Use Eq. (16-6) for the primary shoe.

$$T = \frac{f p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

$$= \frac{0.24[(10)^6](0.075)(0.200)^2(\cos 10° - \cos 75°)}{\sin 75°} = 541 \text{ N}\cdot\text{m}$$

For the secondary shoe, we must first find p_a.

Substituting

$$M_N = \frac{1230}{10^6} p_a \quad \text{and} \quad M_f = \frac{289}{10^6} p_a \quad \text{into Eq. (16-7),}$$

$$5.70 = \frac{(1230/10^6)p_a + (289/10^6)p_a}{165}, \quad \text{solving gives} \quad p_a = 619(10)^3 \text{ Pa}$$

Then

$$T = \frac{0.24[0.619(10)^6](0.075)(0.200)^2(\cos 10° - \cos 75°)}{\sin 75°} = 335 \text{ N}\cdot\text{m}$$

so the braking capacity is $T_{\text{total}} = 2(541) + 2(335) = 1750 \text{ N}\cdot\text{m}$ *Ans.*

(c) *Primary shoes*:

$$R_x = \frac{p_a br}{\sin\theta_a}(C - fB) - F_x$$

$$= \frac{(10^6)(0.075)(0.200)}{\sin 75^\circ}[0.4514 - 0.24(0.528)](10)^{-3} - 5.70 = -0.658 \text{ kN}$$

$$R_y = \frac{p_a br}{\sin\theta_a}(B + fC) - F_y$$

$$= \frac{(10^6)(0.075)(0.200)}{\sin 75^\circ}[0.528 + 0.24(0.4514)](10)^{-3} - 0 = 9.88 \text{ kN}$$

Secondary shoes:

$$R_x = \frac{p_a br}{\sin\theta_a}(C + fB) - F_x$$

$$= \frac{[0.619(10)^6](0.075)(0.200)}{\sin 75^\circ}[0.4514 + 0.24(0.528)](10)^{-3} - 5.70$$

$$= -0.143 \text{ kN}$$

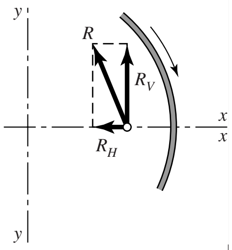
$$R_y = \frac{p_a br}{\sin\theta_a}(B - fC) - F_y$$

$$= \frac{[0.619(10)^6](0.075)(0.200)}{\sin 75^\circ}[0.528 - 0.24(0.4514)](10)^{-3} - 0$$

$$= 4.03 \text{ kN}$$

Note from figure that $+y$ for secondary shoe is opposite to $+y$ for primary shoe.

Combining horizontal and vertical components,



$$R_H = -0.658 - 0.143 = -0.801 \text{ kN}$$

$$R_V = 9.88 - 4.03 = 5.85 \text{ kN}$$

$$R = \sqrt{(0.801)^2 + (5.85)^2}$$

$$= 5.90 \text{ kN} \quad \textit{Ans.}$$

16-5 Preliminaries: $\theta_1 = 45^\circ - \tan^{-1}(150/200) = 8.13^\circ$, $\theta_2 = 98.13^\circ$

$$\theta_a = 90^\circ, \quad a = [(150)^2 + (200)^2]^{1/2} = 250 \text{ mm}$$

Eq. (16-8):

$$A = \frac{1}{2}\left(\sin^2\theta\right)_{8.13^\circ}^{98.13^\circ} = 0.480$$

Let

$$C = \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = -\left(\cos\theta\right)_{8.13^\circ}^{98.13^\circ} = 1.1314$$

Eq. (16-2):

$$M_f = \frac{f p_a b r}{\sin\theta_a}(rC - aA) = \frac{0.25 p_a(0.030)(0.150)}{\sin 90^\circ}[0.15(1.1314) - 0.25(0.48)]$$

$$= 5.59(10^{-5}) p_a \ \text{N}\cdot\text{m}$$

Eq. (16-8):
$$B = \left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)_{8.13\pi/180 \text{ rad}}^{98.13\pi/180 \text{ rad}} = 0.925$$

Eq. (16-3):
$$M_N = \frac{p_a b r a}{\sin\theta_a} B = \frac{p_a(0.030)(0.150)(0.250)}{1}(0.925)$$

$$= 1.0406(10^{-3}) p_a \ \text{N}\cdot\text{m}$$

Using $F = (M_N - M_f)/c$, we obtain

$$400 = \frac{104.06 - 5.59}{0.5(10^5)} p_a \quad \text{or} \quad p_a = 203 \text{ kPa} \quad \textit{Ans.}$$

$$T = \frac{f p_a b r^2 \text{C}}{\sin\theta_a} = \frac{0.25(203)(10^3)(0.030)(0.150)^2}{1}(1.1314)$$

$$= 38.76 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

16-6 For $+3\hat{\sigma}_f$:

$$f = \bar{f} + 3\hat{\sigma}_f = 0.25 + 3(0.025) = 0.325$$

$$M_f = 5.59(10^{-5}) p_a \left(\frac{0.325}{0.25}\right) = 7.267(10^{-5}) p_a$$

Eq. (16-4):

$$400 = \frac{104.06 - 7.267}{10^5(0.500)} p_a$$

$$p_a = 207 \text{ kPa}$$

$$T = 38.75\left(\frac{207}{203}\right)\left(\frac{0.325}{0.25}\right) = 51.4 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

Similarly, for $-3\hat{\sigma}_f$:

$$f = \bar{f} - 3\hat{\sigma}_f = 0.25 - 3(0.025) = 0.175$$

$$M_f = 3.913(10^{-5}) p_a$$

$$p_a = 200 \text{ kPa}$$

$$T = 26.7 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

16-7 Preliminaries: $\theta_2 = 180^\circ - 30^\circ - \tan^{-1}(3/12) = 136^\circ$, $\theta_1 = 20^\circ - \tan^{-1}(3/12) = 6^\circ$, $\theta_a = 90^\circ$, $a = [(3)^2 + (12)^2]^{1/2} = 12.37$ in, $r = 10$ in, $f = 0.30$, $b = 2$ in.

Eq. (16-2):
$$M_f = \frac{0.30(150)(2)(10)}{\sin 90^\circ}\int_{6^\circ}^{136^\circ} \sin\theta(10 - 12.37\cos\theta)\, d\theta$$

$$= 12\,800 \text{ lbf}\cdot\text{in}$$

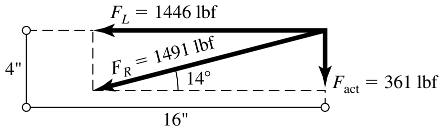
Eq. (16-3): $$M_N = \frac{150(2)(10)(12.37)}{\sin 90^\circ} \int_{6^\circ}^{136^\circ} \sin^2\theta\, d\theta = 53\,300 \text{ lbf} \cdot \text{in}$$

LH shoe:

$$c_L = 12 + 12 + 4 = 28 \text{ in}$$

Now note that M_f is cw and M_N is ccw. Thus,

$$F_L = \frac{53\,300 - 12\,800}{28} = 1446 \text{ lbf}$$



Eq. (16-6): $$T_L = \frac{0.30(150)(2)(10)^2(\cos 6^\circ - \cos 136^\circ)}{\sin 90^\circ} = 15\,420 \text{ lbf} \cdot \text{in}$$

RH shoe:

$$M_N = 53\,300\left(\frac{p_a}{150}\right) = 355.3p_a, \quad M_f = 12\,800\left(\frac{p_a}{150}\right) = 85.3p_a$$

On this shoe, both M_N and M_f are ccw.

Also $$c_R = (24 - 2\tan 14^\circ)\cos 14^\circ = 22.8 \text{ in}$$

$$\text{F}_{act} = F_L \sin 14^\circ = 361 \text{ lbf} \quad \textit{Ans.}$$

$$F_R = F_L / \cos 14^\circ = 1491 \text{ lbf}$$

Thus $$1491 = \frac{355.3 + 85.3}{22.8} p_a \quad \Rightarrow \quad p_a = 77.2 \text{ psi}$$

Then $$T_R = \frac{0.30(77.2)(2)(10)^2(\cos 6^\circ - \cos 136^\circ)}{\sin 90^\circ} = 7940 \text{ lbf} \cdot \text{in}$$

$$T_{\text{total}} = 15\,420 + 7940 = 23\,400 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

16-8

$$M_f = 2\int_0^{\theta_2} (f\,dN)(a'\cos\theta - r) \qquad \text{where } dN = pbr\,d\theta$$

$$= 2fpbr \int_0^{\theta_2} (a'\cos\theta - r)\,d\theta = 0$$

From which

$$a' \int_0^{\theta_2} \cos\theta\,d\theta = r\int_0^{\theta_2} d\theta$$

$$a' = \frac{r\theta_2}{\sin\theta_2} = \frac{r(60^\circ)(\pi/180)}{\sin 60^\circ} = 1.209r$$

Eq. (16-15)

$$a = \frac{4r \sin 60^\circ}{2(60)(\pi/180) + \sin[2(60)]} = 1.170r$$

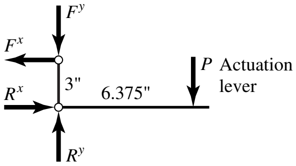
16-9

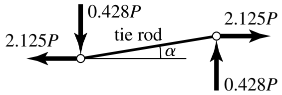
(a) Counter-clockwise rotation, $\theta_2 = \pi/4$ rad, $\quad r = 13.5/2 = 6.75$ in

$$a = \frac{4r\sin\theta_2}{2\theta_2 + \sin 2\theta_2} = \frac{4(6.75)\sin(\pi/4)}{2\pi/4 + \sin(2\pi/4)} = 7.426 \text{ in}$$

$$e = 2(7.426) = 14.85 \text{ in} \quad \textit{Ans.}$$

(b)





$$\alpha = \tan^{-1}(3/14.85) = 11.4^\circ$$

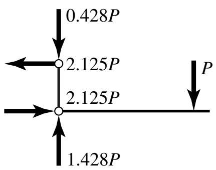
$$\sum M_R = 0 = 3F^x - 6.375P$$

$$F^x = 2.125P$$

$$\sum F_x = 0 = -F^x + R^x$$

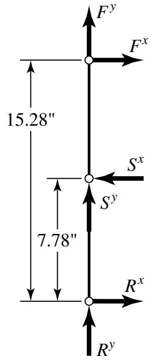
$$R^x = F^x = 2.125P$$

$$F^y = F^x \tan 11.4^\circ = 0.428P$$



$$\sum F_y = -P - F^y + R^y$$

$$R^y = P + 0.428P = 1.428P$$



Left shoe lever.

$$\sum M_R = 0 = 7.78S^x - 15.28F^x$$

$$S^x = \frac{15.28}{7.78}(2.125P) = 4.174P$$

$$S^y = fS^x = 0.30(4.174P)$$

$$= 1.252P$$

$$\sum F_y = 0 = R^y + S^y + F^y$$

$$R^y = -F^y - S^y$$

$$= -0.428P - 1.252P$$

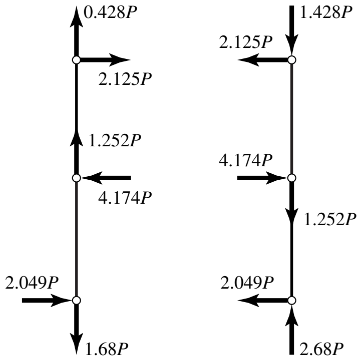
$$= -1.68P$$

$$\sum F_x = 0 = R^x - S^x + F^x$$

$$R^x = S^x - F^x$$

$$= 4.174P - 2.125P$$

$$= 2.049P$$



Left shoe lever   Right shoe lever

Ans.

(c) The direction of brake pulley rotation affects the sense of S^y, which has no effect on the brake shoe lever moment and hence, no effect on S^x or the brake torque.

The brake shoe levers carry identical bending moments but the left lever carries a tension while the right carries compression (column loading). The right lever is designed and used as a left lever, producing interchangeable levers (identical levers). But do not infer from these identical loadings.

16-10 $r = 13.5/2 = 6.75$ in, $\quad b = 7.5$ in, $\quad \theta_2 = 45°$

From Table 16-3 for a rigid, molded nonasbestos use a conservative estimate of $p_a = 100$ psi, $f = 0.31$.

In Eq. (16-16):

$$2\theta_2 + \sin 2\theta_2 = 2(\pi/4) + \sin 2(45°) = 2.571$$

From Prob. 16-9 solution,

$$N = S^x = 4.174P = \frac{p_a br}{2}(2.571) = 1.285 p_a br$$

$$P = \frac{1.285}{4.174}(100)(7.5)(6.75) = 1560 \text{ lbf} \quad Ans.$$

Applying Eq. (16-18) for two shoes,

$$T = 2afN = 2(7.426)(0.31)(4.174)(1560)$$

$$= 29\,980 \text{ lbf} \cdot \text{in} \quad Ans.$$

16-11 From Eq. (16-22),

$$P_1 = \frac{p_a bD}{2} = \frac{90(4)(14)}{2} = 2520 \text{ lbf} \quad Ans.$$

$$f\phi = 0.25(\pi)(270°/180°) = 1.178$$

Eq. (16-19): $P_2 = P_1 \exp(-f\phi) = 2520 \exp(-1.178) = 776 \text{ lbf} \quad Ans.$

$$T = \frac{(P_1 - P_2)D}{2} = \frac{(2520 - 776)14}{2} = 12\,200 \text{ lbf} \cdot \text{in} \quad Ans.$$

16-12 Given: $D = 300$ mm, $f = 0.28$, $b = 80$ mm, $\phi = 270°$, $P_1 = 7600$ N.

$$f\phi = 0.28(\pi)(270°/180°) = 1.319$$

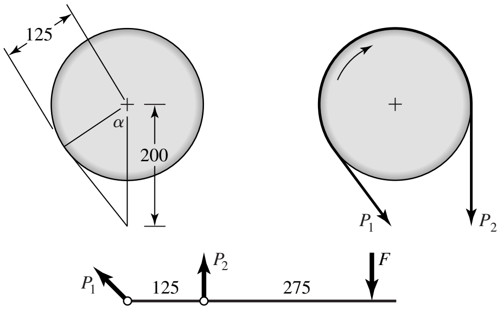
$$P_2 = P_1 \exp(-f\phi) = 7600 \exp(-1.319) = 2032 \text{ N}$$

$$p_a = \frac{2P_1}{bD} = \frac{2(7600)}{80(300)} = 0.6333 \text{ N/mm}^2 \quad \text{or} \quad 633 \text{ kPa} \quad \textit{Ans.}$$

$$T = (P_1 - P_2)\frac{D}{2} = (7600 - 2032)\frac{300}{2}$$

$$= 835\,200 \text{ N} \cdot \text{mm} \quad \text{or} \quad 835.2 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

16-13



$$\alpha = \cos^{-1}\left(\frac{125}{200}\right) = 51.32°$$

$$\phi = 270° - 51.32° = 218.7°$$

$$f\phi = 0.30(218.7)\frac{\pi}{180°} = 1.145$$

$$P_2 = \frac{(125 + 275)F}{125} = \frac{(125 + 275)400}{125} = 1280 \text{ N} \quad \textit{Ans.}$$

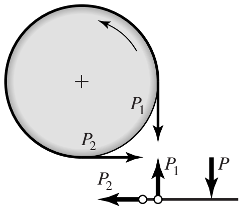
$$P_1 = P_2 \exp(f\phi) = 1280 \exp(1.145) = 4022 \text{ N}$$

$$T = (P_1 - P_2)\frac{D}{2} = (4022 - 1280)\frac{250}{2}$$

$$= 342\,750 \text{ N} \cdot \text{mm} \quad \text{or} \quad 343 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

16-14

(a)



$D = 16"$, $b = 3"$

$n = 200$ rev/min

$f = 0.20$, $p_a = 70$ psi

Eq. (16-22): $$P_1 = \frac{p_a bD}{2} = \frac{70(3)(16)}{2} = 1680 \text{ lbf}$$

$$f\phi = 0.20(3\pi/2) = 0.942$$

Eq. (16-14): $P_2 = P_1 \exp(-f\phi) = 1680\ \exp(-0.942) = 655\text{ lbf}$

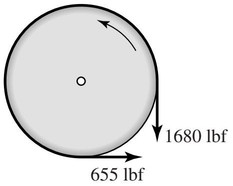
$$T = (P_1 - P_2)\frac{D}{2} = (1680 - 655)\frac{16}{2}$$

$$= 8200\text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$H = \frac{Tn}{63\,025} = \frac{8200(200)}{63\,025} = 26.0\text{ hp} \quad \textit{Ans.}$$

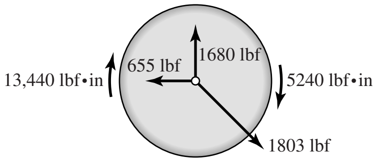
$$P = \frac{3P_1}{10} = \frac{3(1680)}{10} = 504\text{ lbf} \quad \textit{Ans.}$$

(b)



Force of belt on the drum:

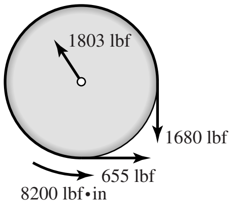
$R = (1680^2 + 655^2)^{1/2} = 1803\text{ lbf}$



Force of shaft on the drum: 1680 and 655 lbf

$T_{P_1} = 1680(8) = 13\,440\text{ lbf}\cdot\text{in}$

$T_{P_2} = 655(8) = 5240\text{ lbf}\cdot\text{in}$



Net torque on drum due to brake band:

$$T = T_{P_1} - T_{P_2}$$
$$= 13\,440 - 5240$$
$$= 8200\text{ lbf}\cdot\text{in}$$

The radial load on the bearing pair is 1803 lbf. If the bearing is straddle mounted with the drum at center span, the bearing radial load is $1803/2 = 901$ lbf.

(c) Eq. (16-22):

$$p = \frac{2P}{bD}$$

$$p|_{\theta=0^\circ} = \frac{2P_1}{3(16)} = \frac{2(1680)}{3(16)} = 70\text{ psi} \quad \textit{Ans.}$$

As it should be

$$p|_{\theta=270^\circ} = \frac{2P_2}{3(16)} = \frac{2(655)}{3(16)} = 27.3\text{ psi} \quad \textit{Ans.}$$

16-15 Given: $\phi = 270^\circ$, $b = 2.125$ in, $f = 0.20$, $T = 150\text{ lbf}\cdot\text{ft}$, $D = 8.25$ in, $c_2 = 2.25$ in

Notice that the pivoting rocker is not located on the vertical centerline of the drum.

(a) To have the band tighten for ccw rotation, it is necessary to have $c_1 < c_2$. When friction is fully developed,

$$\frac{P_1}{P_2} = \exp(f\phi) = \exp[0.2(3\pi/2)] = 2.566$$

If friction is not fully developed

$$P_1/P_2 \leq \exp(f\phi)$$

To help visualize what is going on let's add a force W parallel to P_1, at a lever arm of c_3. Now sum moments about the rocker pivot.

$$\sum M = 0 = c_3 W + c_1 P_1 - c_2 P_2$$

From which

$$W = \frac{c_2 P_2 - c_1 P_1}{c_3}$$

The device is self locking for ccw rotation if W is no longer needed, that is, $W \leq 0$. It follows from the equation above

$$\frac{P_1}{P_2} \geq \frac{c_2}{c_1}$$

When friction is fully developed

$$2.566 = 2.25/c_1$$

$$c_1 = \frac{2.25}{2.566} = 0.877 \text{ in}$$

When P_1/P_2 is less than 2.566, friction is not fully developed. Suppose $P_1/P_2 = 2.25$, then

$$c_1 = \frac{2.25}{2.25} = 1 \text{ in}$$

We don't want to be at the point of slip, and we need the band to tighten.

$$\frac{c_2}{P_1/P_2} \leq c_1 \leq c_2$$

When the developed friction is very small, $P_1/P_2 \rightarrow 1$ and $c_1 \rightarrow c_2$ *Ans.*

(b) Rocker has $c_1 = 1$ in

$$\frac{P_1}{P_2} = \frac{c_2}{c_1} = \frac{2.25}{1} = 2.25$$

$$f = \frac{\ln(P_1/P_2)}{\phi} = \frac{\ln 2.25}{3\pi/2} = 0.172$$

Friction is not fully developed, no slip.

$$T = (P_1 - P_2)\frac{D}{2} = P_2\left(\frac{P_1}{P_2} - 1\right)\frac{D}{2}$$

Solve for P_2

$$P_2 = \frac{2T}{[(P_1/P_2) - 1]D} = \frac{2(150)(12)}{(2.25 - 1)(8.25)} = 349 \text{ lbf}$$

$$P_1 = 2.25P_2 = 2.25(349) = 785 \text{ lbf}$$

$$p = \frac{2P_1}{bD} = \frac{2(785)}{2.125(8.25)} = 89.6 \text{ psi} \quad \textit{Ans.}$$

(c) The torque ratio is $150(12)/100$ or 18-fold.

$$P_2 = \frac{349}{18} = 19.4 \text{ lbf}$$

$$P_1 = 2.25P_2 = 2.25(19.4) = 43.6 \text{ lbf}$$

$$p = \frac{89.6}{18} = 4.98 \text{ psi} \quad \textit{Ans.}$$

Comment:

As the torque opposed by the locked brake increases, P_2 and P_1 increase (although ratio is still 2.25), then p follows. The brake can self-destruct. Protection could be provided by a shear key.

16-16

(a) From Eq. (16-23), since $$F = \frac{\pi p_a d}{2}(D - d)$$

then

$$p_a = \frac{2F}{\pi d(D - d)}$$

and it follows that

$$p_a = \frac{2(5000)}{\pi(225)(300 - 225)}$$

$$= 0.189 \text{ N/mm}^2 \quad \text{or} \quad 189\,000 \text{ N/m}^2 \quad \text{or} \quad 189 \text{ kPa} \quad \textit{Ans.}$$

$$T = \frac{Ff}{4}(D + d) = \frac{5000(0.25)}{4}(300 + 225)$$

$$= 164\,043 \text{ N}\cdot\text{mm} \quad \text{or} \quad 164 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

(b) From Eq. (16-26),

$$F = \frac{\pi p_a}{4}(D^2 - d^2)$$

$$p_a = \frac{4F}{\pi(D^2 - d^2)} = \frac{4(5000)}{\pi(300^2 - 225^2)}$$

$$= 0.162 \text{ N/mm}^2 = 162 \text{ kPa} \quad \textit{Ans.}$$

From Eq. (16-27),

$$T = \frac{\pi}{12} f p_a (D^3 - d^3) = \frac{\pi}{12}(0.25)(162)(10^3)(300^3 - 225^3)(10^{-3})^3$$

$$= 166 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

16-17

(a) Eq. (16-23):

$$F = \frac{\pi p_a d}{2}(D - d) = \frac{\pi(120)(4)}{2}(6.5 - 4) = 1885 \text{ lbf} \quad \textit{Ans.}$$

Eq. (16-24):

$$T = \frac{\pi f p_a d}{8}(D^2 - d^2)N = \frac{\pi(0.24)(120)(4)}{8}(6.5^2 - 4^2)(6)$$

$$= 7125 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

(b)

$$T = \frac{\pi(0.24)(120d)}{8}(6.5^2 - d^2)(6)$$

d, in	T, lbf · in	
2	5191	
3	6769	
4	7125	*Ans.*
5	5853	
6	2545	

(c) The torque-diameter curve exhibits a stationary point maximum in the range of diameter *d*. The clutch has nearly optimal proportions.

16-18

(a)

$$T = \frac{\pi f p_a d(D^2 - d^2)N}{8} = CD^2 d - Cd^3$$

Differentiating with respect to *d* and equating to zero gives

$$\frac{dT}{dd} = CD^2 - 3Cd^2 = 0$$

$$d^* = \frac{D}{\sqrt{3}} \quad \textit{Ans.}$$

$$\frac{d^2T}{dd^2} = -6\,Cd$$

which is negative for all positive *d*. We have a stationary point *maximum*.

(b)

$$d^* = \frac{6.5}{\sqrt{3}} = 3.75 \text{ in} \quad \textit{Ans.}$$

$$T^* = \frac{\pi(0.24)(120)\left(6.5/\sqrt{3}\right)}{8}[6.5^2 - (6.5^2/3)](6) = 7173 \text{ lbf} \cdot \text{in}$$

(c) The table indicates a maximum within the range:

$$3 \le d \le 5 \text{ in}$$

(d) Consider:

$$0.45 \le \frac{d}{D} \le 0.80$$

Multiply through by D

$$0.45D \le d \le 0.80D$$

$$0.45(6.5) \le d \le 0.80(6.5)$$

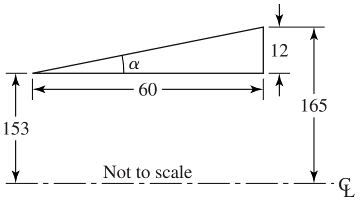
$$2.925 \le d \le 5.2 \text{ in}$$

$$\left(\frac{d}{D}\right)^* = d^*/D = \frac{1}{\sqrt{3}} = 0.577$$

which lies within the common range of clutches.

Yes. *Ans.*

16-19 Given: $d = 0.306\,\text{m}$, $l = 0.060\,\text{m}$, $T = 0.200\text{ kN}\cdot\text{m}$, $D = 0.330\text{ m}$, $f = 0.26$.



$$\alpha = \tan^{-1}\left(\frac{12}{60}\right) = 11.31°$$

Uniform wear

Eq. (16-45):

$$0.200 = \frac{\pi(0.26)(0.306)p_a}{8\sin 11.31°}(0.330^2 - 0.306^2) = 0.002\,432 p_a$$

$$p_a = \frac{0.200}{0.002\,432} = 82.2\text{ kPa} \quad \textit{Ans.}$$

Eq. (16-44):

$$F = \frac{\pi p_a d}{2}(D-d) = \frac{\pi(82.2)(0.306)}{2}(0.330 - 0.306) = 0.949\text{ kN} \quad \textit{Ans.}$$

Uniform pressure

Eq. (16-48):

$$0.200 = \frac{\pi(0.26)p_a}{12\sin 11.31°}(0.330^3 - 0.306^3) = 0.002\,53 p_a$$

$$p_a = \frac{0.200}{0.002\,53} = 79.1\text{ kPa} \quad \textit{Ans.}$$

Eq. (16-47):

$$F = \frac{\pi p_a}{4}(D^2 - d^2) = \frac{\pi(79.1)}{4}(0.330^2 - 0.306^2) = 0.948\text{ kN} \quad \textit{Ans.}$$

16-20 *Uniform wear*

Eq. (16-34):

$$T = \frac{1}{2}(\theta_2 - \theta_1) f p_a r_i \left(r_o^2 - r_i^2\right)$$

Eq. (16-33): $F = (\theta_2 - \theta_1) p_a r_i (r_o - r_i)$

Thus, $$\frac{T}{fFD} = \frac{(1/2)(\theta_2 - \theta_1) f p_a r_i \left(r_o^2 - r_i^2\right)}{f(\theta_2 - \theta_1) p_a r_i (r_o - r_i)(D)}$$

$$= \frac{r_o + r_i}{2D} = \frac{D/2 + d/2}{2D} = \frac{1}{4}\left(1 + \frac{d}{D}\right) \quad \textit{O.K.} \quad \textit{Ans.}$$

Uniform pressure

Eq. (16-38): $$T = \frac{1}{3}(\theta_2 - \theta_1) f p_a \left(r_o^3 - r_i^3\right)$$

Eq. (16-37):

$$F = \frac{1}{2}(\theta_2 - \theta_1) p_a \left(r_o^2 - r_i^2\right)$$

$$\frac{T}{fFD} = \frac{(1/3)(\theta_2 - \theta_1) f p_a \left(r_o^3 - r_i^3\right)}{(1/2) f (\theta_2 - \theta_1) p_a \left(r_o^2 - r_i^2\right) D} = \frac{2}{3}\left\{\frac{(D/2)^3 - (d/2)^3}{[(D/2)^2 - (d/2)^2 D]}\right\}$$

$$= \frac{2(D/2)^3(1 - (d/D)^3)}{3(D/2)^2[1 - (d/D)^2]D} = \frac{1}{3}\left[\frac{1 - (d/D)^3}{1 - (d/D)^2}\right] \quad \textit{O.K.} \quad \textit{Ans.}$$

16-21

$$\omega = 2\pi n/60 = 2\pi\ 500/60 = 52.4 \text{ rad/s}$$

$$T = \frac{H}{\omega} = \frac{2(10^3)}{52.4} = 38.2 \ \text{N} \cdot \text{m}$$

Key:

$$F = \frac{T}{r} = \frac{38.2}{12} = 3.18 \text{ kN}$$

Average shear stress in key is

$$\tau = \frac{3.18(10^3)}{6(40)} = 13.2 \text{ MPa} \quad \textit{Ans.}$$

Average bearing stress is

$$\sigma_b = -\frac{F}{A_b} = -\frac{3.18(10^3)}{3(40)} = -26.5 \text{ MPa} \quad \textit{Ans.}$$

Let one jaw carry the entire load.

$$r_{av} = \frac{1}{2}\left(\frac{26}{2} + \frac{45}{2}\right) = 17.75 \text{ mm}$$

$$F = \frac{T}{r_{av}} = \frac{38.2}{17.75} = 2.15 \text{ kN}$$

The bearing and shear stress estimates are

$$\sigma_b = \frac{-2.15(10^3)}{10(22.5 - 13)} = -22.6 \text{ MPa} \quad \textit{Ans.}$$

$$\tau = \frac{2.15(10^3)}{10[0.25\pi(17.75)^2]} = 0.869 \text{ MPa} \quad \textit{Ans.}$$

16-22

$$\omega_1 = 2\pi n/60 = 2\pi(1800)/60 = 188.5 \text{ rad/s}$$

$$\omega_2 = 0$$

From Eq. (16-51),

$$\frac{I_1 I_2}{I_1 + I_2} = \frac{T t_1}{\omega_1 - \omega_2} = \frac{320(8.3)}{188.5 - 0} = 14.09 \text{ N} \cdot \text{m} \cdot \text{s}^2$$

Eq. (16-52):

$$E = 14.09\left(\frac{188.5^2}{2}\right)(10^{-3}) = 250 \text{ kJ}$$

Eq. (16-55):

$$\Delta T = \frac{E}{C_p m} = \frac{250(10^3)}{500(18)} = 27.8^\circ\text{C} \quad \textit{Ans.}$$

16-23

$$n = \frac{n_1 + n_2}{2} = \frac{260 + 240}{2} = 250 \text{ rev/min}$$

$$C_s = \frac{260 - 240}{250} = 0.08 \quad \textit{Ans.}$$

$$\omega = 2\pi(250)/60 = 26.18 \text{ rad/s}$$

$$I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{5000(12)}{0.08(26.18)^2} = 1094 \text{ lbf} \cdot \text{in} \cdot \text{s}^2$$

$$I_x = \frac{m}{8}\left(d_o^2 + d_i^2\right) = \frac{W}{8g}\left(d_o^2 + d_i^2\right)$$

$$W = \frac{8gI}{d_o^2 + d_i^2} = \frac{8(386)(1094)}{60^2 + 56^2} = 502 \text{ lbf}$$

$$w = 0.260 \text{ lbf/in}^3 \quad \text{for cast iron}$$

$$V = \frac{W}{w} = \frac{502}{0.260} = 1931 \text{ in}^3$$

Also, $$V = \frac{\pi t}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi t}{4}\left(60^2 - 56^2\right) = 364t \text{ in}^3$$

Equating the expressions for volume and solving for t,

$$t = \frac{1931}{364} = 5.3 \text{ in} \quad \textit{Ans.}$$

16-24 **(a)** The useful work performed in one revolution of the crank shaft is

$$U = 35(2000)(8)(0.15) = 84(10^3)\ \text{in}\cdot\text{lbf}$$

Accounting for friction, the total work done in one revolution is

$$U = 84(10^3)/(1-0.16) = 100(10^3)\ \text{in}\cdot\text{lbf}$$

Since 15% of the crank shaft stroke is 7.5% of a crank shaft revolution, the energy fluctuation is

$$E_2 - E_1 = 84(10^3) - 100(10^3)(0.075) = 76.5(10^3)\ \text{in}\cdot\text{lbf} \quad \textit{Ans.}$$

(b) For the flywheel

$$n = 6(90) = 540\ \text{rev/min}$$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(540)}{60} = 56.5\ \text{rad/s}$$

Since $C_s = 0.10$

$$I = \frac{E_2 - E_1}{C_s\omega^2} = \frac{76.5(10^3)}{0.10(56.5)^2} = 239.6\ \text{lbf}\cdot\text{in}\cdot\text{s}^2$$

Assuming all the mass is concentrated at the effective diameter, d,

$$I = \frac{md^2}{4}$$

$$W = \frac{4gI}{d^2} = \frac{4(386)(239.6)}{48^2} = 161\ \text{lbf} \quad \textit{Ans.}$$

16-25 Use Ex. 16-6 and Table 16-6 data for one cylinder of a 3-cylinder engine.

$$C_s = 0.30$$

$$n = 2400\ \text{rev/min}\ \text{ or }\ 251\ \text{rad/s}$$

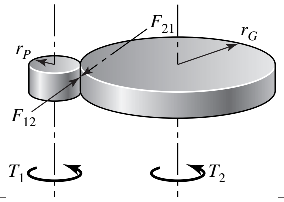
$$T_m = \frac{3(3368)}{4\pi} = 804\ \text{in}\cdot\text{lbf} \quad \textit{Ans.}$$

$$E_2 - E_1 = 3(3531) = 10\,590\ \text{in}\cdot\text{lbf}$$

$$I = \frac{E_2 - E_1}{C_s\omega^2} = \frac{10\,590}{0.30(251^2)} = 0.560\ \text{in}\cdot\text{lbf}\cdot\text{s}^2 \quad \textit{Ans.}$$

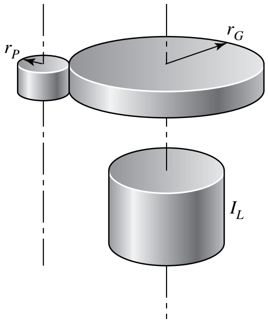
16-26 **(a)**

(1)



$$(T_2)_1 = -F_{21}r_P = -\frac{T_2}{r_G}r_P = \frac{T_2}{-n} \quad \textit{Ans.}$$

(2)



Equivalent energy

$$(1/2)I_2\omega_2^2 = (1/2)(I_2)_1\left(w_1^2\right)$$

$$(I_2)_1 = \frac{\omega_2^2}{\omega_1^2}I_2 = \frac{I_2}{n^2} \quad \textit{Ans.}$$

(3) $$\frac{I_G}{I_P} = \left(\frac{r_G}{r_P}\right)^2\left(\frac{m_G}{m_P}\right) = \left(\frac{r_G}{r_P}\right)^2\left(\frac{r_G}{r_P}\right)^2 = n^4$$

From (2) $$(I_2)_1 = \frac{I_G}{n^2} = \frac{n^4 I_P}{n^2} = n^2 I_P \quad \textit{Ans.}$$

(b) $$I_e = I_M + I_P + n^2 I_P + \frac{I_L}{n^2} \quad \textit{Ans.}$$

(c) $$I_e = 10 + 1 + 10^2(1) + \frac{100}{10^2}$$

$$= 10 + 1 + 100 + 1 = 112$$

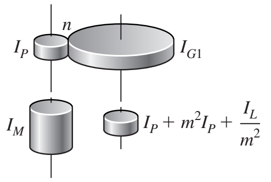
reflected load inertia

reflected gear inertia *Ans.*

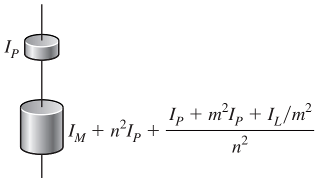
pinion inertia

armature inertia

16-27 **(a)** Reflect I_L, I_{G2} to the center shaft



Reflect the center shaft to the motor shaft



$$I_e = I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{m^2}{n^2}I_P + \frac{I_L}{m^2 n^2} \quad \textit{Ans.}$$

(b) For $R = \text{constant} = nm$, $\quad I_e = I_M + I_P + n^2 I_P + \dfrac{I_P}{n^2} + \dfrac{R^2 I_P}{n^4} + \dfrac{I_L}{R^2}$ *Ans.*

(c) For $R = 10$, $\quad \dfrac{\partial I_e}{\partial n} = 0 + 0 + 2n(1) - \dfrac{2(1)}{n^3} - \dfrac{4(10^2)(1)}{n^5} + 0 = 0$

$$n^6 - n^2 - 200 = 0$$

From which

$$n^* = 2.430 \quad \textit{Ans.}$$

$$m^* = \frac{10}{2.430} = 4.115 \quad \textit{Ans.}$$

Notice that n^* and m^* are independent of I_L.

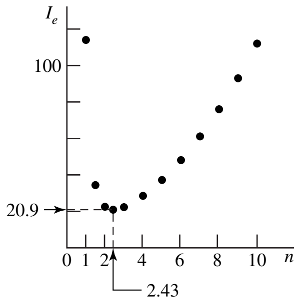
16-28 From Prob. 16-27,

$$I_e = I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{R^2 I_P}{n^4} + \frac{I_L}{R^2}$$

$$= 10 + 1 + n^2(1) + \frac{1}{n^2} + \frac{100(1)}{n^4} + \frac{100}{10^2}$$

$$= 10 + 1 + n^2 + \frac{1}{n^2} + \frac{100}{n^4} + 1$$

n	I_e
1.00	114.00
1.50	34.40
2.00	22.50
2.43	20.90
3.00	22.30
4.00	28.50
5.00	37.20
6.00	48.10
7.00	61.10
8.00	76.00
9.00	93.00
10.00	112.02



Optimizing the partitioning of a double reduction lowered the gear-train inertia to $20.9/112 = 0.187$, or to 19% of that of a single reduction. This includes the two additional gears.

16-29 Figure 16-29 applies,

$$t_2 = 10 \text{ s}, \quad t_1 = 0.5 \text{ s}$$

$$\frac{t_2 - t_1}{t_1} = \frac{10 - 0.5}{0.5} = 19$$

The load torque, as seen by the motor shaft (Rule 1, Prob. 16-26), is

$$T_L = \left|\frac{1300(12)}{10}\right| = 1560 \text{ lbf} \cdot \text{in}$$

The rated motor torque T_r is

$$T_r = \frac{63\,025(3)}{1125} = 168.07 \text{ lbf} \cdot \text{in}$$

For Eqs. (16-65):

$$\omega_r = \frac{2\pi}{60}(1125) = 117.81 \text{ rad/s}$$

$$\omega_s = \frac{2\pi}{60}(1200) = 125.66 \text{ rad/s}$$

$$a = \frac{-T_r}{\omega_s - \omega_r} = -\frac{168.07}{125.66 - 117.81} = -21.41$$

$$b = \frac{T_r\omega_s}{\omega_s - \omega_r} = \frac{168.07(125.66)}{125.66 - 117.81}$$

$$= 2690.4 \text{ lbf} \cdot \text{in}$$

The linear portion of the squirrel-cage motor characteristic can now be expressed as

$$T_M = -21.41\omega + 2690.4 \text{ lbf} \cdot \text{in}$$

Eq. (16-68):

$$T_2 = 168.07\left(\frac{1560 - 168.07}{1560 - T_2}\right)^{19}$$

One root is 168.07 which is for infinite time. The root for 10 s is wanted. Use a successive substitution method

T_2	New T_2
0.00	19.30
19.30	24.40
24.40	26.00
26.00	26.50
26.50	26.67

Continue until convergence.

$$T_2 = 26.771$$

Eq. (16-69):

$$I = \frac{-21.41(10 - 0.5)}{\ln(26.771/168.07)} = 110.72 \text{ in} \cdot \text{lbf} \cdot \text{s/rad}$$

$$\omega = \frac{T - b}{a}$$

$$\omega_{\max} = \frac{T_2 - b}{a} = \frac{26.771 - 2690.4}{-21.41} = 124.41 \text{ rad/s} \quad \textit{Ans.}$$

$$\omega_{\min} = 117.81 \text{ rad/s} \quad \textit{Ans.}$$

$$\bar{\omega} = \frac{124.41 + 117.81}{2} = 121.11 \text{ rad/s}$$

$$C_s = \frac{\omega_{\max} - \omega_{\min}}{(\omega_{\max} + \omega_{\min})/2} = \frac{124.41 - 117.81}{(124.41 + 117.81)/2} = 0.0545 \quad \textit{Ans.}$$

$$E_1 = \frac{1}{2} I \omega_r^2 = \frac{1}{2}(110.72)(117.81)^2 = 768\,352 \text{ in} \cdot \text{lbf}$$

$$E_2 = \frac{1}{2} I \omega_2^2 = \frac{1}{2}(110.72)(124.41)^2 = 856\,854 \text{ in} \cdot \text{lbf}$$

$$\Delta E = E_1 - E_2 = 768\,352 - 856\,854 = -88\,502 \text{ in} \cdot \text{lbf}$$

Eq. (16-64):

$$\Delta E = C_s I \bar{\omega}^2 = 0.0545(110.72)(121.11)^2$$

$$= 88\,508 \text{ in} \cdot \text{lbf}, \quad \text{close enough} \quad \textit{Ans.}$$

During the punch

$$T = \frac{63\,025H}{n}$$

$$H = \frac{T_L \bar{\omega}(60/2\pi)}{63\,025} = \frac{1560(121.11)(60/2\pi)}{63\,025} = 28.6 \text{ hp}$$

The gear train has to be sized for 28.6 hp under shock conditions since the flywheel is on the motor shaft. From Table A-18,

$$I = \frac{m}{8}\left(d_o^2 + d_i^2\right) = \frac{W}{8g}\left(d_o^2 + d_i^2\right)$$

$$W = \frac{8gI}{d_o^2 + d_i^2} = \frac{8(386)(110.72)}{d_o^2 + d_i^2}$$

If a mean diameter of the flywheel rim of 30 in is acceptable, try a rim thickness of 4 in

$$d_i = 30 - (4/2) = 28 \text{ in}$$

$$d_o = 30 + (4/2) = 32 \text{ in}$$

$$W = \frac{8(386)(110.72)}{32^2 + 28^2} = 189.1 \text{ lbf}$$

Rim volume V is given by

$$V = \frac{\pi l}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi l}{4}(32^2 - 28^2) = 188.5l$$

where l is the rim width as shown in Table A-18. The specific weight of cast iron is $\gamma = 0.260 \text{ lbf} \cdot \text{in}^3$, therefore the volume of cast iron is

$$V = \frac{W}{\gamma} = \frac{189.1}{0.260} = 727.3 \text{ in}^3$$

Thus

$$188.5\,l = 727.3$$

$$l = \frac{727.3}{188.5} = 3.86 \text{ in} \quad \text{wide}$$

Proportions can be varied.

16-30 Prob. 16-29 solution has I for the motor shaft flywheel as

$$I = 110.72 \text{ in} \cdot \text{lbf} \cdot \text{s}^2/\text{rad}$$

A flywheel located on the crank shaft needs an inertia of $10^2 I$ (Prob. 16-26, rule 2)

$$I = 10^2(110.72) = 11\,072 \text{ in} \cdot \text{ lbf} \cdot \text{s}^2/\text{rad}$$

A 100-fold inertia increase. On the other hand, the gear train has to transmit 3 hp under shock conditions.

Stating the problem is most of the solution. Satisfy yourself that on the crankshaft:

$$T_L = 1300(12) = 15\,600 \text{ lbf} \cdot \text{in}$$

$$T_r = 10(168.07) = 1680.7 \text{ lbf} \cdot \text{in}$$

$$\omega_r = 117.81/10 = 11.781 \text{ rad/s}$$

$$\omega_s = 125.66/10 = 12.566 \text{ rad/s}$$

$$a = -21.41(100) = -2141$$

$$b = 2690.35(10) = 26903.5$$

$$T_M = -2141\omega_c + 26\,903.5 \text{ lbf} \cdot \text{in}$$

$$T_2 = 1680.6\left(\frac{15\,600 - 1680.5}{15\,600 - T_2}\right)^{19}$$

The root is $10(26.67) = 266.7 \text{ lbf} \cdot \text{in}$

$$\bar{\omega} = 121.11/10 = 12.111 \text{ rad/s}$$

$$C_s = 0.0549 \quad \text{(same)}$$

$$\omega_{\max} = 121.11/10 = 12.111 \text{ rad/s} \quad Ans.$$

$$\omega_{\min} = 117.81/10 = 11.781 \text{ rad/s} \quad Ans.$$

E_1, E_2, ΔE and peak power are the same.

From Table A-18

$$W = \frac{8gI}{d_o^2 + d_i^2} = \frac{8(386)(11\,072)}{d_o^2 + d_i^2}$$

Scaling will affect d_o and d_i, but the gear ratio changed I. Scale up the flywheel in the Prob. 16-29 solution by a factor of 2.5. Thickness becomes $4(2.5) = 10$ in.

$$\bar{d} = 30(2.5) = 75 \text{ in}$$

$$d_o = 75 + (10/2) = 80 \text{ in}$$

$$d_i = 75 - (10/2) = 70 \text{ in}$$

$$W = \frac{8(386)(11\,072)}{80^2 + 70^2} = 3026 \text{ lbf}$$

$$v = \frac{3026}{0.26} = 11\,638 \text{ in}^3$$

$$V = \frac{\pi}{4} l(80^2 - 70^2) = 1178\,l$$

$$l = \frac{11\,638}{1178} = 9.88 \text{ in}$$

Proportions can be varied. The weight has increased $3026/189.1$ or about 16-fold while the moment of inertia I increased 100-fold. The gear train transmits a steady 3 hp. But the motor armature has its inertia magnified 100-fold, and during the punch there are deceleration stresses in the train. With no motor armature information, we cannot comment.

16-31 This can be the basis for a class discussion.

Chapter 17

17-1 Given: F-1 Polyamide, $b = 6$ in, $d = 2$ in @ 1750 rev/min

$C = 9(12) = 108$ in, vel. ratio 0.5, $H_{\text{nom}} = 2$ hp, $K_s = 1.25$, $n_d = 1$

Table 17-2: $t = 0.05$ in, $d_{\min} = 1.0$ in, $F_a = 35$ lbf/in,
$\gamma = 0.035\ \text{lbf/in}^3$, $f = 0.5$

Table 17-4: $C_p = 0.70$

$$w = 12\gamma bt = 12(0.035)(6)(0.05) = 0.126\ \text{lbf/ft}$$

$$\theta_d = 3.123\ \text{rad}, \quad \exp(f\theta) = 4.766 \quad \text{(perhaps)}$$

$$V = \frac{\pi dn}{12} = \frac{\pi(2)(1750)}{12} = 916.3\ \text{ft/min}$$

(a) Eq. (*e*), p. 865: $F_c = \dfrac{w}{32.17}\left(\dfrac{V}{60}\right)^2 = \dfrac{0.126}{32.17}\left(\dfrac{916.3}{60}\right)^2 = 0.913\ \text{lbf}$ *Ans.*

$$T = \frac{63\,025 H_{\text{nom}} K_s n_d}{n} = \frac{63\,025(2)(1.25)(1)}{1750} = 90.0\ \text{lbf} \cdot \text{in}$$

$$\Delta F = \frac{2T}{d} = \frac{2(90)}{2} = 90\ \text{lbf}$$

Eq. (17-12): $(F_1)_a = bF_aC_pC_v = 6(35)(0.70)(1) = 147\ \text{lbf}$ *Ans.*

$$F_2 = F_{1a} - \Delta F = 147 - 90 = 57\ \text{lbf} \quad \textit{Ans.}$$

Do not use Eq. (17-9) because we do not yet know f'.

Eq. (*i*), p. 866: $F_i = \dfrac{F_{1a} + F_2}{2} - F_c = \dfrac{147 + 57}{2} - 0.913 = 101.1\ \text{lbf}$ *Ans.*

Eq. (17-7): $f' = \dfrac{1}{\theta_d}\ln\left[\dfrac{(F_1)_a - F_c}{F_2 - F_c}\right] = \dfrac{1}{3.123}\ln\left(\dfrac{147 - 0.913}{57 - 0.913}\right) = 0.307$

The friction is thus undeveloped.

(b) The transmitted horsepower is,

$$H = \frac{(\Delta F)V}{33\,000} = \frac{90(916.3)}{33\,000} = 2.5\ \text{hp} \quad \textit{Ans.}$$

$$n_{fs} = \frac{H}{H_{\text{nom}}K_s} = \frac{2.5}{2(1.25)} = 1$$

From Eq. (17-2), $L = 225.3$ in *Ans.*

(c) From Eq. (17-13), $\text{dip} = \dfrac{3C^2w}{2F_i}$

where C is the center-to-center distance in feet.

$$\text{dip} = \frac{3(108/12)^2(0.126)}{2(101.1)} = 0.151\ \text{in} \quad \textit{Ans.}$$

Comment: The friction is under-developed. Narrowing the belt width to 5 in (if size is available) will increase f'. The limit of narrowing is $b_{\min} = 4.680$ in, whence

$$\begin{aligned} w &= 0.0983 \text{ lbf/ft} & (F_1)_a &= 114.7 \text{ lbf} \\ F_c &= 0.712 \text{ lbf} & F_2 &= 24.6 \text{ lbf} \\ T &= 90 \text{ lbf} \cdot \text{in} \quad \text{(same)} & f' &= f = 0.50 \\ \Delta F &= (F_1)_a - F_2 = 90 \text{ lbf} & \text{dip} &= 0.173 \text{ in} \\ F_i &= 68.9 \text{ lbf} \end{aligned}$$

Longer life can be obtained with a 6-inch wide belt by reducing F_i to attain $f' = 0.50$. Prob. 17-8 develops an equation we can use here

$$F_i = \frac{(\Delta F + F_c)\exp(f\theta) - F_c}{\exp(f\theta) - 1}$$

$$F_2 = F_1 - \Delta F$$

$$F_i = \frac{F_1 + F_2}{2} - F_c$$

$$f' = \frac{1}{\theta_d} \ln\left(\frac{F_1 - F_c}{F_2 - F_c}\right)$$

$$\text{dip} = \frac{3(CD/12)^2 w}{2F_i}$$

which in this case gives

$$\begin{aligned} F_1 &= 114.9 \text{ lbf} & F_c &= 0.913 \text{ lbf} \\ F_2 &= 24.8 \text{ lbf} & f' &= 0.50 \\ F_i &= 68.9 \text{ lbf} & \text{dip} &= 0.222 \text{ in} \end{aligned}$$

So, reducing F_i from 101.1 lbf to 68.9 lbf will bring the undeveloped friction up to 0.50, with a corresponding dip of 0.222 in. Having reduced F_1 and F_2, the endurance of the belt is improved. Power, service factor and design factor have remained in tack.

17-2 There are practical limitations on doubling the iconic scale. We can double pulley diameters and the center-to-center distance. With the belt we could:

- Use the same A-3 belt and double its width;
- Change the belt to A-5 which has a thickness 0.25 in rather than 2(0.13) = 0.26 in, and an increased F_a;
- Double the thickness and double tabulated F_a which is based on table thickness.

The object of the problem is to reveal where the non-proportionalities occur and the nature of scaling a flat belt drive.

We will utilize the third alternative, choosing an A-3 polyamide belt of double thickness, assuming it is available. We will also remember to double the tabulated F_a from 100 lbf/in to 200 lbf/in.

In assigning this problem, you could outline (or solicit) the three alternatives just mentioned and assign the one of your choice–alternative 3:

Ex. 17-2: $b = 10$ in, $d = 16$ in, $D = 32$ in, Polyamide A-3, $t = 0.13$ in, $\gamma = 0.042$, $F_a = 100$ lbf/in, $C_p = 0.94$, $C_v = 1$, $f = 0.8$

$$T = \frac{63\,025(60)(1.15)(1.05)}{860} = 5313 \text{ lbf} \cdot \text{in}$$

$$w = 12\gamma bt = 12(0.042)(10)(0.13)$$
$$= 0.655 \text{ lbf/ft}$$

$$V = \pi dn/12 = \pi(16)(860/12) = 3602 \text{ ft/min}$$

$$\theta_d = 3.037 \text{ rad}$$

For fully-developed friction:

$$\exp(f\theta_d) = [0.8(3.037)] = 11.35$$

$$F_c = \frac{wV^2}{g} = \frac{0.655(3602/60)^2}{32.174} = 73.4 \text{ lbf}$$

$$(F_1)_a = F_1 = bF_aC_pC_v$$
$$= 10(100)(0.94)(1) = 940 \text{ lbf}$$

$$\Delta F = 2T/D = 2(5313)/(16) = 664 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 940 - 664 = 276 \text{ lbf}$$

$$F_i = \frac{F_1 + F_2}{2} - F_c$$
$$= \frac{940 + 276}{2} - 73.4 = 535 \text{ lbf}$$

Transmitted power H (or H_a):

$$H = \frac{\Delta F(V)}{33\,000} = \frac{664(3602)}{33\,000} = 72.5 \text{ hp}$$

$$f' = \frac{1}{\theta_d}\ln\left(\frac{F_1 - F_c}{F_2 - F_c}\right)$$
$$= \frac{1}{3.037}\ln\left(\frac{940 - 73.4}{276 - 73.4}\right)$$
$$= 0.479 \quad \text{undeveloped}$$

Note, in this as well as in the double-size case, $\exp(f\theta_d)$ is not used. It will show up if we relax F_i (and change other parameters to transmit the required power), in order to bring f' up to $f = 0.80$, and increase belt life.

You may wish to suggest to your students that solving comparison problems in this manner assists in the design process.

Doubled: $b = 20$ in, $d = 32$ in, $D = 72$ in, Polyamide A-3, $t = 0.26$ in, $\gamma = 0.042$, $F_a = 2(100) = 200$ lbf/in, $C_p = 1$, $C_v = 1$, $f = 0.8$

$$T = 4(5313) = 21\,252 \text{ lbf} \cdot \text{in}$$

$$w = 12(0.042)(20)(0.26) = 2.62 \text{ lbf/ft}$$

$$V = \pi(32)(860/12) = 7205 \text{ ft/min}$$

$$\theta = 3.037 \text{ rad}$$

For fully-developed friction:

$$\exp(f\theta_d) = \exp[0.8(3.037)] = 11.35$$

$$F_c = \frac{wV^2}{g} = \frac{0.262(7205/60)^2}{32.174} = 1174.3 \text{ lbf}$$

$$(F_1)_a = 20(200)(1)(1)$$
$$= 4000 \text{ lbf} = F_1$$

$$\Delta F = 2T/D = 2(21\,252)/(32) = 1328.3 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 4000 - 1328.3 = 2671.7 \text{ lbf}$$

$$F_i = \frac{F_1 + F_2}{2} - F_c$$
$$= \frac{4000 + 2671.7}{2} - 1174.3 = 2161.6 \text{ lbf}$$

Transmitted power H:

$$H = \frac{\Delta F(V)}{33\,000} = \frac{1328.3(7205)}{33\,000} = 290 \text{ hp}$$

$$f' = \frac{1}{\theta_d}\ln\left(\frac{F_1 - F_c}{F_2 - F_c}\right)$$
$$= \frac{1}{3.037}\ln\left(\frac{4000 - 1174.3}{2671.7 - 1174.3}\right)$$
$$= 0.209 \quad \text{undeveloped}$$

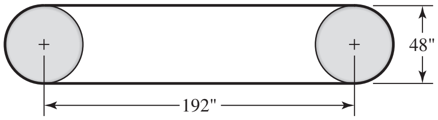
There was a small change in C_p.

Parameter	Change
V	2-fold
F_c	16-fold
F_1	4.26-fold
F_2	9.7-fold

Parameter	Change
ΔF	2-fold
F_i	4-fold
H_t	4-fold
f'	0.48-fold

Note the change in F_c!

17-3



As a design task, the decision set on p. 873 is useful.

A priori decisions:

- Function: $H_{\text{nom}} = 60$ hp, $n = 380$ rev/min, $C = 192$ in, $K_s = 1.1$
- Design factor: $n_d = 1$
- Initial tension: Catenary
- Belt material: Polyamide A-3, $F_a = 100$ lbf/in, $\gamma = 0.042$ lbf/in^3, $f = 0.8$
- Drive geometry: $d = D = 48$ in
- Belt thickness: $t = 0.13$ in

Design variable: Belt width of 6 in

Use a method of trials. Initially choose $b = 6$ in

$$V = \frac{\pi dn}{12} = \frac{\pi(48)(380)}{12} = 4775 \text{ ft/min}$$

$$w = 12\gamma bt = 12(0.042)(6)(0.13) = 0.393 \text{ lbf/ft}$$

$$F_c = \frac{wV^2}{g} = \frac{0.393(4775/60)^2}{32.174} = 77.4 \text{ lbf}$$

$$T = \frac{63\,025 H_{\text{nom}} K_s n_d}{n} = \frac{63\,025(60)(1.1)(1)}{380} = 10\,946 \text{ lbf} \cdot \text{in}$$

$$\Delta F = \frac{2T}{d} = \frac{2(10\,946)}{48} = 456.1 \text{ lbf}$$

$$F_1 = (F_1)_a = bF_aC_pC_v = 6(100)(1)(1) = 600 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 600 - 456.1 = 143.9 \text{ lbf}$$

Transmitted power H

$$H = \frac{\Delta F(V)}{33\,000} = \frac{456.1(4775)}{33\,000} = 66 \text{ hp}$$

$$F_i = \frac{F_1 + F_2}{2} - F_c = \frac{600 + 143.9}{2} - 77.4 = 294.6 \text{ lbf}$$

$$f' = \frac{1}{\theta_d} \ln\left(\frac{F_1 - F_c}{F_2 - F_c}\right) = \frac{1}{\pi} \ln\left(\frac{600 - 77.4}{143.9 - 77.4}\right) = 0.656$$

Eq. (17-2): $L = [4(192)^2 - (48 - 48)^2]^{1/2} + 0.5[48(\pi) + 48(\pi)] = 534.8$ in

Friction is not fully developed, so $b_{\min}$ is just a little smaller than 6 in (5.7 in). Not having a figure of merit, we choose the most narrow belt available (6 in). We can improve the design by reducing the initial tension, which reduces F_1 and F_2, thereby increasing belt life. This will bring f' to 0.80

$$F_1 = \frac{(\Delta F + F_c)\exp(f\theta) - F_c}{\exp(f\theta) - 1}$$

$$\exp(f\theta) = \exp(0.80\pi) = 12.345$$

Therefore

$$F_1 = \frac{(456.1 + 77.4)(12.345) - 77.4}{12.345 - 1} = 573.7 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 573.7 - 456.1 = 117.6 \text{ lbf}$$

$$F_i = \frac{F_1 + F_2}{2} - F_c = \frac{573.7 + 117.6}{2} - 77.4 = 268.3 \text{ lbf}$$

These are small reductions since f' is close to f, but improvements nevertheless.

$$\text{dip} = \frac{3C^2 w}{2F_i} = \frac{3(192/12)^2(0.393)}{2(268.3)} = 0.562 \text{ in}$$

17-4 From the last equation given in the Problem Statement,

$$\exp(f\phi) = \frac{1}{1 - \{2T/[d(a_0 - a_2)b]\}}$$

$$\left[1 - \frac{2T}{d(a_0 - a_2)b}\right]\exp(f\phi) = 1$$

$$\left[\frac{2T}{d(a_0 - a_2)b}\right]\exp(f\phi) = \exp(f\phi) - 1$$

$$b = \frac{1}{a_0 - a_2}\left(\frac{2T}{d}\right)\left[\frac{\exp(f\phi)}{\exp(f\phi) - 1}\right]$$

But $2T/d = 33\,000 H_d/V$

Thus,

$$b = \frac{1}{a_0 - a_2}\left(\frac{33\,000 H_d}{V}\right)\left[\frac{\exp(f\phi)}{\exp(f\phi) - 1}\right] \quad Q.E.D.$$

17-5 Refer to Ex. 17-1 on p. 870 for the values used below.

(a) The maximum torque prior to slip is,

$$T = \frac{63\,025 H_{\text{nom}} K_s n_d}{n} = \frac{63\,025(15)(1.25)(1.1)}{1750} = 742.8 \text{ lbf} \cdot \text{in} \quad Ans.$$

The corresponding initial tension is,

$$F_i = \frac{T}{D}\left(\frac{\exp(f\theta) + 1}{\exp(f\theta) - 1}\right) = \frac{742.8}{6}\left(\frac{11.17 + 1}{11.17 - 1}\right) = 148.1 \text{ lbf} \quad Ans.$$

(b) See Prob. 17-4 statement. The final relation can be written

$$b_{\min} = \frac{1}{F_a C_p C_v - (12\gamma t/32.174)(V/60)^2} \left\{ \frac{33\,000 H_a \exp(f\theta)}{V[\exp(f\theta) - 1]} \right\}$$

$$= \frac{1}{100(0.7)(1) - \{[12(0.042)(0.13)]/32.174\}(2749/60)^2} \left[\frac{33\,000(20.6)(11.17)}{2749(11.17 - 1)} \right]$$

$$= 4.13 \text{ in} \quad Ans.$$

This is the minimum belt width since the belt is at the point of slip. The design must round up to an available width.

Eq. (17-1):

$$\theta_d = \pi - 2\sin^{-1}\left(\frac{D-d}{2C}\right) = \pi - 2\sin^{-1}\left[\frac{18-6}{2(96)}\right]$$

$$= 3.016\,511 \text{ rad}$$

$$\theta_D = \pi + 2\sin^{-1}\left(\frac{D-d}{2C}\right) = \pi + 2\sin^{-1}\left[\frac{18-6}{2(96)}\right]$$

$$= 3.266\,674$$

Eq. (17-2):

$$L = [4(96)^2 - (18-6)^2]^{1/2} + \frac{1}{2}[18(3.266\,674) + 6(3.016\,511)]$$

$$= 230.074 \text{ in} \quad Ans.$$

(c)

$$\Delta F = \frac{2T}{d} = \frac{2(742.8)}{6} = 247.6 \text{ lbf}$$

$$(F_1)_a = bF_a C_p C_v = F_1 = 4.13(100)(0.70)(1) = 289.1 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 289.1 - 247.6 = 41.5 \text{ lbf}$$

$$F_c = 25.6\left(\frac{0.271}{0.393}\right) = 17.7 \text{ lbf}$$

$$F_i = \frac{F_1 + F_2}{2} - F_c = \frac{289.1 + 41.5}{2} - 17.7 = 147.6 \text{ lbf}$$

Transmitted belt power H

$$H = \frac{\Delta F(V)}{33\,000} = \frac{247.6(2749)}{33\,000} = 20.6 \text{ hp}$$

$$n_{fs} = \frac{H}{H_{\text{nom}} K_s} = \frac{20.6}{15(1.25)} = 1.1$$

If you only change the belt width, the parameters in the following table change as shown.

	Ex. 17-1	This Problem
b	6.00	4.13
w	0.393	0.271
F_c	25.6	17.6
$(F_1)_a$	420	289
F_2	172.4	42
F_i	270.6	147.7
f'	0.33*	0.80**
dip	0.139	0.176

*Friction underdeveloped
**Friction fully developed

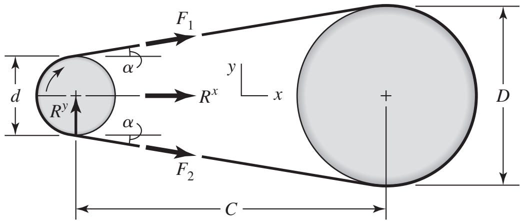
17-6 The transmitted power is the same.

	$b = 6$ in	$b = 12$ in	n-Fold Change
F_c	25.65	51.3	2
F_i	270.35	664.9	2.46
$(F_1)_a$	420	840	2
F_2	172.4	592.4	3.44
H_a	20.62	20.62	1
n_{fs}	1.1	1.1	1
f'	0.139	0.125	0.90
dip	0.328	0.114	0.34

If we relax F_i to develop full friction ($f = 0.80$) and obtain longer life, then

	$b = 6$ in	$b = 12$ in	n-Fold Change
F_c	25.6	51.3	2
F_i	148.1	148.1	1
F_1	297.6	323.2	1.09
F_2	50	75.6	1.51
f'	0.80	0.80	1
dip	0.255	0.503	2

17-7



Find the resultant of F_1 and F_2:

$$\alpha = \sin^{-1} \frac{D-d}{2C}$$

$$\sin\alpha = \frac{D-d}{2C}$$

$$\cos\alpha \doteq 1 - \frac{1}{2}\left(\frac{D-d}{2C}\right)^2$$

$$R^x = F_1\cos\alpha + F_2\cos\alpha = (F_1+F_2)\left[1 - \frac{1}{2}\left(\frac{D-d}{2C}\right)^2\right] \quad Ans.$$

$$R^y = F_1\sin\alpha - F_2\sin\alpha = (F_1-F_2)\frac{D-d}{2C} \quad Ans.$$

From Ex. 17-2, $d = 16$ in, $D = 36$ in, $C = 16(12) = 192$ in, $F_1 = 940$ lbf, $F_2 = 276$ lbf

$$\alpha = \sin^{-1}\left[\frac{36-16}{2(192)}\right] = 2.9855^\circ$$

$$R^x = (940+276)\left[1 - \frac{1}{2}\left(\frac{36-16}{2(192)}\right)^2\right] = 1214.4 \text{ lbf}$$

$$R^y = (940-276)\left[\frac{36-16}{2(192)}\right] = 34.6 \text{ lbf}$$

$$T = (F_1-F_2)\left(\frac{d}{2}\right) = (940-276)\left(\frac{16}{2}\right) = 5312 \text{ lbf}\cdot\text{in}$$

17-8 Begin with Eq. (17-10),

$$F_1 = F_c + F_i\frac{2\exp(f\theta)}{\exp(f\theta)-1}$$

Introduce Eq. (17-9):

$$F_1 = F_c + \frac{T}{D}\left[\frac{\exp(f\theta)+1}{\exp(f\theta)-1}\right]\left[\frac{2\exp(f\theta)}{\exp(f\theta)+1}\right] = F_c + \frac{2T}{D}\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right]$$

$$F_1 = F_c + \Delta F\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right]$$

Now add and subtract $F_c \left[\dfrac{\exp(f\theta)}{\exp(f\theta)-1}\right]$

$$F_1 = F_c + F_c\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right] + \Delta F\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right] - F_c\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right]$$

$$F_1 = (F_c + \Delta F)\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right] + F_c - F_c\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right]$$

$$F_1 = (F_c + \Delta F)\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right] - \frac{F_c}{\exp(f\theta)-1}$$

$$F_1 = \frac{(F_c + \Delta F)\exp(f\theta) - F_c}{\exp(f\theta)-1} \quad \textit{Q.E.D.}$$

From Ex. 17-2: $\theta_d = 3.037$ rad, $\Delta F = 664$ lbf, $\exp(f\theta) = \exp[0.80(3.037)] = 11.35$, and $F_c = 73.4$ lbf.

$$F_1 = \frac{(73.4 + 664)(11.35 - 73.4)}{(11.35 - 1)} = 802 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 802 - 664 = 138 \text{ lbf}$$

$$F_i = \frac{802 + 138}{2} - 73.4 = 396.6 \text{ lbf}$$

$$f' = \frac{1}{\theta_d}\ln\left(\frac{F_1 - F_c}{F_2 - F_c}\right) = \frac{1}{3.037}\ln\left(\frac{802 - 73.4}{138 - 73.4}\right) = 0.80 \quad \textit{Ans.}$$

17-9 This is a good class project. Form four groups, each with a belt to design. Once each group agrees internally, all four should report their designs including the forces and torques on the line shaft. If you give them the pulley locations, they could design the line shaft.

17-10 If you have the students implement a computer program, the design problem selections may differ, and the students will be able to explore them. For $K_s = 1.25$, $n_d = 1.1$, $d = 14$ in and $D = 28$ in, a polyamide A-5 belt, 8 inches wide, will do ($b_{\min} = 6.58$ in)

17-11 An efficiency of less than unity lowers the output for a given input. Since the object of the drive is the output, the efficiency must be incorporated such that the belt's capacity is increased. The design power would thus be expressed as

$$H_d = \frac{H_{\text{nom}} K_s n_d}{\text{eff}} \quad \textit{Ans.}$$

17-12 Some perspective on the size of F_c can be obtained from

$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{12\gamma bt}{g}\left(\frac{V}{60}\right)^2$$

An approximate comparison of non-metal and metal belts is presented in the table below.

	Non-metal	Metal
γ, lbf/in^3	0.04	0.280
b, in	5.00	1.000
t, in	0.20	0.005

The ratio w/w_m is

$$\frac{w}{w_m} = \frac{12(0.04)(5)(0.2)}{12(0.28)(1)(0.005)} \doteq 29$$

The second contribution to F_c is the belt peripheral velocity which tends to be low in metal belts used in instrument, printer, plotter and similar drives. The velocity ratio squared influences any $F_c/(F_c)_m$ ratio.

It is common for engineers to treat F_c as negligible compared to other tensions in the belting problem. However, when developing a computer code, one should include F_c.

17-13 Eq. (17-8):

$$\Delta F = F_1 - F_2 = (F_1 - F_c)\frac{\exp(f\theta) - 1}{\exp(f\theta)}$$

Assuming negligible centrifugal force and setting $F_1 = ab$ from step 3,

$$b_{\min} = \frac{\Delta F}{a}\left[\frac{\exp(f\theta)}{\exp(f\theta) - 1}\right] \tag{1}$$

Also, $$H_d = H_{\text{nom}} K_s n_d = \frac{(\Delta F)V}{33\,000}$$

$$\Delta F = \frac{33\,000 H_{\text{nom}} K_s n_d}{V}$$

Substituting into (1), $$b_{\min} = \frac{1}{a}\left(\frac{33\,000 H_d}{V}\right)\frac{\exp(f\theta)}{\exp(f\theta) - 1} \quad \textit{Ans.}$$

17-14 The decision set for the friction metal flat-belt drive is:

A priori decisions

- Function: $H_{\text{nom}} = 1$ hp, $n = 1750$ rev/min, $VR = 2$, $C \doteq 15$ in, $K_s = 1.2$, $N_p = 10^6$ belt passes.
- Design factor: $n_d = 1.05$
- Belt material and properties: 301/302 stainless steel
  Table 17-8: $S_y = 175\,000$ psi, $E = 28$ Mpsi, $\nu = 0.285$

- Drive geometry: $d = 2$ in, $D = 4$ in
- Belt thickness: $t = 0.003$ in

Design variables:

- Belt width b
- Belt loop periphery

Preliminaries

$$H_d = H_{\text{nom}} K_s n_d = 1(1.2)(1.05) = 1.26 \text{ hp}$$

$$T = \frac{63\,025(1.26)}{1750} = 45.38 \text{ lbf} \cdot \text{in}$$

A 15 in center-to-center distance corresponds to a belt loop periphery of 39.5 in. The 40 in loop available corresponds to a 15.254 in center distance.

$$\theta_d = \pi - 2\sin^{-1}\left[\frac{4-2}{2(15.254)}\right] = 3.010 \text{ rad}$$

$$\theta_D = \pi + 2\sin^{-1}\left[\frac{4-2}{2(15.274)}\right] = 3.273 \text{ rad}$$

For full friction development

$$\exp(f\theta_d) = \exp[0.35(3.010)] = 2.868$$

$$V = \frac{\pi d n}{12} = \frac{\pi(2)(1750)}{12} = 916.3 \text{ ft/s}$$

$$S_y = 175\,000 \text{ psi}$$

Eq. (17-15):

$$S_f = 14.17(10^6)(10^6)^{-0.407} = 51\,212 \text{ psi}$$

From selection step 3

$$a = \left[S_f - \frac{Et}{(1-\nu^2)d}\right]t = \left[51\,212 - \frac{28(10^6)(0.003)}{(1-0.285^2)(2)}\right](0.003)$$

$$= 16.50 \text{ lbf/in} \quad \text{of belt width}$$

$$(F_1)_a = ab = 16.50b$$

For full friction development, from Prob. 17-13,

$$b_{\min} = \frac{\Delta F}{a} \frac{\exp(f\theta_d)}{\exp(f\theta_d) - 1}$$

$$\Delta F = \frac{2T}{d} = \frac{2(45.38)}{2} = 45.38 \text{ lbf}$$

So

$$b_{\min} = \frac{45.38}{16.50}\left(\frac{2.868}{2.868 - 1}\right) = 4.23 \text{ in}$$

Decision #1: $b = 4.5$ in

$$F_1 = (F_1)_a = ab = 16.5(4.5) = 74.25 \text{ lbf}$$
$$F_2 = F_1 - \Delta F = 74.25 - 45.38 = 28.87 \text{ lbf}$$
$$F_i = \frac{F_1 + F_2}{2} = \frac{74.25 + 28.87}{2} = 51.56 \text{ lbf}$$

Existing friction

$$f' = \frac{1}{\theta_d} \ln\left(\frac{F_1}{F_2}\right) = \frac{1}{3.010} \ln\left(\frac{74.25}{28.87}\right) = 0.314$$
$$H_t = \frac{(\Delta F)V}{33\,000} = \frac{45.38(916.3)}{33\,000} = 1.26 \text{ hp}$$
$$n_{fs} = \frac{H_t}{H_{\text{nom}} K_s} = \frac{1.26}{1(1.2)} = 1.05$$

This is a non-trivial point. The methodology preserved the factor of safety corresponding to $n_d = 1.1$ *even* as we rounded $b_{\min}$ up to b.

Decision #2 was taken care of with the adjustment of the center-to-center distance to accommodate the belt loop. Use Eq. (17-2) as is and solve for C to assist in this. Remember to subsequently recalculate θ_d and θ_D.

17-15 Decision set:

A priori decisions

- Function: $H_{\text{nom}} = 5$ hp, $N = 1125$ rev/min, $VR = 3$, $C \doteq 20$ in, $K_s = 1.25$, $N_p = 10^6$ belt passes
- Design factor: $n_d = 1.1$
- Belt material: BeCu, $S_y = 170\,000$ psi, $E = 17(10^6)$ psi, $\nu = 0.220$
- Belt geometry: $d = 3$ in, $D = 9$ in
- Belt thickness: $t = 0.003$ in

Design decisions

- Belt loop periphery
- Belt width b

Preliminaries:

$$H_d = H_{\text{nom}} K_s n_d = 5(1.25)(1.1) = 6.875 \text{ hp}$$
$$T = \frac{63\,025(6.875)}{1125} = 385.2 \text{ lbf} \cdot \text{in}$$

Decision #1: Choose a 60-in belt loop with a center-to-center distance of 20.3 in.

$$\theta_d = \pi - 2\sin^{-1}\left[\frac{9-3}{2(20.3)}\right] = 2.845 \text{ rad}$$
$$\theta_D = \pi + 2\sin^{-1}\left[\frac{9-3}{2(20.3)}\right] = 3.438 \text{ rad}$$

For full friction development:

$$\exp(f\theta_d) = \exp[0.32(2.845)] = 2.485$$

$$V = \frac{\pi dn}{12} = \frac{\pi(3)(1125)}{12} = 883.6 \text{ ft/min}$$

$$S_f = 56\,670 \text{ psi}$$

From selection step 3

$$a = \left[S_f - \frac{Et}{(1-\nu^2)d}\right]t = \left[56\,670 - \frac{17(10^6)(0.003)}{(1-0.22^2)(3)}\right](0.003) = 116.4 \text{ lbf/in}$$

$$\Delta F = \frac{2T}{d} = \frac{2(385.2)}{3} = 256.8 \text{ lbf}$$

$$b_{\min} = \frac{\Delta F}{a}\left[\frac{\exp(f\theta_d)}{\exp(f\theta_d)-1}\right] = \frac{256.8}{116.4}\left(\frac{2.485}{2.485-1}\right) = 3.69 \text{ in}$$

Decision #2: $b = 4$ in

$$F_1 = (F_1)_a = ab = 116.4(4) = 465.6 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 465.6 - 256.8 = 208.8 \text{ lbf}$$

$$F_i = \frac{F_1 + F_2}{2} = \frac{465.6 + 208.8}{2} = 337.3 \text{ lbf}$$

Existing friction

$$f' = \frac{1}{\theta_d}\ln\left(\frac{F_1}{F_2}\right) = \frac{1}{2.845}\ln\left(\frac{465.6}{208.8}\right) = 0.282$$

$$H = \frac{(\Delta F)V}{33\,000} = \frac{256.8(883.6)}{33\,000} = 6.88 \text{ hp}$$

$$n_{fs} = \frac{H}{5(1.25)} = \frac{6.88}{5(1.25)} = 1.1$$

F_i can be reduced only to the point at which $f' = f = 0.32$. From Eq. (17-9)

$$F_i = \frac{T}{d}\left[\frac{\exp(f\theta_d)+1}{\exp(f\theta_d)-1}\right] = \frac{385.2}{3}\left(\frac{2.485+1}{2.485-1}\right) = 301.3 \text{ lbf}$$

Eq. (17-10):

$$F_1 = F_i\left[\frac{2\exp(f\theta_d)}{\exp(f\theta_d)+1}\right] = 301.3\left[\frac{2(2.485)}{2.485+1}\right] = 429.7 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 429.7 - 256.8 = 172.9 \text{ lbf}$$

and $f' = f = 0.32$

17-16 This solution is the result of a series of five design tasks involving different belt thicknesses. The results are to be compared as a matter of perspective. These design tasks are accomplished in the same manner as in Probs. 17-14 and 17-15 solutions.

The details will not be presented here, but the table is provided as a means of learning. Five groups of students could each be assigned a belt thickness. You can form a table from their results or use the table below

	t, in				
	0.002	0.003	0.005	0.008	0.010
b	4.000	3.500	4.000	1.500	1.500
CD	20.300	20.300	20.300	18.700	20.200
a	109.700	131.900	110.900	194.900	221.800
d	3.000	3.000	3.000	5.000	6.000
D	9.000	9.000	9.000	15.000	18.000
F_i	310.600	333.300	315.200	215.300	268.500
F_1	439.000	461.700	443.600	292.300	332.700
F_2	182.200	209.000	186.800	138.200	204.300
n_{fs}	1.100	1.100	1.100	1.100	1.100
L	60.000	60.000	60.000	70.000	80.000
f'	0.309	0.285	0.304	0.288	0.192
F_i	301.200	301.200	301.200	195.700	166.600
F_1	429.600	429.600	429.600	272.700	230.800
F_2	172.800	172.800	172.800	118.700	102.400
f	0.320	0.320	0.320	0.320	0.320

The first three thicknesses result in the same adjusted F_i, F_1 and F_2 (why?). We have no figure of merit, but the costs of the belt and the pulleys is about the same for these three thicknesses. Since the same power is transmitted and the belts are widening, belt forces are lessening.

17-17 This is a design task. The decision variables would be belt length and belt section, which could be combined into one, such as B90. The number of belts is not an issue.

We have no figure of merit, which is not practical in a text for this application. I suggest you gather sheave dimensions and costs and V-belt costs from a principal vendor and construct a figure of merit based on the costs. Here is one trial.

Preliminaries: For a single V-belt drive with $H_{\text{nom}} = 3$ hp, $n = 3100$ rev/min, $D = 12$ in, and $d = 6.2$ in, choose a B90 belt, $K_s = 1.3$ and $n_d = 1$.

$$L_p = 90 + 1.8 = 91.8 \text{ in}$$

Eq. (17-16*b*):

$$C = 0.25\left\{\left[91.8 - \frac{\pi}{2}(12 + 6.2)\right] + \sqrt{\left[91.8 - \frac{\pi}{2}(12 + 6.2)\right]^2 - 2(12 - 6.2)^2}\right\}$$

$$= 31.47 \text{ in}$$

$$\theta_d = \pi - 2\sin^{-1}\left[\frac{12 - 6.2}{2(31.47)}\right] = 2.9570 \text{ rad}$$

$$\exp(f\theta_d) = \exp[0.5123(2.9570)] = 4.5489$$

$$V = \frac{\pi dn}{12} = \frac{\pi(6.2)(3100)}{12} = 5031.8 \text{ ft/min}$$

Table 17-13:

$$\text{Angle } \theta = \theta_d \frac{180°}{\pi} = (2.957 \text{ rad}) \left(\frac{180°}{\pi}\right) = 169.42°$$

The footnote regression equation gives K_1 without interpolation:

$$K_1 = 0.143\,543 + 0.007\,468(169.42°) - 0.000\,015\,052(169.42°)^2 = 0.9767$$

The design power is

$$H_d = H_{\text{nom}} K_s n_d = 3(1.3)(1) = 3.9 \text{ hp}$$

From Table 17-14 for B90, $K_2 = 1$. From Table 17-12 take a marginal entry of $H_{\text{tab}} = 4$, although extrapolation would give a slightly lower H_{tab}.

Eq. (17-17):

$$\begin{aligned} H_a &= K_1 K_2 H_{\text{tab}} \\ &= 0.9767(1)(4) = 3.91 \text{ hp} \end{aligned}$$

The allowable ΔF_a is given by

$$\Delta F_a = \frac{63\,025 H_a}{n(d/2)} = \frac{63\,025(3.91)}{3100(6.2/2)} = 25.6 \text{ lbf}$$

The allowable torque T_a is

$$T_a = \frac{\Delta F_a d}{2} = \frac{25.6(6.2)}{2} = 79.4 \text{ lbf} \cdot \text{in}$$

From Table 17-16, $K_c = 0.965$. Thus, Eq. (17-21) gives,

$$F_c = 0.965 \left(\frac{5031.8}{1000}\right)^2 = 24.4 \text{ lbf}$$

At incipient slip, Eq. (17-9) provides:

$$F_i = \left(\frac{T}{d}\right) \left[\frac{\exp(f\theta) + 1}{\exp(f\theta) - 1}\right] = \left(\frac{79.4}{6.2}\right) \left(\frac{4.5489 + 1}{4.5489 - 1}\right) = 20.0 \text{ lbf}$$

Eq. (17-10):

$$F_1 = F_c + F_i \left[\frac{2\exp(f\theta)}{\exp(f\theta) + 1}\right] = 24.4 + 20 \left[\frac{2(4.5489)}{4.5489 + 1}\right] = 57.2 \text{ lbf}$$

Thus, $F_2 = F_1 - \Delta F_a = 57.2 - 25.6 = 31.6$ lbf

Eq. (17-26):

$$n_{fs} = \frac{H_a N_b}{H_d} = \frac{(3.91)(1)}{3.9} = 1.003 \quad \textit{Ans.}$$

If we had extrapolated for H_{tab}, the factor of safety would have been slightly less than one.

Life Use Table 17-16 to find equivalent tensions T_1 and T_2.

$$T_1 = F_1 + (F_b)_1 = F_1 + \frac{K_b}{d} = 57.2 + \frac{576}{6.2} = 150.1 \text{ lbf}$$

$$T_2 = F_1 + (F_b)_2 = F_1 + \frac{K_b}{D} = 57.2 + \frac{576}{12} = 105.2 \text{ lbf}$$

From Eq. (17-27), the number of belt passes is:

$$N_P = \left[\left(\frac{1193}{150.1}\right)^{-10.929} + \left(\frac{1193}{105.2}\right)^{-10.929}\right]^{-1} = 6.76(10^9)$$

From Eq. (17-28) for $N_P > 10^9$,

$$t = \frac{N_P L_p}{720V} > \frac{10^9(91.8)}{720(5031.8)}$$

$$t > 25\,340 \text{ h} \quad Ans.$$

Suppose n_{fs} was too small. Compare these results with a 2-belt solution.

$$H_{\text{tab}} = 4 \text{ hp/belt}, \quad T_a = 39.6 \text{ lbf} \cdot \text{in/belt},$$

$$\Delta F_a = 12.8 \text{ lbf/belt}, \quad H_a = 3.91 \text{ hp/belt}$$

$$n_{fs} = \frac{N_b H_a}{H_d} = \frac{N_b H_a}{H_{\text{nom}} K_s} = \frac{2(3.91)}{3(1.3)} = 2.0$$

Also,

$$F_1 = 40.8 \text{ lbf/belt}, \qquad F_2 = 28.0 \text{ lbf/belt},$$

$$F_i = 9.99 \text{ lbf/belt}, \qquad F_c = 24.4 \text{ lbf/belt}$$

$$(F_b)_1 = 92.9 \text{ lbf/belt}, \quad (F_b)_2 = 48 \text{ lbf/belt}$$

$$T_1 = 133.7 \text{ lbf/belt}, \qquad T_2 = 88.8 \text{ lbf/belt}$$

$$N_P = 2.39(10^{10}) \text{ passes}, \quad t > 605\,600 \text{ h}$$

Initial tension of the drive:

$$(F_i)_{\text{drive}} = N_b F_i = 2(9.99) = 20 \text{ lbf}$$

17-18 Given: two B85 V-belts with $d = 5.4$ in, $D = 16$ in, $n = 1200$ rev/min, and $K_s = 1.25$

Table 17-11: $L_p = 85 + 1.8 = 86.8$ in

Eq. (17-17*b*):

$$C = 0.25\left\{\left[86.8 - \frac{\pi}{2}(16+5.4)\right] + \sqrt{\left[86.8 - \frac{\pi}{2}(16+5.4)\right]^2 - 2(16-5.4)^2}\right\}$$

$$= 26.05 \text{ in} \quad Ans.$$

Eq. (17-1):

$$\theta_d = 180^\circ - 2\sin^{-1}\left[\frac{16-5.4}{2(26.05)}\right] = 156.5^\circ$$

From table 17-13 footnote:

$$K_1 = 0.143\,543 + 0.007\,468(156.5^\circ) - 0.000\,015\,052(156.5^\circ)^2 = 0.944$$

Table 17-14: $K_2 = 1$

Belt speed: $V = \dfrac{\pi(5.4)(1200)}{12} = 1696$ ft/min

Use Table 17-12 to interpolate for H_{tab}.

$$H_{\text{tab}} = 1.59 + \left(\frac{2.62 - 1.59}{2000 - 1000}\right)(1696 - 1000) = 2.31 \text{ hp/belt}$$

$$H_a = K_1 K_2 N_b H_{\text{tab}} = 1(0.944)(2)(2.31) = 4.36 \text{ hp}$$

Assuming $n_d = 1$

$$H_d = K_s H_{\text{nom}} n_d = 1.25(1) H_{\text{nom}}$$

For a factor of safety of one,

$$H_a = H_d$$

$$4.36 = 1.25 H_{\text{nom}}$$

$$H_{\text{nom}} = \frac{4.36}{1.25} = 3.49 \text{ hp} \quad \textit{Ans.}$$

17-19 Given: $H_{\text{nom}} = 60$ hp, $n = 400$ rev/min, $K_s = 1.4$, $d = D = 26$ in on 12 ft centers. Design task: specify V-belt and number of strands (belts). *Tentative decision*: Use D360 belts.

Table 17-11: $L_p = 360 + 3.3 = 363.3$ in

Eq. (17-16*b*):

$$C = 0.25 \left\{ \left[363.3 - \frac{\pi}{2}(26 + 26) \right] + \sqrt{\left[363.3 - \frac{\pi}{2}(26 + 26) \right]^2 - 2(26 - 26)^2} \right\}$$

$$= 140.8 \text{ in (nearly 144 in)}$$

$$\theta_d = \pi, \quad \theta_D = \pi, \quad \exp[0.5123\pi] = 5.0,$$

$$V = \frac{\pi d n}{12} = \frac{\pi(26)(400)}{12} = 2722.7 \text{ ft/min}$$

Table 17-13: For $\theta = 180°$, $K_1 = 1$

Table 17-14: For D360, $K_2 = 1.10$

Table 17-12: $H_{\text{tab}} = 16.94$ hp by interpolation

Thus, $H_a = K_1 K_2 H_{\text{tab}} = 1(1.1)(16.94) = 18.63$ hp

$$H_d = K_s H_{\text{nom}} = 1.4(60) = 84 \text{ hp}$$

Number of belts, N_b

$$N_b = \frac{K_s H_{\text{nom}}}{K_1 K_2 H_{\text{tab}}} = \frac{H_d}{H_a} = \frac{84}{18.63} = 4.51$$

Round up to five belts. It is left to the reader to repeat the above for belts such as C360 and E360.

$$\Delta F_a = \frac{63\,025 H_a}{n(d/2)} = \frac{63\,025(18.63)}{400(26/2)} = 225.8 \text{ lbf/belt}$$

$$T_a = \frac{(\Delta F_a) d}{2} = \frac{225.8(26)}{2} = 2935 \text{ lbf} \cdot \text{in/belt}$$

Eq. (17-21):

$$F_c = 3.498\left(\frac{V}{1000}\right)^2 = 3.498\left(\frac{2722.7}{1000}\right)^2 = 25.9 \text{ lbf/belt}$$

At fully developed friction, Eq. (17-9) gives

$$F_i = \frac{T}{d}\left[\frac{\exp(f\theta)+1}{\exp(f\theta)-1}\right] = \frac{2935}{26}\left(\frac{5+1}{5-1}\right) = 169.3 \text{ lbf/belt}$$

Eq. (17-10): $F_1 = F_c + F_i\left[\dfrac{2\exp(f\theta)}{\exp(f\theta)+1}\right] = 25.9 + 169.3\left[\dfrac{2(5)}{5+1}\right] = 308.1 \text{ lbf/belt}$

$$F_2 = F_1 - \Delta F_a = 308.1 - 225.8 = 82.3 \text{ lbf/belt}$$

$$n_{fs} = \frac{H_a N_b}{H_d} = \frac{(185.63)}{84} = 1.109 \quad \textit{Ans.}$$

Reminder: Initial tension is for the *drive*

$$(F_i)_{\text{drive}} = N_b F_i = 5(169.3) = 846.5 \text{ lbf}$$

A 360 belt is at the right-hand edge of the range of center-to-center pulley distances.

$$D \le C \le 3(D+d)$$
$$26 \le C \le 3(26+26)$$

17-20 Preliminaries: $D \doteq 60$ in, 14-in wide rim, $H_{\text{nom}} = 50$ hp, $n = 875$ rev/min, $K_s = 1.2$, $n_d = 1.1$, $m_G = 875/170 = 5.147$, $d \doteq 60/5.147 = 11.65$ in

(a) From Table 17-9, an 11-in sheave exceeds C-section minimum diameter and precludes D- and E-section V-belts.

Decision: Use $d = 11$ in, C270 belts

Table 17-11: $L_p = 270 + 2.9 = 272.9$ in

$$C = 0.25\left\{\left[272.9 - \frac{\pi}{2}(60+11)\right] + \sqrt{\left[272.9 - \frac{\pi}{2}(60+11)\right]^2 - 2(60-11)^2}\right\}$$
$$= 76.78 \text{ in}$$

This fits in the range

$$D < C < 3(D+d)$$
$$60 < C < 3(60+11)$$
$$60 \text{ in} < C < 213 \text{ in}$$

$$\theta_d = \pi - 2\sin^{-1}\left[\frac{60-11}{2(76.78)}\right] = 2.492 \text{ rad}$$

$$\theta_D = \pi + 2\sin^{-1}\left[\frac{60-11}{2(76.78)}\right] = 3.791 \text{ rad}$$

$$\exp[0.5123(2.492)] = 3.5846$$

For the flat on flywheel

$$\exp[0.13(3.791)] = 1.637$$

$$V = \frac{\pi dn}{12} = \frac{\pi(11)(875)}{12} = 2519.8 \text{ ft/min}$$

Table 17-13: Regression equation gives $K_1 = 0.90$

Table 17-14: $K_2 = 1.15$

Table 17-12: $H_{\text{tab}} = 7.83$ hp/belt by interpolation

Eq. (17-17): $H_a = K_1 K_2 H_{\text{tab}} = 0.905(1.15)(7.83) = 8.15$ hp

Eq. (17-19): $H_d = H_{\text{nom}} K_s n_d = 50(1.2)(1.1) = 66$ hp

Eq. (17-20): $N_b = \dfrac{H_d}{H_a} = \dfrac{66}{8.15} = 8.1$ belts

Decision: Use 9 belts. On a per belt basis,

$$\Delta F_a = \frac{63\,025 H_a}{n(d/2)} = \frac{63\,025(8.15)}{875(11/2)} = 106.7 \text{ lbf/belt}$$

$$T_a = \frac{\Delta F_a d}{2} = \frac{106.7(11)}{2} = 586.9 \text{ lbf per belt}$$

$$F_c = 1.716\left(\frac{V}{1000}\right)^2 = 1.716\left(\frac{2519.8}{1000}\right)^2 = 10.9 \text{ lbf/belt}$$

At fully developed friction, Eq. (17-9) gives

$$F_i = \frac{T}{d}\left[\frac{\exp(f\theta_d)+1}{\exp(f\theta_d)-1}\right] = \frac{586.9}{11}\left(\frac{3.5846+1}{3.5846-1}\right) = 94.6 \text{ lbf/belt}$$

Eq. (17-10):

$$F_1 = F_c + F_i\left[\frac{2\exp(f\theta_d)}{\exp(f\theta_d)+1}\right] = 10.9 + 94.6\left[\frac{2(3.5846)}{3.5846+1}\right] = 158.8 \text{ lbf/belt}$$

$$F_2 = F_1 - \Delta F_a = 158.8 - 106.7 = 52.1 \text{ lbf/belt}$$

$$n_{fs} = \frac{N_b H_a}{H_d} = \frac{9(8.15)}{66} = 1.11 \quad \textit{O.K.} \quad \textit{Ans.}$$

Durability:

$$(F_b)_1 = 145.45 \text{ lbf/belt}, \quad (F_b)_2 = 76.7 \text{ lbf/belt}$$

$$T_1 = 304.4 \text{ lbf/belt}, \qquad T_2 = 185.6 \text{ lbf/belt}$$

and $t > 150\,000$ h

Remember: $(F_i)_{\text{drive}} = 9(94.6) = 851.4$ lbf

Table 17-9: C-section belts are 7/8" wide. Check sheave groove spacing to see if 14"-width is accommodating.

(b) The fully developed friction torque on the flywheel using the flats of the V-belts is

$$T_{\text{flat}} = \Delta F_i \left[\frac{\exp(f\theta) - 1}{\exp(f\theta) + 1}\right] = 60(94.6)\left(\frac{1.637 - 1}{1.637 + 1}\right) = 1371 \text{ lbf} \cdot \text{in} \quad \text{per belt}$$

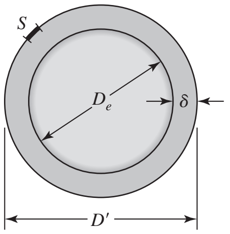
The flywheel torque should be

$$T_{\text{fly}} = m_G T_a = 5.147(586.9) = 3021 \text{ lbf} \cdot \text{in} \quad \text{per belt}$$

but it is not. There are applications, however, in which it will work. For example, make the flywheel controlling. Yes. *Ans.*

17-21

(a)



S is the spliced-in string segment length

D_e is the equatorial diameter

D' is the spliced string diameter

δ is the radial clearance

$$S + \pi D_e = \pi D' = \pi(D_e + 2\delta) = \pi D_e + 2\pi\delta$$

From which

$$\delta = \frac{S}{2\pi}$$

The radial clearance is thus *independent* of D_e.

$$\delta = \frac{12(6)}{2\pi} = 11.5 \text{ in} \quad Ans.$$

This is true whether the sphere is the earth, the moon or a marble. Thinking in terms of a radial or diametral increment removes the basic size from the problem. *Viewpoint again!*

(b) and **(c)**

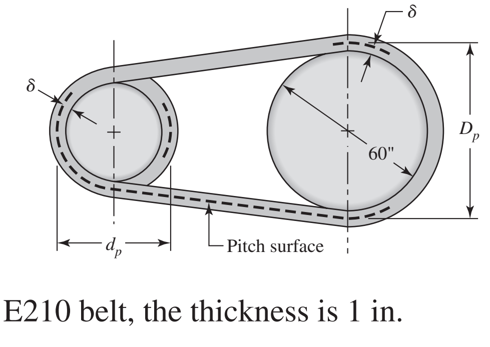
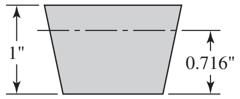


Table 17-9: For an E210 belt, the thickness is 1 in.



$$d_P - d_i = \frac{210 + 4.5}{\pi} - \frac{210}{\pi} = \frac{4.5}{\pi}$$

$$2\delta = \frac{4.5}{\pi}$$

$$\delta = \frac{4.5}{2\pi} = 0.716 \text{ in}$$

The pitch diameter of the flywheel is

$$D_P - 2\delta = D$$

$$D_P = D + 2\delta = 60 + 2(0.716) = 61.43 \text{ in}$$

We could make a table:

Diametral Growth	Section				
	A	*B*	*C*	*D*	*E*
2δ	$\frac{1.3}{\pi}$	$\frac{1.8}{\pi}$	$\frac{2.9}{\pi}$	$\frac{3.3}{\pi}$	$\frac{4.5}{\pi}$

The velocity ratio for the D-section belt of Prob. 17-20 is

$$m'_G = \frac{D + 2\delta}{d} = \frac{60 + 3.3/\pi}{11} = 5.55 \quad Ans.$$

for the V-flat drive as compared to $m_a = 60/11 = 5.455$ for the VV drive.

The pitch diameter of the pulley is still $d = 11$ in, so the new angle of wrap, θ_d, is

$$\theta_d = \pi - 2\sin^{-1}\left(\frac{D + 2\delta - d}{2C}\right) \quad Ans.$$

$$\theta_D = \pi + 2\sin^{-1}\left(\frac{D + 2\delta - d}{2C}\right) \quad Ans.$$

Equations (17-16*a*) and (17-16*b*) are modified as follows

$$L_p = 2C + \frac{\pi}{2}(D + 2\delta + d) + \frac{(D + \delta - d)^2}{4C} \quad Ans.$$

$$C_p = 0.25\left\{\left[L_p - \frac{\pi}{2}(D + 2\delta + d)\right] + \sqrt{\left[L_p - \frac{\pi}{2}(D + 2\delta + d)\right]^2 - 2(D + 2\delta - d)^2}\right\} \quad Ans.$$

The changes are small, but if you are writing a computer code for a V-flat drive, remember that θ_d and θ_D changes are exponential.

17-22 This design task involves specifying a drive to couple an electric motor running at 1720 rev/min to a blower running at 240 rev/min, transmitting two horsepower with a center distance of at least 22 inches. Instead of focusing on the steps, we will display two different designs side-by-side for study. Parameters are in a "per belt" basis with per drive quantities shown along side, where helpful.

Parameter	Four A-90 Belts	Two A-120 Belts
m_G	7.33	7.142
K_s	1.1	1.1
n_d	1.1	1.1
K_1	0.877	0.869
K_2	1.05	1.15
d, in	3.0	4.2
D, in	22	30
θ_d, rad	2.333	2.287
V, ft/min	1350.9	1891
$\exp(f\theta_d)$	3.304	3.2266
L_p, in	91.3	101.3
C, in	24.1	31
H_{tab}, uncorr.	0.783	1.662
$N_b H_{\text{tab}}$, uncorr.	3.13	3.326
T_a, lbf · in	26.45(105.8)	60.87(121.7)
ΔF_a, lbf	17.6(70.4)	29.0(58)
H_a, hp	0.721(2.88)	1.667(3.33)
n_{fs}	1.192	1.372
F_1, lbf	26.28(105.2)	44(88)
F_2, lbf	8.67(34.7)	15(30)
$(F_b)_1$, lbf	73.3(293.2)	52.4(109.8)
$(F_b)_2$, lbf	10(40)	7.33(14.7)
F_c, lbf	1.024	2.0
F_i, lbf	16.45(65.8)	27.5(55)
T_1, lbf · in	99.2	96.4
T_2, lbf · in	36.3	57.4
N', passes	$1.61(10^9)$	$2.3(10^9)$
$t > h$	93 869	89 080

Conclusions:

- Smaller sheaves lead to more belts.
- Larger sheaves lead to larger *D* and larger *V*.
- Larger sheaves lead to larger tabulated power.
- The discrete numbers of belts obscures some of the variation. The factors of safety exceed the design factor by differing amounts.

17-23 In Ex. 17-5 the selected chain was 140-3, making the pitch of this 140 chain $14/8 = 1.75$ in. Table 17-19 confirms.

17-24

(a) Eq. (17-32): $$H_1 = 0.004N_1^{1.08}n_1^{0.9}p^{(3-0.07p)}$$

Eq. (17-33): $$H_2 = \frac{1000K_r N_1^{1.5}p^{0.8}}{n_1^{1.5}}$$

Equating and solving for n_1 gives

$$n_1 = \left[\frac{0.25(10^6)K_r N_1^{0.42}}{p^{(2.2-0.07p)}}\right]^{1/2.4} \qquad Ans.$$

(b) For a No. 60 chain, $p = 6/8 = 0.75$ in, $N_1 = 17$, $K_r = 17$

$$n_1 = \left\{\frac{0.25(10^6)(17)(17)^{0.42}}{0.75^{[2.2-0.07(0.75)]}}\right\}^{1/2.4} = 1227 \text{ rev/min} \quad Ans.$$

Table 17-20 confirms that this point occurs at 1200 ± 200 rev/min.

(c) Life predictions using Eq. (17-40) are possible at speeds greater than 1227 rev/min. *Ans.*

17-25 Given: a double strand No. 60 roller chain with $p = 0.75$ in, $N_1 = 13$ teeth at 300 rev/min, $N_2 = 52$ teeth.

(a) Table 17-20: $H_{\text{tab}} = 6.20$ hp

Table 17-22: $K_1 = 0.75$

Table 17-23: $K_2 = 1.7$

Use $K_s = 1$

Eq. (17-37):

$$H_a = K_1 K_2 H_{\text{tab}} = 0.75(1.7)(6.20) = 7.91 \text{ hp} \quad Ans.$$

(b) Eqs. (17-35) and (17-36) with $L/p = 82$

$$A = \frac{13+52}{2} - 82 = -49.5$$

$$C = \frac{p}{4}\left[49.5 + \sqrt{49.5^2 - 8\left(\frac{52-13}{2\pi}\right)^2}\right] = 23.95p$$

$$C = 23.95(0.75) = 17.96 \text{ in}, \quad \text{round up to 18 in} \quad Ans.$$

(c) For 30 percent less power transmission,

$$H = 0.7(7.91) = 5.54 \text{ hp}$$

$$T = \frac{63\,025(5.54)}{300} = 1164 \text{ lbf} \cdot \text{in} \quad Ans.$$

Eq. (17-29):

$$D = \frac{0.75}{\sin(180^\circ/13)} = 3.13 \text{ in}$$

$$F = \frac{T}{r} = \frac{1164}{3.13/2} = 744 \text{ lbf} \quad Ans.$$

17-26 Given: No. 40-4 chain, $N_1 = 21$ teeth for $n = 2000$ rev/min, $N_2 = 84$ teeth, $h = 20\,000$ hours.

(a) Chain pitch is $p = 4/8 = 0.500$ in and $C \doteq 20$ in.

Eq. (17-34):
$$\frac{L}{p} = \frac{2(20)}{0.5} + \frac{21+84}{2} + \frac{(84-21)^2}{4\pi^2(20/0.5)} = 135 \text{ pitches} \quad \text{(or links)}$$
$$L = 135(0.500) = 67.5 \text{ in} \quad Ans.$$

(b) Table 17-20: $H_{\text{tab}} = 7.72$ hp (post-extreme power)

Eq. (17-40): Since K_1 is required, the $N_1^{3.75}$ term is omitted.
$$\text{const} = \frac{\left(7.72^{2.5}\right)(15\,000)}{135} = 18\,399$$
$$H'_{\text{tab}} = \left[\frac{18\,399(135)}{20\,000}\right]^{1/2.5} = 6.88 \text{ hp} \quad Ans.$$

(c) Table 17-22:
$$K_1 = \left(\frac{21}{17}\right)^{1.5} = 1.37$$

Table 17-23: $K_2 = 3.3$
$$H_a = K_1K_2H'_{\text{tab}} = 1.37(3.3)(6.88) = 31.1 \text{ hp} \quad Ans.$$

(d)
$$V = \frac{N_1pn}{12} = \frac{21(0.5)(2000)}{12} = 1750 \text{ ft/min}$$
$$F_1 = \frac{33\,000(31.1)}{1750} = 586 \text{ lbf} \quad Ans.$$

17-27 This is our first design/selection task for chain drives. A possible decision set:

A priori decisions

- Function: H_{nom}, n_1, space, life, K_s
- Design factor: n_d
- Sprockets: Tooth counts N_1 and N_2, factors K_1 and K_2

Decision variables

- Chain number
- Strand count
- Lubrication type
- Chain length in pitches

Function: Motor with $H_{\text{nom}} = 25$ hp at $n = 700$ rev/min; pump at $n = 140$ rev/min; $m_G = 700/140 = 5$

Design Factor: $n_d = 1.1$

Sprockets: Tooth count $N_2 = m_GN_1 = 5(17) = 85$ teeth–odd and unavailable. Choose 84 teeth. *Decision*: $N_1 = 17$, $N_2 = 84$

Evaluate K_1 and K_2

Eq. (17-38): $H_d = H_{\text{nom}} K_s n_d$

Eq. (17-37): $H_a = K_1 K_2 H_{\text{tab}}$

Equate H_d to H_a and solve for H_{tab}:

$$H_{\text{tab}} = \frac{K_s n_d H_{\text{nom}}}{K_1 K_2}$$

Table 17-22: $K_1 = 1$

Table 17-23: $K_2 = 1$, 1.7, 2.5, 3.3 for 1 through 4 strands

$$H'_{\text{tab}} = \frac{1.5(1.1)(25)}{(1)K_2} = \frac{41.25}{K_2}$$

Prepare a table to help with the design decisions:

Strands	K_2	H'_{tab}	Chain No.	H_{tab}	n_{fs}	Lub. Type
1	1.0	41.3	100	59.4	1.58	B
2	1.7	24.3	80	31.0	1.40	B
3	2.5	16.5	80	31.0	2.07	B
4	3.3	12.5	60	13.3	1.17	B

Design Decisions

We need a figure of merit to help with the choice. If the best was 4 strands of No. 60 chain, then

Decision #1 and #2: Choose four strand No. 60 roller chain with $n_{fs} = 1.17$.

$$n_{fs} = \frac{K_1 K_2 H_{\text{tab}}}{K_s H_{\text{nom}}} = \frac{1(3.3)(13.3)}{1.5(25)} = 1.17$$

Decision #3: Choose Type B lubrication

Analysis:

Table 17-20: $H_{\text{tab}} = 13.3$ hp

Table 17-19: $p = 0.75$ in

Try $C = 30$ in in Eq. (17-34):

$$\begin{aligned}\frac{L}{p} &= \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p} \\ &= 2(30/0.75) + \frac{17 + 84}{2} + \frac{(84 - 17)^2}{4\pi^2(30/0.75)} \\ &= 133.3 \to 134\end{aligned}$$

From Eq. (17-35) with $p = 0.75$ in, $C = 30.26$ in.

Decision #4: Choose $C = 30.26$ in.

17-28 Follow the decision set outlined in Prob. 17-27 solution. We will form two tables, the first for a 15 000 h life goal, and a second for a 50 000 h life goal. The comparison is useful.

Function: $H_{\text{nom}} = 50$ hp at $n = 1800$ rev/min, $n_{\text{pump}} = 900$ rev/min
$m_G = 1800/900 = 2, \quad K_s = 1.2$
life $= 15\,000$ h, then repeat with life $= 50\,000$ h

Design factor: $n_d = 1.1$

Sprockets: $N_1 = 19$ teeth, $N_2 = 38$ teeth

Table 17-22 (post extreme):

$$K_1 = \left(\frac{N_1}{17}\right)^{1.5} = \left(\frac{19}{17}\right)^{1.5} = 1.18$$

Table 17-23:

$$K_2 = 1,\ 1.7,\ 2.5,\ 3.3,\ 3.9,\ 4.6,\ 6.0$$

Decision variables for 15 000 h life goal:

$$H'_{\text{tab}} = \frac{K_s n_d H_{\text{nom}}}{K_1 K_2} = \frac{1.2(1.1)(50)}{1.18 K_2} = \frac{55.9}{K_2} \tag{1}$$

$$n_{fs} = \frac{K_1 K_2 H_{\text{tab}}}{K_s H_{\text{nom}}} = \frac{1.18 K_2 H_{\text{tab}}}{1.2(50)} = 0.0197 K_2 H_{\text{tab}}$$

Form a table for a 15 000 h life goal using these equations.

	K_2	H'_{tab}	Chain #	H_{tab}	n_{fs}	Lub
1	1.0	55.90	120	21.6	0.423	C′
2	1.7	32.90	120	21.6	0.923	C′
3	2.5	22.40	120	21.6	1.064	C′
4	3.3	16.90	120	21.6	1.404	C′
5	3.9	14.30	80	15.6	1.106	C′
6	4.6	12.20	60	12.4	1.126	C′
8	6.0	9.32	60	12.4	1.416	C′

There are 4 possibilities where $n_{fs} \geq 1.1$

Decision variables for 50 000 h life goal

From Eq. (17-40), the power-life tradeoff is:

$$(H'_{\text{tab}})^{2.5} 15\,000 = (H''_{\text{tab}})^{2.5} 50\,000$$

$$H''_{\text{tab}} = \left[\frac{15\,000}{50\,000}(H'_{\text{tab}})^{2.5}\right]^{1/2.5} = 0.618\ H'_{\text{tab}}$$

Substituting from (1),

$$H''_{\text{tab}} = 0.618\left(\frac{55.9}{K_2}\right) = \frac{34.5}{K_2}$$

The H'' notation is only necessary because we constructed the first table, which we normally would not do.

$$n_{fs} = \frac{K_1 K_2 H''_{\text{tab}}}{K_s H_{\text{nom}}} = \frac{K_1 K_2 (0.618 H'_{\text{tab}})}{K_s H_{\text{nom}}} = 0.618[(0.0197)K_2 H_{\text{tab}}] = 0.0122 K_2 H_{\text{tab}}$$

Form a table for a 50 000 h life goal.

	K_2	H''_{tab}	Chain #	H_{tab}	n_{fs}	Lub
1	1.0	34.50	120	21.6	0.264	C′
2	1.7	20.30	120	21.6	0.448	C′
3	2.5	13.80	120	21.6	0.656	C′
4	3.3	10.50	120	21.6	0.870	C′
5	3.9	8.85	120	21.6	1.028	C′
6	4.6	7.60	120	21.6	1.210	C′
8	6.0	5.80	80	15.6	1.140	C′

There are two possibilities in the second table with $n_{fs} \geq 1.1$. (The tables allow for the identification of a longer life one of the outcomes.) We need a figure of merit to help with the choice; costs of sprockets and chains are thus needed, but is more information than we have.

Decision #1: #80 Chain (smaller installation) *Ans.*

$$n_{fs} = 0.0122 K_2 H_{\text{tab}} = 0.0122(8.0)(15.6) = 1.14 \quad O.K.$$

Decision #2: 8-Strand, No. 80 *Ans.*

Decision #3: Type C′ Lubrication *Ans.*

Decision #4: $p = 1.0$ in, C is in midrange of 40 pitches

$$\begin{aligned}\frac{L}{p} &= \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p}\\ &= 2(40) + \frac{19+38}{2} + \frac{(38-19)^2}{4\pi^2(40)}\\ &= 108.7 \quad \Rightarrow \quad 110 \text{ even integer} \quad Ans.\end{aligned}$$

Eq. (17-36):

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p} = \frac{19+38}{2} - 110 = -81.5$$

Eq. (17-35): $$\frac{C}{p} = \frac{1}{4}\left[81.5 + \sqrt{81.5^2 - 8\left(\frac{38-19}{2\pi}\right)^2}\right] = 40.64$$

$$C = p(C/p) = 1.0(40.64) = 40.64 \text{ in (for reference)} \quad Ans.$$

17-29 The objective of the problem is to explore factors of safety in wire rope. We will express strengths as tensions.

(a) Monitor steel 2-in 6×19 rope, 480 ft long

Table 17-2: Minimum diameter of a sheave is $30d = 30(2) = 60$ in, preferably $45(2) = 90$ in. The hoist abuses the wire when it is bent around a sheave. Table 17-24 gives the nominal tensile strength as 106 kpsi. The ultimate load is

$$F_u = (S_u)_{\text{nom}} A_{\text{nom}} = 106 \left[\frac{\pi (2)^2}{4} \right] = 333 \text{ kip} \quad Ans.$$

The tensile loading of the wire is given by Eq. (17-46)

$$F_t = \left(\frac{W}{m} + wl \right) \left(1 + \frac{a}{g} \right)$$

$$W = 4(2) = 8 \text{ kip}, \quad m = 1$$

Table (17-24):

$$wl = 1.60 d^2 l = 1.60(2^2)(480) = 3072 \text{ lbf} \quad \text{or} \quad 3.072 \text{ kip}$$

Therefore,

$$F_t = (8 + 3.072) \left(1 + \frac{2}{32.2} \right) = 11.76 \text{ kip} \quad Ans.$$

Eq. (17-48):

$$F_b = \frac{E_r d_w A_m}{D}$$

and for the 72-in drum

$$F_b = \frac{12(10^6)(2/13)(0.38)(2^2)(10^{-3})}{72} = 39 \text{ kip} \quad Ans.$$

For use in Eq. (17-44), from Fig. 17-21

$$(p/S_u) = 0.0014$$

$$S_u = 240 \text{ kpsi}, \quad \text{p. 908}$$

$$F_f = \frac{0.0014(240)(2)(72)}{2} = 24.2 \text{ kip} \quad Ans.$$

(b) Factors of safety

Static, no bending:

$$n = \frac{F_u}{F_t} = \frac{333}{11.76} = 28.3 \quad Ans.$$

Static, with bending:

Eq. (17-49): $$n_s = \frac{F_u - F_b}{F_t} = \frac{333 - 39}{11.76} = 25.0 \quad Ans.$$

Fatigue without bending:

$$n_f = \frac{F_f}{F_t} = \frac{24.2}{11.76} = 2.06 \quad Ans.$$

Fatigue, with bending: For a life of $0.1(10^6)$ cycles, from Fig. 17-21

$$(p/S_u) = 4/1000 = 0.004$$

$$F_f = \frac{0.004(240)(2)(72)}{2} = 69.1 \text{ kip}$$

Eq. (17-50):

$$n_f = \frac{69.1 - 39}{11.76} = 2.56 \quad Ans.$$

If we were to use the endurance strength at 10^6 cycles ($F_f = 24.2$ kip) the factor of safety would be less than 1 indicating 10^6 cycle life impossible.

Comments:

- There are a number of factors of safety used in wire rope analysis. They are different, with different meanings. There is no substitute for knowing exactly which factor of safety is written or spoken.
- Static performance of a rope in tension is impressive.
- In this problem, at the drum, we have a finite life.
- The remedy for fatigue is the use of smaller diameter ropes, with multiple ropes supporting the load. See Ex. 17-6 for the effectiveness of this approach. It will also be used in Prob. 17-30.
- Remind students that wire ropes do not fail suddenly due to fatigue. The outer wires gradually show wear and breaks; such ropes should be retired. Periodic inspections prevent fatigue failures by parting of the rope.

17-30 Since this is a design task, a decision set is useful.

A priori decisions

- Function: load, height, acceleration, velocity, life goal
- Design Factor: n_d
- Material: IPS, PS, MPS or other
- Rope: Lay, number of strands, number of wires per strand

Decision variables:

- Nominal wire size: d
- Number of load-supporting wires: m

From experience with Prob. 17-29, a 1-in diameter rope is not likely to have much of a life, so approach the problem with the d and m decisions open.

Function: 5000 lbf load, 90 foot lift, acceleration = 4 ft/s^2, velocity = 2 ft/s, life goal = 10^5 cycles

Design Factor: $n_d = 2$

Material: IPS

Rope: Regular lay, 1-in plow-steel 6×19 hoisting

Design variables

Choose 30-in D_{min}. Table 17-27: $w = 1.60d^2$ lbf/ft

$$wl = 1.60d^2 l = 1.60d^2(90) = 144d^2 \text{ lbf, ea.}$$

Eq. (17-46):

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right) = \left(\frac{5000}{m} + 144d^2\right)\left(1 + \frac{4}{32.2}\right)$$

$$= \frac{5620}{m} + 162d^2 \text{ lbf,} \quad \text{each wire}$$

Eq. (17-47):

$$F_f = \frac{(p/S_u)S_u Dd}{2}$$

From Fig. 17-21 for 10^5 cycles, $p/S_u = 0.004$; from p. 908, $S_u = 240\,000$ psi, based on metal area.

$$F_f = \frac{0.004(240\,000)(30d)}{2} = 14\,400d \text{ lbf} \quad \text{each wire}$$

Eq. (17-48) and Table 17-27:

$$F_b = \frac{E_w d_w A_m}{D} = \frac{12(10^6)(0.067d)(0.4d^2)}{30} = 10\,720d^3 \text{ lbf,} \quad \text{each wire}$$

Eq. (17-45):

$$n_f = \frac{F_f - F_b}{F_t} = \frac{14\,400d - 10\,720d^3}{(5620/m) + 162d^2}$$

We could use a computer program to build a table similar to that of Ex. 17-6. Alternatively, we could recognize that $162d^2$ is small compared to $5620/m$, and therefore eliminate the $162d^2$ term.

$$n_f \doteq \frac{14\,400d - 10\,720d^3}{5620/m} = \frac{m}{5620}(14\,400d - 10\,720d^3)$$

Maximize n_f,

$$\frac{\partial n_f}{\partial d} = 0 = \frac{m}{5620}\,[14\,400 - 3(10\,720)d^2]$$

From which

$$d* = \sqrt{\frac{14\,400}{32\,160}} = 0.669 \text{ in}$$

Back-substituting

$$n_f = \frac{m}{5620}[14\,400(0.669) - 10\,720(0.669^3)] = 1.14 \text{ m}$$

Thus $n_f = 1.14$, 2.28, 3.42, 4.56 for $m = 1$, 2, 3, 4 respectively. If we choose $d = 0.50$ in, then $m = 2$.

$$n_f = \frac{14\,400(0.5) - 10\,720(0.5^3)}{(5620/2) + 162(0.5)^2} = 2.06$$

This exceeds $n_d = 2$

Decision #1: $d = 1/2$ in

Decision #2: $m = 2$ ropes supporting load. Rope should be inspected weekly for any signs of fatigue (broken outer wires).

Comment: Table 17-25 gives n for freight elevators in terms of velocity.

$$F_u = (S_u)_{\text{nom}} A_{\text{nom}} = 106\,000\left(\frac{\pi d^2}{4}\right) = 83\,252 d^2 \text{ lbf,} \quad \text{each wire}$$

$$n = \frac{F_u}{F_t} = \frac{83\,452(0.5)^2}{(5620/2) + 162(0.5)^2} = 7.32$$

By comparison, interpolation for 120 ft/min gives 7.08-close. The category of construction hoists is not addressed in Table 17-25. We should investigate this before proceeding further.

17-31 2000 ft lift, 72 in drum, 6 × 19 MS rope. Cage and load 8000 lbf, acceleration = 2 ft/s^2.

(a) Table 17-24: $(S_u)_{\text{nom}} = 106$ kpsi; $S_u = 240$ kpsi (p. 1093, metal area); Fig. 17-22: $(p/S_u)_{10^6} = 0.0014$

$$F_f = \frac{0.0014(240)(72)d}{2} = 12.1d \text{ kip}$$

Table 17-24: $$wl = 1.6d^2 2000(10^{-3}) = 3.2d^2 \text{ kip}$$

Eq. (17-46): $$F_t = (W + wl)\left(1 + \frac{a}{g}\right)$$

$$= (8 + 3.2d^2)\left(1 + \frac{2}{32.2}\right)$$

$$= 8.5 + 3.4d^2 \text{ kip}$$

Note that bending is not included.

$$n = \frac{F_f}{F_t} = \frac{12.1d}{8.5 + 3.4d^2}$$

d, in	n	
0.500	0.650	
1.000	1.020	
1.500	1.124	
1.625	1.125	← maximum n *Ans.*
1.750	1.120	
2.000	1.095	

(b) Try $m = 4$ strands

$$F_t = \left(\frac{8}{4} + 3.2d^2\right)\left(1 + \frac{2}{32.2}\right)$$

$$= 2.12 + 3.4d^2 \text{ kip}$$

$$F_f = 12.1d \text{ kip}$$

$$n = \frac{12.1d}{2.12 + 3.4d^2}$$

d, in	n	
0.5000	2.037	
0.5625	2.130	
0.6250	2.193	
0.7500	2.250	← maximum n *Ans.*
0.8750	2.242	
1.0000	2.192	

Comparing tables, multiple ropes supporting the load increases the factor of safety, and reduces the corresponding wire rope diameter, a useful perspective.

17-32

$$n = \frac{ad}{b/m + cd^2}$$

$$\frac{dn}{dd} = \frac{(b/m + cd^2)a - ad(2cd)}{(b/m + cd^2)^2} = 0$$

From which

$$d^* = \sqrt{\frac{b}{mc}} \quad \textit{Ans.}$$

$$n^* = \frac{a\sqrt{b/(mc)}}{(b/m) + c[b/(mc)]} = \frac{a}{2}\sqrt{\frac{m}{bc}} \quad \textit{Ans.}$$

These results agree closely with Prob. 17-31 solution. The small differences are due to rounding in Prob. 17-31.

17-33 From Prob. 17-32 solution:

$$n_1 = \frac{ad}{b/m + cd^2}$$

Solve the above equation for m

$$m = \frac{b}{ad/n_1 - cd^2} \tag{1}$$

$$\frac{dm}{ad} = 0 = \frac{[(ad/n_1) - ad^2](0) - b[(a/n_1) - 2cd]}{[(ad/n_1) - cd^2]^2}$$

From which $$d^* = \frac{a}{2cn_1} \quad Ans.$$

Substituting this result for d in Eq. (1) gives

$$m^* = \frac{4bcn_1}{a^2} \quad Ans.$$

17-34

$$\begin{aligned} A_m &= 0.40d^2 = 0.40(2^2) = 1.6 \text{ in}^2 \\ E_r &= 12 \text{ Mpsi}, \quad w = 1.6d^2 = 1.6(2^2) = 6.4 \text{ lbf/ft} \\ wl &= 6.4(480) = 3072 \text{ lbf} \end{aligned}$$

Treat the rest of the system as rigid, so that all of the stretch is due to the cage weighing 1000 lbf and the wire's weight. From Prob. 4-6

$$\begin{aligned} \delta_1 &= \frac{Pl}{AE} + \frac{(wl)l}{2AE} \\ &= \frac{1000(480)(12)}{1.6(12)(10^6)} + \frac{3072(480)(12)}{2(1.6)(12)(10^6)} \\ &= 0.3 + 0.461 = 0.761 \text{ in} \end{aligned}$$

due to cage and wire. The stretch due to the wire, the cart and the cage is

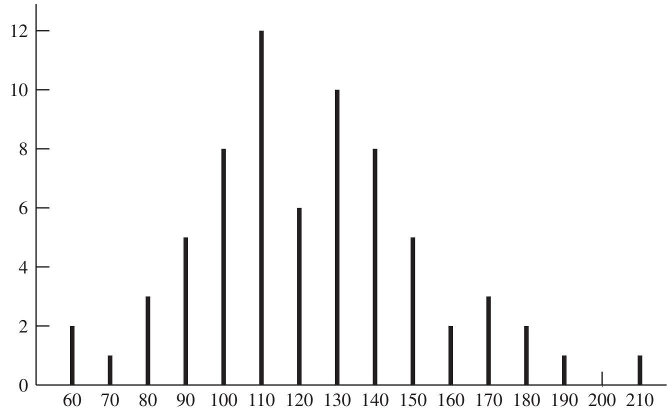
$$\delta_2 = \frac{9000(480)(12)}{1.6(12)(10^6)} + 0.761 = 3.461 \text{ in} \quad Ans.$$

17-35 to 17-38 Computer programs will vary.

Chapter 20

20-1

(a)



(b) $f/(N\Delta x) = f/(69 \cdot 10) = f/690$

x	f	fx	fx^2	$f/(N\Delta x)$
60	2	120	7200	0.0029
70	1	70	4900	0.0015
80	3	240	19 200	0.0043
90	5	450	40 500	0.0072
100	8	800	80 000	0.0116
110	12	1320	145 200	0.0174
120	6	720	86 400	0.0087
130	10	1300	169 000	0.0145
140	8	1120	156 800	0.0116
150	5	750	112 500	0.0174
160	2	320	51 200	0.0029
170	3	510	86 700	0.0043
180	2	360	64 800	0.0029
190	1	190	36 100	0.0015
200	0	0	0	0
210	1	210	44 100	0.0015
	69	8480	1 104 600	

Eq. (20-9) $$\bar{x} = \frac{8480}{69} = 122.9 \text{ kcycles}$$

Eq. (20-10) $$s_x = \left[\frac{1\,104\,600 - 8480^2/69}{69 - 1}\right]^{1/2}$$

$$= 30.3 \text{ kcycles} \quad \textit{Ans.}$$

20-2 Data represents a 7-class histogram with $N = 197$.

x	f	fx	fx^2
174	6	1044	181 656
182	9	1638	298 116
190	44	8360	1 588 400
198	67	13 266	2 626 688
206	53	10 918	2 249 108
214	12	2568	549 552
220	6	1320	290 400
	197	39 114	7 789 900

$$\bar{x} = \frac{39\,114}{197} = 198.55 \text{ kpsi} \quad \textit{Ans.}$$

$$s_x = \left[\frac{7\,783\,900 - 39\,114^2/197}{197 - 1}\right]^{1/2}$$

$$= 9.55 \text{ kpsi} \quad \textit{Ans.}$$

20-3

Form a table:

x	f	fx	fx^2
64	2	128	8192
68	6	408	27 744
72	6	432	31 104
76	9	684	51 984
80	19	1520	121 600
84	10	840	70 560
88	4	352	30 976
92	2	184	16 928
	58	4548	359 088

$$\bar{x} = \frac{4548}{58} = 78.4 \text{ kpsi}$$

$$s_x = \left[\frac{359\,088 - 4548^2/58}{58 - 1}\right]^{1/2} = 6.57 \text{ kpsi}$$

From Eq. (20-14)

$$f(x) = \frac{1}{6.57\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - 78.4}{6.57}\right)^2\right]$$

20-4 (a)

y	f	fy	fy^2	y	$f/(Nw)$	$f(y)$	$g(y)$
5.625	1	5.625	31.640 63	5.625	0.072 727	0.001 262	0.000 295
5.875	0	0	0	5.875	0	0.008 586	0.004 088
6.125	0	0	0	6.125	0	0.042 038	0.031 194
6.375	3	19.125	121.9219	6.375	0.218 182	0.148 106	0.140 262
6.625	3	19.875	131.6719	6.625	0.218 182	0.375 493	0.393 667
6.875	6	41.25	283.5938	6.875	0.436 364	0.685 057	0.725 002
7.125	14	99.75	710.7188	7.125	1.018 182	0.899 389	0.915 128
7.375	15	110.625	815.8594	7.375	1.090 909	0.849 697	0.822 462
7.625	10	76.25	581.4063	7.625	0.727 273	0.577 665	0.544 251
7.875	2	15.75	124.0313	7.875	0.145 455	0.282 608	0.273 138
8.125	1	8.125	66.015 63	8.125	0.072 727	0.099 492	0.106 72
	55	396.375	2866.859				

For a normal distribution,

$$\bar{y} = 396.375/55 = 7.207, \qquad s_y = \left(\frac{2866.859 - (396.375^2/55)}{55 - 1}\right)^{1/2} = 0.4358$$

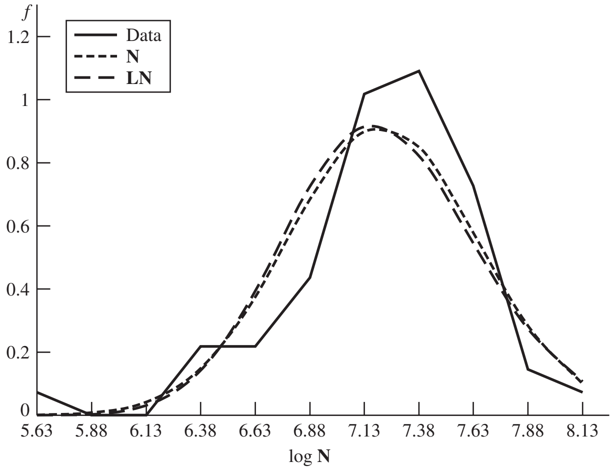
$$f(y) = \frac{1}{0.4358\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - 7.207}{0.4358}\right)^2\right]$$

For a lognormal distribution,

$$\bar{x} = \ln 7.206\,818 - \ln\sqrt{1 + 0.060\,474^2} = 1.9732, \qquad s_x = \ln\sqrt{1 + 0.060\,474^2} = 0.0604$$

$$g(y) = \frac{1}{x(0.0604)(\sqrt{2\pi})} \exp\left[-\frac{1}{2}\left(\frac{\ln x - 1.9732}{0.0604}\right)^2\right]$$

(b) Histogram



20-5 Distribution is uniform in interval 0.5000 to 0.5008 in, range numbers are $a = 0.5000$, $b = 0.5008$ in.

(a) Eq. (20-22) $$\mu_x = \frac{a+b}{2} = \frac{0.5000 + 0.5008}{2} = 0.5004$$

Eq. (20-23) $$\sigma_x = \frac{b-a}{2\sqrt{3}} = \frac{0.5008 - 0.5000}{2\sqrt{3}} = 0.000\,231$$

(b) PDF from Eq. (20-20)

$$f(x) = \begin{cases} 1250 & 0.5000 \le x \le 0.5008 \text{ in} \\ 0 & \text{otherwise} \end{cases}$$

(c) CDF from Eq. (20-21)

$$F(x) = \begin{cases} 0 & x < 0.5000 \\ (x - 0.5)/0.0008 & 0.5000 \le x \le 0.5008 \\ 1 & x > 0.5008 \end{cases}$$

If all smaller diameters are removed by inspection, $a = 0.5002$, $b = 0.5008$

$$\mu_x = \frac{0.5002 + 0.5008}{2} = 0.5005 \text{ in}$$

$$\hat{\sigma}_x = \frac{0.5008 - 0.5002}{2\sqrt{3}} = 0.000\,173 \text{ in}$$

$$f(x) = \begin{cases} 1666.7 & 0.5002 \le x \le 0.5008 \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 0.5002 \\ 1666.7(x - 0.5002) & 0.5002 \le x \le 0.5008 \\ 1 & x > 0.5008 \end{cases}$$

20-6 Dimensions produced are due to tool dulling and wear. When parts are mixed, the distribution is uniform. From Eqs. (20-22) and (20-23),

$$a = \mu_x - \sqrt{3}s = 0.6241 - \sqrt{3}(0.000\,581) = 0.6231 \text{ in}$$

$$b = \mu_x + \sqrt{3}s = 0.6241 + \sqrt{3}(0.000\,581) = 0.6251 \text{ in}$$

We suspect the dimension was $\dfrac{0.623}{0.625}$ in *Ans.*

20-7 $F(x) = 0.555x - 33$ mm

(a) Since $F(x)$ is linear, the distribution is uniform at $x = a$

$$F(a) = 0 = 0.555(a) - 33$$

$\therefore a = 59.46$ mm. Therefore, at $x = b$

$$F(b) = 1 = 0.555b - 33$$

$\therefore b = 61.26$ mm. Therefore,

$$F(x) = \begin{cases} 0 & x < 59.46 \text{ mm} \\ 0.555x - 33 & 59.46 \le x \le 61.26 \text{ mm} \\ 1 & x > 61.26 \text{ mm} \end{cases}$$

The PDF is dF/dx, thus the range numbers are:

$$f(x) = \begin{cases} 0.555 & 59.46 \le x \le 61.26 \text{ mm} \\ 0 & \text{otherwise} \end{cases} \quad \textit{Ans.}$$

From the range numbers,

$$\mu_x = \frac{59.46 + 61.26}{2} = 60.36 \text{ mm} \quad \textit{Ans.}$$

$$\hat{\sigma}_x = \frac{61.26 - 59.46}{2\sqrt{3}} = 0.520 \text{ mm} \quad \textit{Ans.}$$

(b) $\boldsymbol{\sigma}$ is an uncorrelated quotient $\bar{F} = 3600$ lbf, $\bar{A} = 0.112$ in^2

$$C_F = 300/3600 = 0.083\,33, \qquad C_A = 0.001/0.112 = 0.008\,929$$

From Table 20-6, for $\boldsymbol{\sigma}$

$$\bar{\sigma} = \frac{\mu_F}{\mu_A} = \frac{3600}{0.112} = 32\,143 \text{ psi} \quad \textit{Ans.}$$

$$\hat{\sigma}_\sigma = 32\,143\left[\frac{(0.08333^2 + 0.008929^2)}{(1 + 0.008929^2)}\right]^{1/2} = 2694 \text{ psi} \quad \textit{Ans.}$$

$$C_\sigma = 2694/32\,143 = 0.0838 \quad \textit{Ans.}$$

Since **F** and **A** are lognormal, division is closed and $\boldsymbol{\sigma}$ is lognormal too.

$$\boldsymbol{\sigma} = \mathbf{LN}(32\,143,\ 2694) \text{ psi} \quad \textit{Ans.}$$

20-8 Cramer's rule

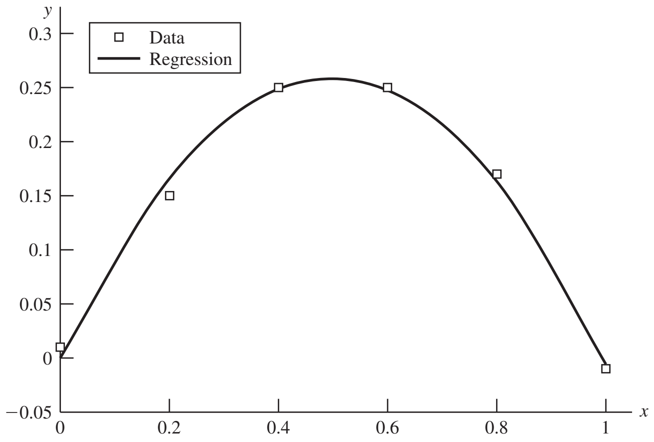
$$a_1 = \frac{\begin{vmatrix} \Sigma y & \Sigma x^2 \\ \Sigma xy & \Sigma x^3 \end{vmatrix}}{\begin{vmatrix} \Sigma x & \Sigma x^2 \\ \Sigma x^2 & \Sigma x^3 \end{vmatrix}} = \frac{\Sigma y \Sigma x^3 - \Sigma xy \Sigma x^2}{\Sigma x \Sigma x^3 - (\Sigma x^2)^2} \quad \textit{Ans.}$$

$$a_2 = \frac{\begin{vmatrix} \Sigma x & \Sigma y \\ \Sigma x^2 & \Sigma xy \end{vmatrix}}{\begin{vmatrix} \Sigma x & \Sigma x^2 \\ \Sigma x^2 & \Sigma x^3 \end{vmatrix}} = \frac{\Sigma x \Sigma xy - \Sigma y \Sigma x^2}{\Sigma x \Sigma x^3 - (\Sigma x^2)^2} \quad \textit{Ans.}$$

x	y	x^2	x^3	xy
0	0.01	0	0	0
0.2	0.15	0.04	0.008	0.030
0.4	0.25	0.16	0.064	0.100
0.6	0.25	0.36	0.216	0.150
0.8	0.17	0.64	0.512	0.136
1.0	−0.01	1.00	1.000	−0.010
3.0	0.82	2.20	1.800	0.406

$a_1 = 1.040\,714 \qquad a_2 = -1.046\,43 \quad \textit{Ans.}$

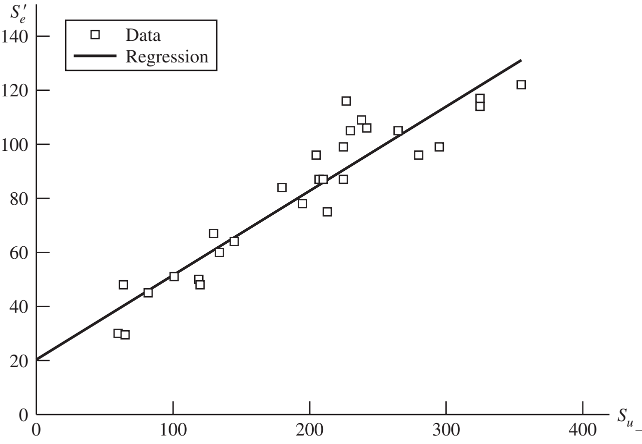
	Data	Regression
x	y	y
0	0.01	0
0.2	0.15	0.166 286
0.4	0.25	0.248 857
0.6	0.25	0.247 714
0.8	0.17	0.162 857
1.0	−0.01	−0.005 71



20-9

	Data	Regression		
S_u	S'_e	S'_e	S_u^2	$S_u S'_e$
0		20.356 75		
60	30	39.080 78	3 600	1 800
64	48	40.329 05	4 096	3 072
65	29.5	40.641 12	4 225	1 917.5
82	45	45.946 26	6 724	3 690
101	51	51.875 54	10 201	5 151
119	50	57.492 75	14 161	5 950
120	48	57.804 81	14 400	5 760
130	67	60.925 48	16 900	8 710
134	60	62.173 75	17 956	8 040
145	64	65.606 49	21 025	9 280
180	84	76.528 84	32 400	15 120
195	78	81.209 85	38 025	15 210
205	96	84.330 52	42 025	19 680
207	87	84.954 66	42 849	18 009
210	87	85.890 86	44 100	18 270
213	75	86.827 06	45 369	15 975
225	99	90.571 87	50 625	22 275
225	87	90.571 87	50 625	19 575
227	116	91.196	51 529	26 332
230	105	92.132 2	52 900	24 150
238	109	94.628 74	56 644	25 942
242	106	95.877 01	58 564	25 652
265	105	103.054 6	70 225	27 825
280	96	107.735 6	78 400	26 880
295	99	112.416 6	87 025	29 205
325	114	121.778 6	105 625	37 050
325	117	121.778 6	105 625	38 025
355	122	131.140 6	126 025	43 310
5462	2274.5		1 251 868	501 855.5

$m = 0.312\,067 \qquad b = 20.356\,75$ *Ans.*



20-10

$$\mathcal{E} = \sum \left(y - a_0 - a_2x^2\right)^2$$

$$\frac{\partial \mathcal{E}}{\partial a_0} = -2\sum\left(y - a_0 - a_2x^2\right) = 0$$

$$\sum y - na_0 - a_2\sum x^2 = 0 \quad \Rightarrow \quad \sum y = na_0 + a_2\sum x^2$$

Ans.

$$\frac{\partial \mathcal{E}}{\partial a_2} = 2\sum\left(y - a_0 - a_2x^2\right)(2x) = 0 \quad \Rightarrow \quad \sum xy = a_0\sum x + a_2\sum x^3$$

Cramer's rule

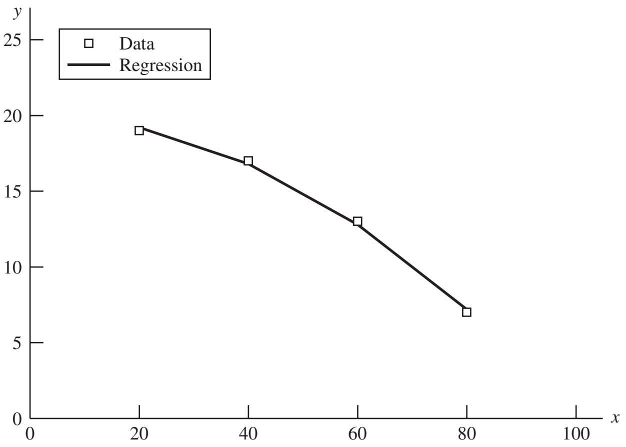
$$a_0 = \frac{\begin{vmatrix} \Sigma y & \Sigma x^2 \\ \Sigma xy & \Sigma x^3 \end{vmatrix}}{\begin{vmatrix} n & \Sigma x^2 \\ \Sigma x & \Sigma x^3 \end{vmatrix}} = \frac{\Sigma x^3 \Sigma y - \Sigma x^2 \Sigma xy}{n\Sigma x^3 - \Sigma x \Sigma x^2}$$

$$a_2 = \frac{\begin{vmatrix} n & \Sigma y \\ \Sigma x & \Sigma xy \end{vmatrix}}{\begin{vmatrix} n & \Sigma x^2 \\ \Sigma x & \Sigma x^3 \end{vmatrix}} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^3 - \Sigma x \Sigma x^2}$$

	Data	Regression			
x	y	y	x^2	x^3	xy
20	19	19.2	400	8 000	380
40	17	16.8	1600	64 000	680
60	13	12.8	3600	216 000	780
80	7	7.2	6400	512 000	560
200	56		12 000	800 000	2400

$$a_0 = \frac{800\,000(56) - 12\,000(2400)}{4(800\,000) - 200(12\,000)} = 20$$

$$a_2 = \frac{4(2400) - 200(56)}{4(800\,000) - 200(12\,000)} = -0.002$$

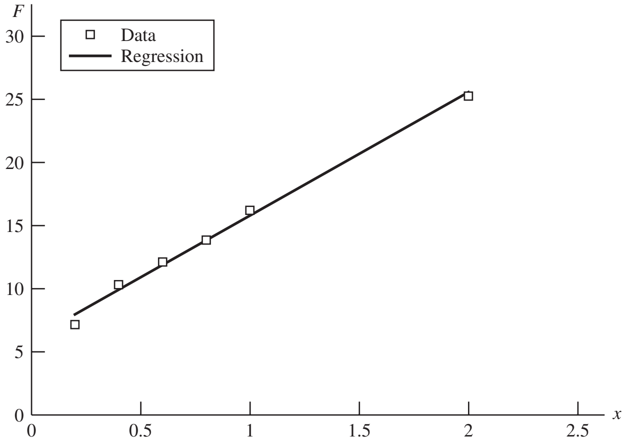


20-11

	Data	Regression					
x	y	y	x^2	y^2	xy	$x-\bar{x}$	$(x-\bar{x})^2$
0.2	7.1	7.931 803	0.04	50.41	1.42	−0.633 333	0.401 111 111
0.4	10.3	9.884 918	0.16	106.09	4.12	−0.433 333	0.187 777 778
0.6	12.1	11.838 032	0.36	146.41	7.26	−0.233 333	0.054 444 444
0.8	13.8	13.791 147	0.64	190.44	11.04	−0.033 333	0.001 111 111
1	16.2	15.744 262	1.00	262.44	16.20	0.166 666	0.027 777 778
2	25.2	25.509 836	4.00	635.04	50.40	1.166 666	1.361 111 111
5	84.7		6.2	1390.83	90.44	0	2.033 333 333

$$\hat{m} = \bar{k} = \frac{6(90.44) - 5(84.7)}{6(6.2) - (5)^2} = 9.7656$$

$$\hat{b} = \bar{F}_i = \frac{84.7 - 9.7656(5)}{6} = 5.9787$$



(a) $$\bar{x} = \frac{5}{6}; \quad \bar{y} = \frac{84.7}{6} = 14.117$$

Eq. (20-37)

$$s_{yx} = \sqrt{\frac{1390.83 - 5.9787(84.7) - 9.7656(90.44)}{6-2}}$$

$$= 0.556$$

Eq. (20-36)

$$s_{\hat{b}} = 0.556\sqrt{\frac{1}{6} + \frac{(5/6)^2}{2.0333}} = 0.3964 \text{ lbf}$$

$$F_i = (5.9787, 0.3964) \text{ lbf} \quad \textit{Ans.}$$

(b) Eq. (20-35)

$$s_{\hat{m}} = \frac{0.556}{\sqrt{2.0333}} = 0.3899 \text{ lbf/in}$$

$$k = (9.7656,\ 0.3899) \text{ lbf/in} \quad \textit{Ans.}$$

20-12 The expression $\boldsymbol{\epsilon} = \boldsymbol{\delta}/\mathbf{l}$ is of the form $\boldsymbol{x}/\boldsymbol{y}$. Now $\boldsymbol{\delta} = (0.0015,\ 0.000\,092)$ in, unspecified distribution; $\mathbf{l} = (2.000,\ 0.0081)$ in, unspecified distribution;

$$C_x = 0.000\,092/0.0015 = 0.0613$$

$$C_y = 0.0081/2.000 = 0.000\,75$$

From Table 20-6, $\bar{\epsilon} = 0.0015/2.000 = 0.000\,75$

$$\hat{\sigma}_\epsilon = 0.000\,75\left[\frac{0.0613^2 + 0.004\,05^2}{1 + 0.004\,05^2}\right]^{1/2}$$

$$= 4.607(10^{-5}) = 0.000\,046$$

We can predict $\bar{\epsilon}$ and $\hat{\sigma}_\epsilon$ but not the distribution of $\boldsymbol{\epsilon}$.

20-13 $\boldsymbol{\sigma} = \boldsymbol{\epsilon}\mathbf{E}$

$\boldsymbol{\epsilon} = (0.0005,\ 0.000\,034)$ distribution unspecified; $\mathbf{E} = (29.5,\ 0.885)$ Mpsi, distribution unspecified;

$$C_x = 0.000\,034/0.0005 = 0.068,$$

$$C_y = 0.0885/29.5 = 0.030$$

$\boldsymbol{\sigma}$ is of the form $\boldsymbol{x}, \boldsymbol{y}$

Table 20-6

$$\begin{aligned} \bar{\sigma} &= \bar{\epsilon}\bar{E} = 0.0005(29.5)10^6 = 14\,750 \text{ psi} \\ \hat{\sigma}_\sigma &= 14\,750(0.068^2 + 0.030^2 + 0.068^2 + 0.030^2)^{1/2} \\ &= 1096.7 \text{ psi} \\ C_\sigma &= 1096.7/14\,750 = 0.074\,35 \end{aligned}$$

20-14

$$\delta = \frac{\mathbf{Fl}}{\mathbf{AE}}$$

$\mathbf{F} = (14.7,\ 1.3)$ kip, $\mathbf{A} = (0.226,\ 0.003)\,\text{in}^2$, $\mathbf{l} = (1.5,\ 0.004)$ in, $\mathbf{E} = (29.5,\ 0.885)$ Mpsi distributions unspecified.

$C_F = 1.3/14.7 = 0.0884$; $\quad C_A = 0.003/0.226 = 0.0133$; $\quad C_l = 0.004/1.5 = 0.00267$;

$C_E = 0.885/29.5 = 0.03$

Mean of δ:

$$\boldsymbol{\delta} = \frac{\mathbf{Fl}}{\mathbf{AE}} = \mathbf{Fl}\left(\frac{1}{\mathbf{A}}\right)\left(\frac{1}{\mathbf{E}}\right)$$

From Table 20-6,

$$\bar{\delta} = \bar{F}\bar{l}(1/\bar{A})(1/\bar{E})$$

$$\bar{\delta} = 14\,700(1.5)\frac{1}{0.226}\frac{1}{29.5(10^6)}$$

$$= 0.003\,31 \text{ in} \quad \textit{Ans.}$$

For the standard deviation, using the first-order terms in Table 20-6,

$$\hat{\sigma}_\delta \doteq \frac{\bar{F}\bar{l}}{\bar{A}\bar{E}}\left(C_F^2 + C_l^2 + C_A^2 + C_E^2\right)^{1/2} = \bar{\delta}\left(C_F^2 + C_l^2 + C_A^2 + C_E^2\right)^{1/2}$$

$$\hat{\sigma}_\delta = 0.003\,31(0.0884^2 + 0.00267^2 + 0.0133^2 + 0.03^2)^{1/2}$$

$$= 0.000\,313 \text{ in} \quad \textit{Ans.}$$

COV

$$C_\delta = 0.000\,313/0.003\,31 = 0.0945 \quad \textit{Ans.}$$

Force COV dominates. There is no distributional information on $\boldsymbol{\delta}$.

20-15 $\mathbf{M} = (15\,000,\ 1350)$ lbf · in, distribution unspecified; $\mathbf{d} = (2.00,\ 0.005)$ in distribution unspecified.

$$\boldsymbol{\sigma} = \frac{32\mathbf{M}}{\pi \mathbf{d}^3}, \quad C_M = \frac{1350}{15\,000} = 0.09, \quad C_d = \frac{0.005}{2.00} = 0.0025$$

$\boldsymbol{\sigma}$ is of the form $\mathbf{x}/\mathbf{y}$, Table 20-6.

Mean:

$$\bar{\sigma} = \frac{32\bar{M}}{\pi \overline{d^3}} \doteq \frac{32\bar{M}}{\pi \bar{d}^3} = \frac{32(15\,000)}{\pi(2^3)}$$

$$= 19\,099 \text{ psi} \quad \textit{Ans.}$$

Standard Deviation:

$$\hat{\sigma}_\sigma = \bar{\sigma}\left[\left(C_M^2 + C_{d^3}^2\right)\big/\left(1 + C_{d^3}^2\right)\right]^{1/2}$$

From Table 20-6, $C_{d^3} \doteq 3C_d = 3(0.0025) = 0.0075$

$$\hat{\sigma}_\sigma = \bar{\sigma}\left[\left(C_M^2 + (3C_d)^2\right)\big/(1 + (3C_d))^2\right]^{1/2}$$

$$= 19\,099[(0.09^2 + 0.0075^2)/(1 + 0.0075^2)]^{1/2}$$

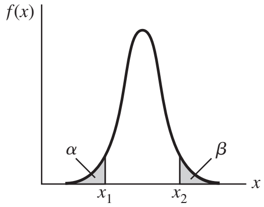
$$= 1725 \text{ psi} \quad \textit{Ans.}$$

COV:

$$C_\sigma = \frac{1725}{19\,099} = 0.0903 \quad \textit{Ans.}$$

Stress COV dominates. No information of distribution of $\boldsymbol{\sigma}$.

20-16



Fraction discarded is $\alpha + \beta$. The area under the PDF was unity. Having discarded $\alpha + \beta$ fraction, the ordinates to the truncated PDF are multiplied by a.

$$a = \frac{1}{1 - (\alpha + \beta)}$$

New PDF, $g(x)$, is given by

$$g(x) = \begin{cases} f(x)/[1 - (\alpha + \beta)] & x_1 \le x \le x_2 \\ 0 & \text{otherwise} \end{cases}$$

More formal proof: $g(x)$ has the property

$$1 = \int_{x_1}^{x_2} g(x)\,dx = a \int_{x_1}^{x_2} f(x)\,dx$$

$$1 = a \left[\int_{-\infty}^{\infty} f(x)\,dx - \int_{0}^{x_1} f(x)\,dx - \int_{x_2}^{\infty} f(x)\,dx \right]$$

$$1 = a \{1 - F(x_1) - [1 - F(x_2)]\}$$

$$a = \frac{1}{F(x_2) - F(x_1)} = \frac{1}{(1 - \beta) - \alpha} = \frac{1}{1 - (\alpha + \beta)}$$

20-17

(a) $\mathbf{d} = \mathbf{U}[0.748, 0.751]$

$$\mu_d = \frac{0.751 + 0.748}{2} = 0.7495 \text{ in}$$

$$\hat{\sigma}_d = \frac{0.751 - 0.748}{2\sqrt{3}} = 0.000\,866 \text{ in}$$

$$f(x) = \frac{1}{b - a} = \frac{1}{0.751 - 0.748} = 333.3 \text{ in}^{-1}$$

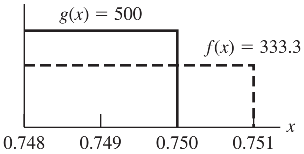
$$F(x) = \frac{x - 0.748}{0.751 - 0.748} = 333.3(x - 0.748)$$

(b)

$$F(x_1) = F(0.748) = 0$$

$$F(x_2) = (0.750 - 0.748)333.3 = 0.6667$$

If $g(x)$ is truncated, PDF becomes



$$g(x) = \frac{f(x)}{F(x_2) - F(x_1)} = \frac{333.3}{0.6667 - 0} = 500 \text{ in}^{-1}$$

$$\mu_x = \frac{a' + b'}{2} = \frac{0.748 + 0.750}{2} = 0.749 \text{ in}$$

$$\hat{\sigma}_x = \frac{b' - a'}{2\sqrt{3}} = \frac{0.750 - 0.748}{2\sqrt{3}} = 0.000\,577 \text{ in}$$

20-18 From Table A-10, 8.1% corresponds to $z_1 = -1.4$ and 5.5% corresponds to $z_2 = +1.6$.

$$k_1 = \mu + z_1\hat{\sigma}$$

$$k_2 = \mu + z_2\hat{\sigma}$$

From which

$$\mu = \frac{z_2 k_1 - z_1 k_2}{z_2 - z_1} = \frac{1.6(9) - (-1.4)11}{1.6 - (-1.4)}$$

$$= 9.933$$

$$\hat{\sigma} = \frac{k_2 - k_1}{z_2 - z_1}$$

$$= \frac{11 - 9}{1.6 - (-1.4)} = 0.6667$$

The original density function is

$$f(k) = \frac{1}{0.6667\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{k - 9.933}{0.6667}\right)^2\right] \quad \textit{Ans.}$$

20-19 From Prob. 20-1, $\mu = 122.9$ kcycles and $\hat{\sigma} = 30.3$ kcycles.

$$z_{10} = \frac{x_{10} - \mu}{\hat{\sigma}} = \frac{x_{10} - 122.9}{30.3}$$

$$x_{10} = 122.9 + 30.3 z_{10}$$

From Table A-10, for 10 percent failure, $z_{10} = -1.282$

$$x_{10} = 122.9 + 30.3(-1.282)$$

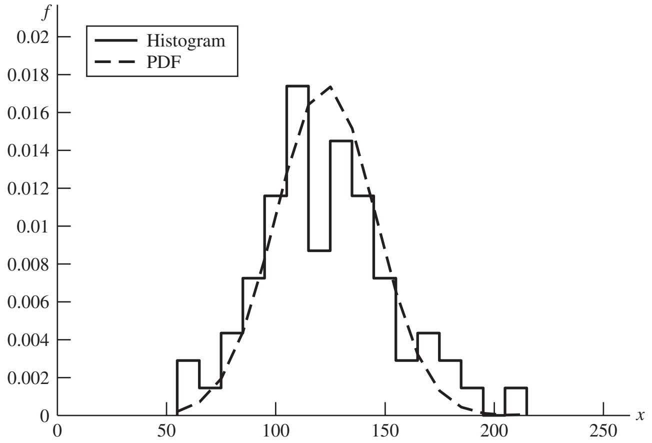
$$= 84.1 \text{ kcycles} \quad \textit{Ans.}$$

20-20

x	f	fx	fx^2	x	$f/(Nw)$	$f(x)$
60	2	120	7200	60	0.002 899	0.000 399
70	1	70	4900	70	0.001 449	0.001 206
80	3	240	19 200	80	0.004 348	0.003 009
90	5	450	40 500	90	0.007 246	0.006 204
100	8	800	80 000	100	0.011 594	0.010 567
110	12	1320	145 200	110	0.017 391	0.014 871
120	6	720	86 400	120	0.008 696	0.017 292
130	10	1300	169 000	130	0.014 493	0.016 612
140	8	1120	156 800	140	0.011 594	0.013 185
150	5	750	112 500	150	0.007 246	0.008 647
160	2	320	51 200	160	0.002 899	0.004 685
170	3	510	86 700	170	0.004 348	0.002 097
180	2	360	64 800	180	0.002 899	0.000 776
190	1	190	36 100	190	0.001 449	0.000 237
200	0	0	0	200	0	5.98E-05
210	1	210	44 100	210	0.001 449	1.25E-05
	69	8480				

$\bar{x} = 122.8986 \qquad s_x = 22.887\,19$

x	$f/(Nw)$	$f(x)$	x	$f/(Nw)$	$f(x)$
55	0	0.000 214	145	0.011 594	0.010 935
55	0.002 899	0.000 214	145	0.007 246	0.010 935
65	0.002 899	0.000 711	155	0.007 246	0.006 518
65	0.001 449	0.000 711	155	0.002 899	0.006 518
75	0.001 449	0.001 951	165	0.002 899	0.003 21
75	0.004 348	0.001 951	165	0.004 348	0.003 21
85	0.004 348	0.004 425	175	0.004 348	0.001 306
85	0.007 246	0.004 425	175	0.002 899	0.001 306
95	0.007 246	0.008 292	185	0.002 899	0.000 439
95	0.011 594	0.008 292	185	0.001 449	0.000 439
105	0.011 594	0.012 839	195	0.001 449	0.000 122
105	0.017 391	0.012 839	195	0	0.000 122
115	0.017 391	0.016 423	205	0	2.8E-05
115	0.008 696	0.016 423	205	0.001 499	2.8E-05
125	0.008 696	0.017 357	215	0.001 499	5.31E-06
125	0.014 493	0.017 357	215	0	5.31E-06
135	0.014 493	0.015 157			
135	0.011 594	0.015 157			

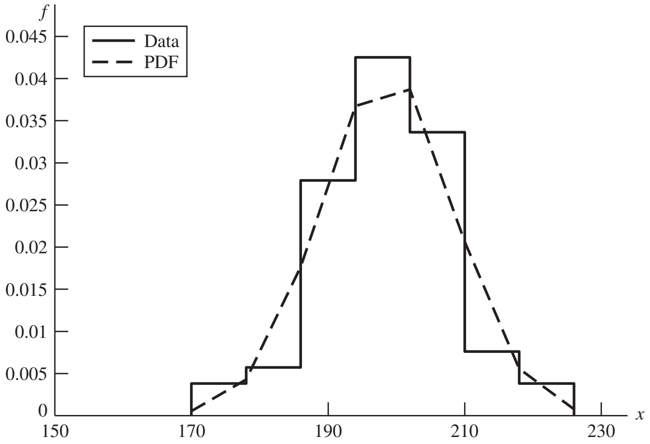


20-21

x	f	fx	fx^2	$f/(Nw)$	$f(x)$
174	6	1044	181 656	0.003 807	0.001 642
182	9	1638	298 116	0.005 711	0.009 485
190	44	8360	1 588 400	0.027 919	0.027 742
198	67	13 266	2 626 668	0.042 513	0.041 068
206	53	10 918	2 249 108	0.033 629	0.030 773
214	12	2568	549 552	0.007 614	0.011 671
222	6	1332	295 704	0.003 807	0.002 241
1386	197	39 126	7 789 204		

$\bar{x} = 198.6091 \qquad s_x = 9.695\,071$

x	$f/(Nw)$	$f(x)$
170	0	0.000 529
170	0.003 807	0.000 529
178	0.003 807	0.004 297
178	0.005 711	0.004 297
186	0.005 711	0.017 663
186	0.027 919	0.017 663
194	0.027 919	0.036 752
194	0.042 513	0.036 752
202	0.042 513	0.038 708
202	0.033 629	0.038 708
210	0.033 629	0.020 635
210	0.007 614	0.020 635
218	0.007 614	0.005 568
218	0.003 807	0.005 568
226	0.003 807	0.000 76
226	0	0.000 76

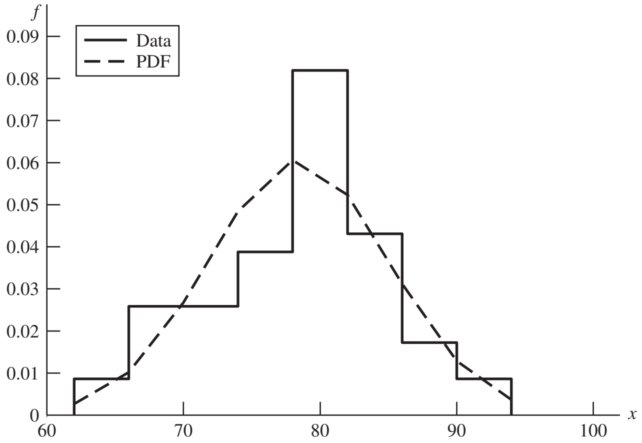


20-22

x	f	fx	fx^2	$f/(Nw)$	$f(x)$
64	2	128	8192	0.008621	0.00548
68	6	408	27744	0.025862	0.017299
72	6	432	31104	0.025862	0.037705
76	9	684	51984	0.038793	0.056742
80	19	1520	121600	0.081897	0.058959
84	10	840	70560	0.043103	0.042298
88	4	352	30976	0.017241	0.020952
92	2	184	16928	0.008621	0.007165
624	58	4548	359088		

$\bar{x} = 78.41379 \qquad s_x = 6.572229$

x	$f/(Nw)$	$f(x)$	x	$f/(Nw)$	$f(x)$
62	0	0.002684	82	0.081897	0.052305
62	0.008621	0.002684	82	0.043103	0.052305
66	0.008621	0.010197	86	0.043103	0.03118
66	0.025862	0.010197	86	0.017241	0.03118
70	0.025862	0.026749	90	0.017241	0.012833
70	0.025862	0.026749	90	0.008621	0.012833
74	0.025862	0.048446	94	0.008621	0.003647
74	0.038793	0.048446	94	0	0.003647
78	0.038793	0.060581			
78	0.081897	0.060581			



20-23

$$\bar{\sigma} = \frac{4\bar{P}}{\pi d^2} = \frac{4(40)}{\pi(1^2)} = 50.93 \text{ kpsi}$$

$$\hat{\sigma}_\sigma = \frac{4\,\hat{\sigma}_P}{\pi d^2} = \frac{4(8.5)}{\pi(1^2)} = 10.82 \text{ kpsi}$$

$$\hat{\sigma}_{s_y} = 5.9 \text{ kpsi}$$

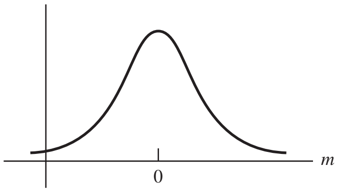
For no yield, $m = S_y - \sigma \geq 0$

$$z = \frac{m - \mu_m}{\hat{\sigma}_m} = \frac{0 - \mu_m}{\hat{\sigma}_m} = -\frac{\mu_m}{\hat{\sigma}_m}$$

$$\mu_m = \bar{S}_y - \bar{\sigma} = 27.47 \text{ kpsi},$$

$$\hat{\sigma}_m = \left(\hat{\sigma}_\sigma^2 + \hat{\sigma}_{S_y}^2\right)^{1/2} = 12.32 \text{ kpsi}$$

$$z = \frac{-27.47}{12.32} = -2.230$$



From Table A-10, $p_f = 0.0129$

$$R = 1 - p_f = 1 - 0.0129 = 0.987 \quad \textit{Ans.}$$

20-24 For a lognormal distribution,

Eq. (20-18) $\qquad \mu_y = \ln \mu_x - \ln \sqrt{1 + C_x^2}$

Eq. (20-19) $\qquad \hat{\sigma}_y = \sqrt{\ln\left(1 + C_x^2\right)}$

From Prob. (20-23)

$$\mu_m = \bar{S}_y - \bar{\sigma} = \mu_x$$

$$\mu_y = \left(\ln \bar{S}_y - \ln \sqrt{1 + C_{S_y}^2}\,\right) - \left(\ln \bar{\sigma} - \ln \sqrt{1 + C_\sigma^2}\right)$$

$$= \ln\left[\frac{\bar{S}_y}{\bar{\sigma}} \sqrt{\frac{1 + C_\sigma^2}{1 + C_{S_y}^2}}\,\right]$$

$$\hat{\sigma}_y = \left[\ln\left(1 + C_{S_y}^2\right) + \ln\left(1 + C_\sigma^2\right)\right]^{1/2}$$

$$= \sqrt{\ln\left[\left(1 + C_{S_y}^2\right)\left(1 + C_\sigma^2\right)\right]}$$

$$z = -\frac{\mu}{\hat{\sigma}} = -\frac{\ln\left(\dfrac{\bar{S}_y}{\bar{\sigma}} \sqrt{\dfrac{1 + C_\sigma^2}{1 + C_{S_y}^2}}\right)}{\sqrt{\ln\left[\left(1 + C_{S_y}^2\right)\left(1 + C_\sigma^2\right)\right]}}$$

$$\bar{\sigma} = \frac{4\bar{P}}{\pi d^2} = \frac{4(30)}{\pi(1^2)} = 38.197 \text{ kpsi}$$

$$\hat{\sigma}_\sigma = \frac{4\,\hat{\sigma}_P}{\pi d^2} = \frac{4(5.1)}{\pi(1^2)} = 6.494 \text{ kpsi}$$

$$C_\sigma = \frac{6.494}{38.197} = 0.1700$$

$$C_{S_y} = \frac{3.81}{49.6} = 0.076\,81$$

$$z = -\frac{\ln\left[\dfrac{49.6}{38.197}\sqrt{\dfrac{1+0.170^2}{1+0.076\,81^2}}\right]}{\sqrt{\ln\left[(1+0.076\,81^2)(1+0.170^2)\right]}} = -1.470$$

From Table A-10

$$p_f = 0.0708$$
$$R = 1 - p_f = 0.929 \quad \textit{Ans.}$$

20-25

x	n	nx	nx^2
93	19	1767	164 311
95	25	2375	225 625
97	38	3685	357 542
99	17	1683	166 617
101	12	1212	122 412
103	10	1030	106 090
105	5	525	55 125
107	4	428	45 796
109	4	436	47 524
111	2	222	24 624
	136	13 364	1315 704

$$\bar{x} = 13\,364/136 = 98.26 \text{ kpsi}$$
$$s_x = \left(\frac{1\,315\,704 - 13\,364^2/136}{135}\right)^{1/2} = 4.30 \text{ kpsi}$$

Under normal hypothesis,

$$\begin{aligned} z_{0.01} &= (x_{0.01} - 98.26)/4.30 \\ x_{0.01} &= 98.26 + 4.30 z_{0.01} \\ &= 98.26 + 4.30(-2.3267) \\ &= 88.26 \doteq 88.3 \text{ kpsi} \quad \textit{Ans.} \end{aligned}$$

20-26 From Prob. 20-25, $\mu_x = 98.26$ kpsi, and $\hat{\sigma}_x = 4.30$ kpsi.

$$C_x = \hat{\sigma}_x/\mu_x = 4.30/98.26 = 0.043\,76$$

From Eqs. (20-18) and (20-19),

$$\mu_y = \ln(98.26) - 0.043\,76^2/2 = 4.587$$
$$\hat{\sigma}_y = \sqrt{\ln(1 + 0.043\,76^2)} = 0.043\,74$$

For a yield strength exceeded by 99% of the population,

$$z_{0.01} = (\ln x_{0.01} - \mu_y)/\hat{\sigma}_y \quad \Rightarrow \quad \ln x_{0.01} = \mu_y + \hat{\sigma}_y z_{0.01}$$

From Table A-10, for 1% failure, $z_{0.01} = -2.326$. Thus,

$$\ln x_{0.01} = 4.587 + 0.043\,74(-2.326) = 4.485$$
$$x_{0.01} = 88.7 \text{ kpsi} \quad \textit{Ans.}$$

The normal PDF is given by Eq. (20-14) as

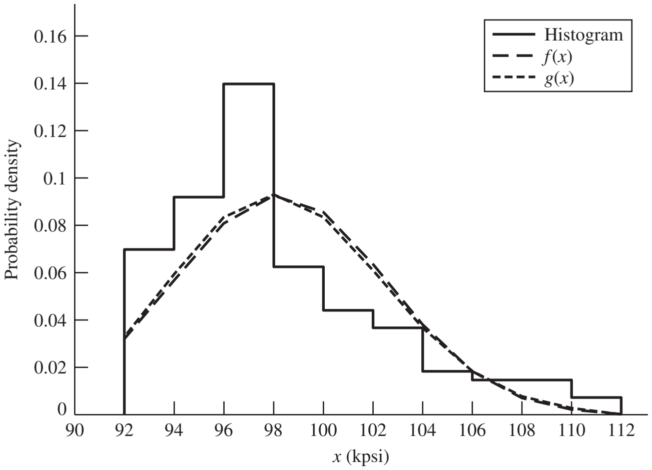
$$f(x) = \frac{1}{4.30\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - 98.26}{4.30}\right)^2\right]$$

For the lognormal distribution, from Eq. (20-17), defining $g(x)$,

$$g(x) = \frac{1}{x(0.043\,74)\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - 4.587}{0.043\,74}\right)^2\right]$$

x (kpsi)	$f/(Nw)$	$f(x)$	$g(x)$	x (kpsi)	$f/(Nw)$	$f(x)$	$g(x)$
92	0.000 00	0.032 15	0.032 63	102	0.036 76	0.063 56	0.061 34
92	0.069 85	0.032 15	0.032 63	104	0.036 76	0.038 06	0.037 08
94	0.069 85	0.056 80	0.058 90	104	0.018 38	0.038 06	0.037 08
94	0.091 91	0.056 80	0.058 90	106	0.018 38	0.018 36	0.018 69
96	0.091 91	0.080 81	0.083 08	106	0.014 71	0.018 36	0.018 69
96	0.139 71	0.080 81	0.083 08	108	0.014 71	0.007 13	0.007 93
98	0.139 71	0.092 61	0.092 97	108	0.014 71	0.007 13	0.007 93
98	0.062 50	0.092 61	0.092 97	110	0.014 71	0.002 23	0.002 86
100	0.062 50	0.085 48	0.083 67	110	0.007 35	0.002 23	0.002 86
100	0.044 12	0.085 48	0.083 67	112	0.007 35	0.000 56	0.000 89
102	0.044 12	0.063 56	0.061 34	112	0.000 00	0.000 56	0.000 89

Note: rows are repeated to draw histogram



The normal and lognormal are almost the same. However the data is quite skewed and perhaps a Weibull distribution should be explored. For a method of establishing the

Weibull parameters see Shigley, J. E., and C. R. Mischke, *Mechanical Engineering Design,* McGraw-Hill, 5th ed., 1989, Sec. 4-12.

20-27 Let $\mathbf{x} = (\mathbf{S}'_{fe})_{10^4}$

$x_0 = 79$ kpsi, $\theta = 86.2$ kpsi, $b = 2.6$

Eq. (20-28)

$$\bar{x} = x_0 + (\theta - x_0)\Gamma(1 + 1/b)$$
$$\bar{x} = 79 + (86.2 - 79)\Gamma(1 + 1/2.6)$$
$$= 79 + 7.2\,\Gamma(1.38)$$

From Table A-34, $\Gamma(1.38) = 0.888\,54$

$$\bar{x} = 79 + 7.2(0.888\,54) = 85.4 \text{ kpsi} \quad \textit{Ans.}$$

Eq. (20-29)

$$\hat{\sigma}_x = (\theta - x_0)[\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)]^{1/2}$$
$$= (86.2 - 79)[\Gamma(1 + 2/2.6) - \Gamma^2(1 + 1/2.6)]^{1/2}$$
$$= 7.2[0.923\,76 - 0.888\,54^2]^{1/2}$$
$$= 2.64 \text{ kpsi} \quad \textit{Ans.}$$
$$C_x = \frac{\hat{\sigma}_x}{\bar{x}} = \frac{2.64}{85.4} = 0.031 \quad \textit{Ans.}$$

20-28

$\mathbf{x} = \mathbf{S}_{ut}$

$$x_0 = 27.7, \quad \theta = 46.2, \quad b = 4.38$$
$$\mu_x = 27.7 + (46.2 - 27.7)\Gamma(1 + 1/4.38)$$
$$= 27.7 + 18.5\,\Gamma(1.23)$$
$$= 27.7 + 18.5(0.910\,75)$$
$$= 44.55 \text{ kpsi} \quad \textit{Ans.}$$
$$\hat{\sigma}_x = (46.2 - 27.7)[\Gamma(1 + 2/4.38) - \Gamma^2(1 + 1/4.38)]^{1/2}$$
$$= 18.5[\Gamma(1.46) - \Gamma^2(1.23)]^{1/2}$$
$$= 18.5[0.8856 - 0.910\,75^2]^{1/2}$$
$$= 4.38 \text{ kpsi} \quad \textit{Ans.}$$
$$C_x = \frac{4.38}{44.55} = 0.098 \quad \textit{Ans.}$$

From the Weibull survival equation

$$R = \exp\left[-\left(\frac{x - x_0}{\theta - x_0}\right)^b\right] = 1 - p$$

$$R_{40} = \exp\left[-\left(\frac{x_{40} - x_0}{\theta - x_0}\right)^b\right] = 1 - p_{40}$$

$$= \exp\left[-\left(\frac{40 - 27.7}{46.2 - 27.7}\right)^{4.38}\right] = 0.846$$

$p_{40} = 1 - R_{40} = 1 - 0.846 = 0.154 = 15.4\%$ *Ans.*

20-29

$\mathbf{x} = \mathbf{S}_{ut}$

$$\begin{aligned}
x_0 &= 151.9, \theta = 193.6, b = 8\\
\mu_x &= 151.9 + (193.6 - 151.9)\Gamma(1 + 1/8)\\
&= 151.9 + 41.7\,\Gamma(1.125)\\
&= 151.9 + 41.7(0.941\,76)\\
&= 191.2 \text{ kpsi} \quad \textit{Ans.}\\
\hat{\sigma}_x &= (193.6 - 151.9)[\Gamma(1 + 2/8) - \Gamma^2(1 + 1/8)]^{1/2}\\
&= 41.7[\Gamma(1.25) - \Gamma^2(1.125)]^{1/2}\\
&= 41.7[0.906\,40 - 0.941\,76^2]^{1/2}\\
&= 5.82 \text{ kpsi} \quad \textit{Ans.}\\
C_x &= \frac{5.82}{191.2} = 0.030
\end{aligned}$$

20-30

$\mathbf{x} = \mathbf{S}_{ut}$

$$\begin{aligned}
x_0 &= 47.6, \theta = 125.6, b = 11.84\\
\bar{x} &= 47.6 + (125.6 - 47.6)\Gamma(1 + 1/11.84)\\
\bar{x} &= 47.6 + 78\,\Gamma(1.08)\\
&= 47.6 + 78(0.959\,73) = 122.5 \text{ kpsi}\\
\hat{\sigma}_x &= (125.6 - 47.6)[\Gamma(1 + 2/11.84) - \Gamma^2(1 + 1/11.84)]^{1/2}\\
&= 78[\Gamma(1.08) - \Gamma^2(1.17)]^{1/2}\\
&= 78(0.959\,73 - 0.936\,70^2)^{1/2}\\
&= 22.4 \text{ kpsi}
\end{aligned}$$

From Prob. 20-28

$$\begin{aligned}
p &= 1 - \exp\left[-\left(\frac{x - x_0}{\theta - \theta_0}\right)^b\right]\\
&= 1 - \exp\left[-\left(\frac{100 - 47.6}{125.6 - 47.6}\right)^{11.84}\right]\\
&= 0.0090 \quad \textit{Ans.}
\end{aligned}$$

$\mathbf{y} = \mathbf{S}_y$

$$y_0 = 64.1, \theta = 81.0, b = 3.77$$

$$\begin{aligned}\bar{y} &= 64.1 + (81.0 - 64.1)\Gamma(1 + 1/3.77)\\ &= 64.1 + 16.9\,\Gamma(1.27)\\ &= 64.1 + 16.9(0.902\,50)\\ &= 79.35 \text{ kpsi}\end{aligned}$$

$$\sigma_y = (81 - 64.1)[\Gamma(1 + 2/3.77) - \Gamma(1 + 1/3.77)]^{1/2}$$

$$\begin{aligned}\sigma_y &= 16.9[(0.887\,57) - 0.902\,50^2]^{1/2}\\ &= 4.57 \text{ kpsi}\end{aligned}$$

$$p = 1 - \exp\left[-\left(\frac{y - y_0}{\theta - y_0}\right)^{3.77}\right]$$

$$p = 1 - \exp\left[-\left(\frac{70 - 64.1}{81 - 64.1}\right)^{3.77}\right] = 0.019 \quad \textit{Ans.}$$

20-31 $\mathbf{x} = \mathbf{S}_{ut} = \mathbf{W}[122.3, 134.6, 3.64]$ kpsi, $p(x > 120) = 1 = 100\%$ since $x_0 > 120$ kpsi

$$p(x > 133) = \exp\left[-\left(\frac{133 - 122.3}{134.6 - 122.3}\right)^{3.64}\right]$$

$$= 0.548 = 54.8\% \quad \textit{Ans.}$$

20-32 Using Eqs. (20-28) and (20-29) and Table A-34,

$$\mu_n = n_0 + (\theta - n_0)\Gamma(1 + 1/b) = 36.9 + (133.6 - 36.9)\Gamma(1 + 1/2.66) = 122.85 \text{ kcycles}$$

$$\hat{\sigma}_n = (\theta - n_0)[\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)] = 34.79 \text{ kcycles}$$

For the Weibull density function, Eq. (2-27),

$$f_W(n) = \frac{2.66}{133.6 - 36.9}\left(\frac{n - 36.9}{133.6 - 36.9}\right)^{2.66-1} \exp\left[-\left(\frac{n - 36.9}{133.6 - 36.9}\right)^{2.66}\right]$$

For the lognormal distribution, Eqs. (20-18) and (20-19) give,

$$\mu_y = \ln(122.85) - (34.79/122.85)^2/2 = 4.771$$

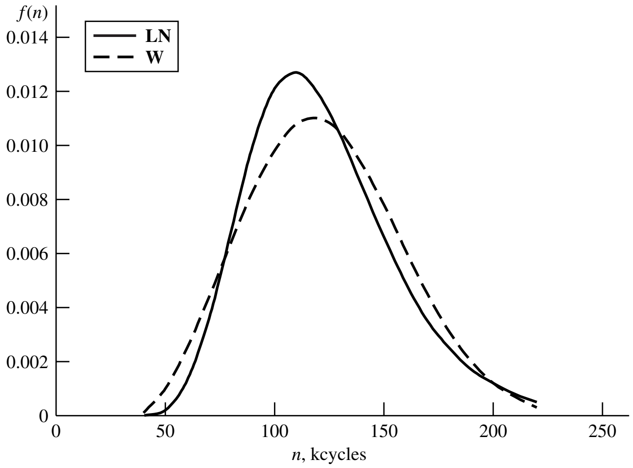
$$\hat{\sigma}_y = \sqrt{[1 + (34.79/122.85)^2]} = 0.2778$$

From Eq. (20-17), the lognormal PDF is

$$f_{LN}(n) = \frac{1}{0.2778\, n\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln n - 4.771}{0.2778}\right)^2\right]$$

We form a table of densities $f_W(n)$ and $f_{LN}(n)$ and plot.

n (kcycles)	$f_W(n)$	$f_{LN}(n)$
40	9.1E-05	1.82E-05
50	0.000 991	0.000 241
60	0.002 498	0.001 233
70	0.004 380	0.003 501
80	0.006 401	0.006 739
90	0.008 301	0.009 913
100	0.009 822	0.012 022
110	0.010 750	0.012 644
120	0.010 965	0.011 947
130	0.010 459	0.010 399
140	0.009 346	0.008 492
150	0.007 827	0.006 597
160	0.006 139	0.004 926
170	0.004 507	0.003 564
180	0.003 092	0.002 515
190	0.001 979	0.001 739
200	0.001 180	0.001 184
210	0.000 654	0.000 795
220	0.000 336	0.000 529



The Weibull L10 life comes from Eq. (20-26) with a reliability of $R = 0.90$. Thus,

$$n_{0.10} = 36.9 + (133 - 36.9)[\ln(1/0.90)]^{1/2.66} = 78.1 \text{ kcycles} \quad \textit{Ans.}$$

The lognormal L10 life comes from the definition of the z variable. That is,

$$\ln n_0 = \mu_y + \hat{\sigma}_y z \quad \text{or} \quad n_0 = \exp(\mu_y + \hat{\sigma}_y z)$$

From Table A-10, for $R = 0.90$, $z = -1.282$. Thus,

$$n_0 = \exp[4.771 + 0.2778(-1.282)] = 82.7 \text{ kcycles} \quad \textit{Ans.}$$

20-33 Form a table

i	x $L(10^{-5})$	f_i	$f_i x(10^{-5})$	$f_i x^2(10^{-10})$	$g(x)$ (10^5)
1	3.05	3	9.15	27.9075	0.0557
2	3.55	7	24.85	88.2175	0.1474
3	4.05	11	44.55	180.4275	0.2514
4	4.55	16	72.80	331.24	0.3168
5	5.05	21	106.05	535.5525	0.3216
6	5.55	13	72.15	400.4325	0.2789
7	6.05	13	78.65	475.8325	0.2151
8	6.55	6	39.30	257.415	0.1517
9	7.05	2	14.10	99.405	0.1000
10	7.55	0	0	0	0.0625
11	8.05	4	32.20	259.21	0.0375
12	8.55	3	25.65	219.3075	0.0218
13	9.05	0	0	0	0.0124
14	9.55	0	0	0	0.0069
15	10.05	1	10.05	101.0025	0.0038
		100	529.50	2975.95	

$$\bar{x} = 529.5(10^5)/100 = 5.295(10^5) \text{ cycles} \quad \textit{Ans.}$$

$$s_x = \left[\frac{2975.95(10^{10}) - [529.5(10^5)]^2/100}{100 - 1}\right]^{1/2}$$

$$= 1.319(10^5) \text{ cycles} \quad \textit{Ans.}$$

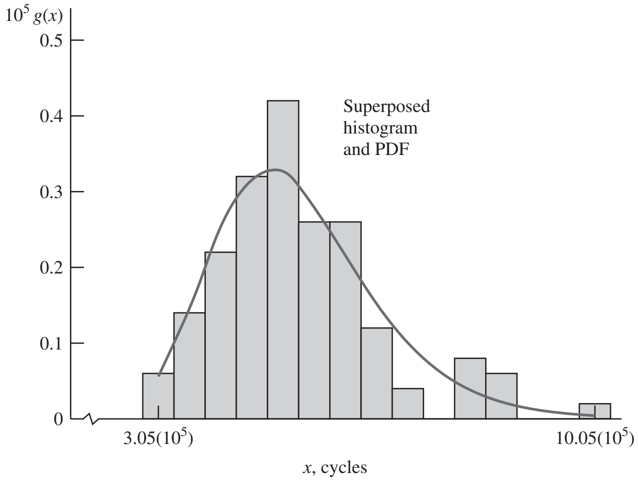
$$C_x = s/\bar{x} = 1.319/5.295 = 0.249$$

$$\mu_y = \ln 5.295(10^5) - 0.249^2/2 = 13.149$$

$$\hat{\sigma}_y = \sqrt{\ln(1 + 0.249^2)} = 0.245$$

$$g(x) = \frac{1}{x\hat{\sigma}_y\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu_y}{\hat{\sigma}_y}\right)^2\right]$$

$$g(x) = \frac{1.628}{x} \exp\left[-\frac{1}{2}\left(\frac{\ln x - 13.149}{0.245}\right)^2\right]$$



20-34

$$\mathbf{x} = \mathbf{S}_u = \mathbf{W}[70.3, 84.4, 2.01]$$

Eq. (20-28)
$$\begin{aligned}\mu_x &= 70.3 + (84.4 - 70.3)\Gamma(1 + 1/2.01)\\ &= 70.3 + (84.4 - 70.3)\Gamma(1.498)\\ &= 70.3 + (84.4 - 70.3)0.886\,17\\ &= 82.8 \text{ kpsi} \quad \textit{Ans.}\end{aligned}$$

Eq. (20-29)
$$\hat{\sigma}_x = (84.4 - 70.3)[\Gamma(1 + 2/2.01) - \Gamma^2(1 + 1/2.01)]^{1/2}$$

$$\begin{aligned}\hat{\sigma}_x &= 14.1[0.997\,91 - 0.886\,17^2]^{1/2}\\ &= 6.502 \text{ kpsi}\end{aligned}$$

$$C_x = \frac{6.502}{82.8} = 0.079 \quad \textit{Ans.}$$

20-35 Take the Weibull equation for the standard deviation

$$\hat{\sigma}_x = (\theta - x_0)[\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)]^{1/2}$$

and the mean equation solved for $\bar{x} - x_0$

$$\bar{x} - x_0 = (\theta - x_0)\Gamma(1 + 1/b)$$

Dividing the first by the second,

$$\frac{\hat{\sigma}_x}{\bar{x} - x_0} = \frac{[\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)]^{1/2}}{\Gamma(1 + 1/b)}$$

$$\frac{4.2}{49 - 33.8} = \sqrt{\frac{\Gamma(1 + 2/b)}{\Gamma^2(1 + 1/b)} - 1} = \sqrt{R} = 0.2763$$

Make a table and solve for b iteratively

b	$1+2/b$	$1+1/b$	$\Gamma(1+2/b)$	$\Gamma(1+1/b)$	
3	1.67	1.33	0.903 30	0.893 38	0.363
4	1.5	1.25	0.886 23	0.906 40	0.280
4.1	1.49	1.24	0.885 95	0.908 52	0.271

$$b \doteq 4.068 \quad \text{Using MathCad} \quad \textit{Ans.}$$

$$\theta = x_0 + \frac{\bar{x} - x_0}{\Gamma(1+1/b)} = 33.8 + \frac{49 - 33.8}{\Gamma(1+1/4.068)}$$

$$= 49.8 \text{ kpsi} \quad \textit{Ans.}$$

20-36

$$\mathbf{x} = \mathbf{S}_y = \mathbf{W}[34.7, 39, 2.93] \text{ kpsi}$$

$$\bar{x} = 34.7 + (39 - 34.7)\Gamma(1 + 1/2.93)$$

$$= 34.7 + 4.3\Gamma(1.34)$$

$$= 34.7 + 4.3(0.892\,22) = 38.5 \text{ kpsi}$$

$$\hat{\sigma}_x = (39 - 34.7)[\Gamma(1 + 2/2.93) - \Gamma^2(1 + 1/2.93)]^{1/2}$$

$$= 4.3[\Gamma(1.68) - \Gamma^2(1.34)]^{1/2}$$

$$= 4.3[0.905\,00 - 0.892\,22^2]^{1/2}$$

$$= 1.42 \text{ kpsi} \quad \textit{Ans.}$$

$$C_x = 1.42/38.5 = 0.037 \quad \textit{Ans.}$$

20-37

	x (Mrev)	f	fx	fx^2
	1	11	11	11
	2	22	44	88
	3	38	114	342
	4	57	228	912
	5	31	155	775
	6	19	114	684
	7	15	105	735
	8	12	96	768
	9	11	99	891
	10	9	90	900
	11	7	77	847
	12	5	60	720
Sum	78	237	1193	7673

$$\mu_x = 1193(10^6)/237 = 5.034(10^6) \text{ cycles}$$

$$\hat{\sigma}_x = \sqrt{\frac{7673(10^{12}) - [1193(10^6)]^2/237}{237 - 1}} = 2.658(10^6) \text{ cycles}$$

$$C_x = 2.658/5.034 = 0.528$$

From Eqs. (20-18) and (20-19),

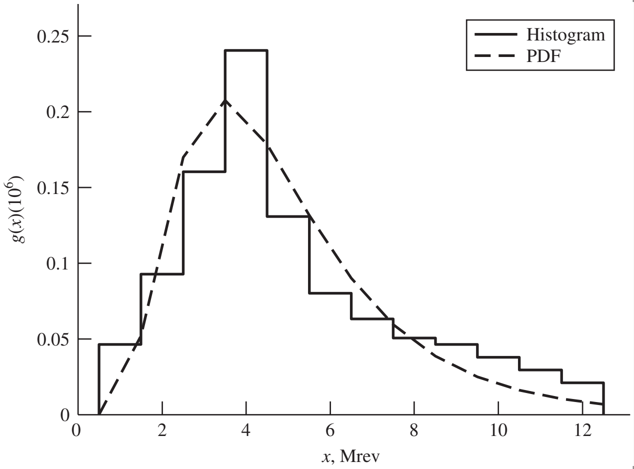
$$\mu_y = \ln[5.034(10^6)] - 0.528^2/2 = 15.292$$

$$\hat{\sigma}_y = \sqrt{\ln(1 + 0.528^2)} = 0.496$$

From Eq. (20-17), defining $g(x)$,

$$g\,(x) = \frac{1}{x(0.496)\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - 15.292}{0.496}\right)^2\right]$$

x (Mrev)	$f/(Nw)$	$g(x) \cdot (10^6)$
0.5	0.00000	0.00011
0.5	0.04641	0.00011
1.5	0.04641	0.05204
1.5	0.09283	0.05204
2.5	0.09283	0.16992
2.5	0.16034	0.16992
3.5	0.16034	0.20754
3.5	0.24051	0.20754
4.5	0.24051	0.17848
4.5	0.13080	0.17848
5.5	0.13080	0.13158
5.5	0.08017	0.13158
6.5	0.08017	0.09011
6.5	0.06329	0.09011
7.5	0.06329	0.05953
7.5	0.05063	0.05953
8.5	0.05063	0.03869
8.5	0.04641	0.03869
9.5	0.04641	0.02501
9.5	0.03797	0.02501
10.5	0.03797	0.01618
10.5	0.02954	0.01618
11.5	0.02954	0.01051
11.5	0.02110	0.01051
12.5	0.02110	0.00687
12.5	0.00000	0.00687



$$z = \frac{\ln x - \mu_y}{\hat{\sigma}_y} \quad \Rightarrow \quad \ln x = \mu_y + \hat{\sigma}_y z = 15.292 + 0.496z$$

L_{10} life, where 10% of bearings fail, from Table A-10, $z = -1.282$. Thus,

$$\ln x = 15.292 + 0.496(-1.282) = 14.66$$

$$\therefore\ x = 2.32 \times 10^6 \text{ rev} \quad \textit{Ans.}$$