Stability of Columns

- Bending due to a compressive axial load is called Buckling.
- Structural members that support compressive axial loads are called Columns.
- Buckling is the study of stability of a structure's equilibrium.

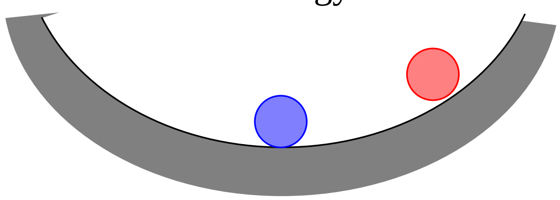
Learning objectives

- Develop an appreciation of the phenomena of buckling and the various types of structure instabilities.
- Understand the development and use of buckling formulas in analysis and design of structures.

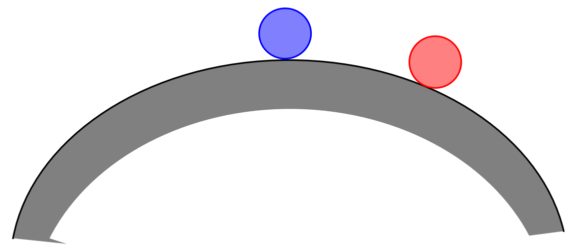
Buckling Phenomenon

Energy Approach

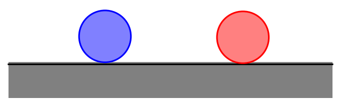
Stable Equilibrium
Potential Energy is Minimum



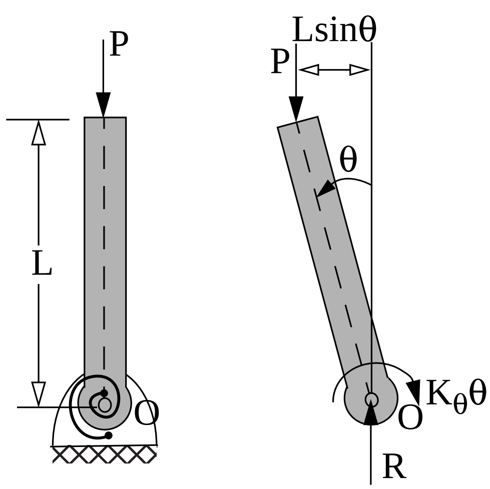
Unstable Equilibrium



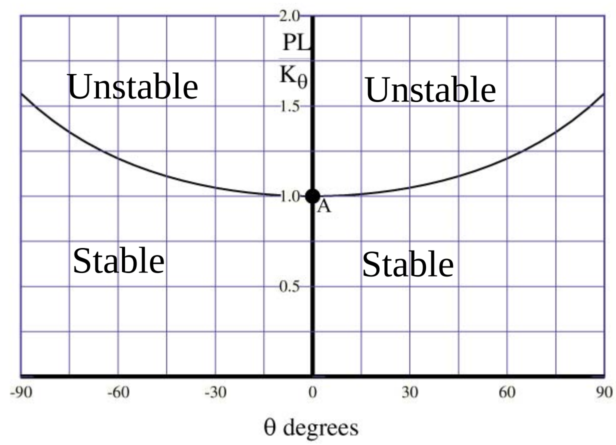
Neutral Equilibrium



Bifurcation/Eigenvalue Problem

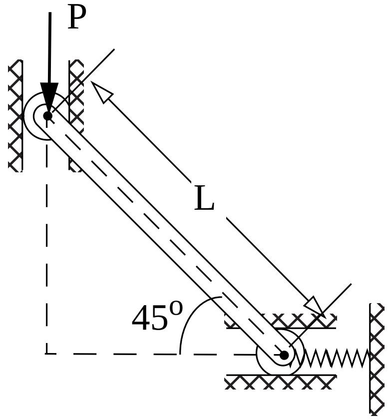


$$PL/K_\theta = \theta/\sin\theta$$

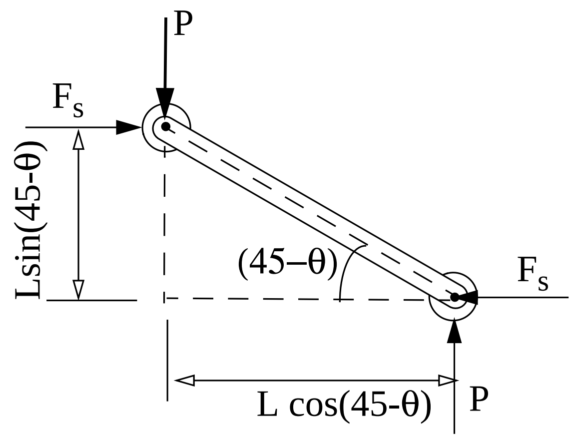


Snap Buckling Problem

$\theta = 0$



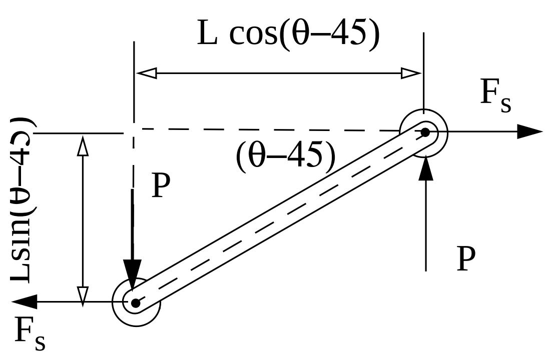
$0 < \theta < 45^{o}$

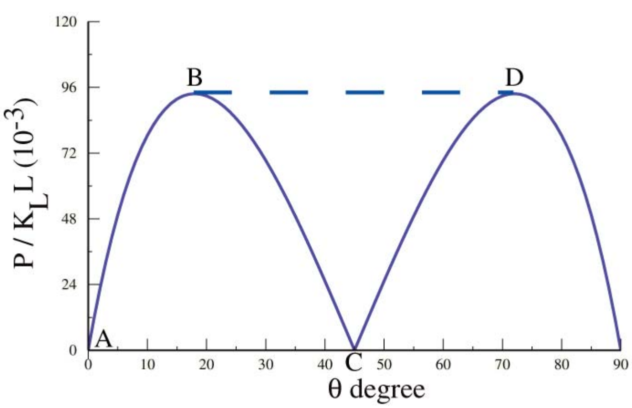


$$\frac{P}{K_L L} = (\cos(45-\theta) - \cos 45)\tan(45-\theta) \qquad 0 < \theta < 45^{0}$$

$$\frac{P}{K_L L} = (\cos(\theta-45) - \cos 45)\tan(\theta-45) \qquad \theta > 45^{0}$$

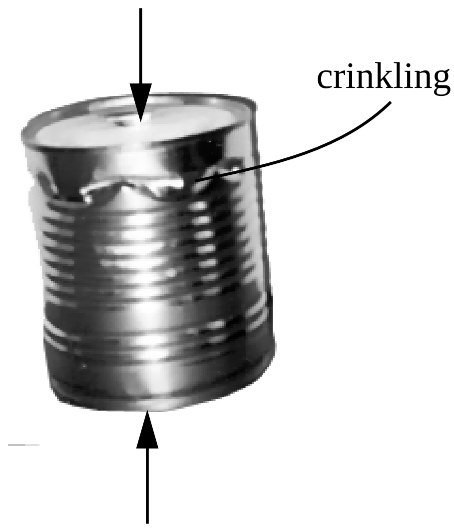
$\theta > 45^{0}$



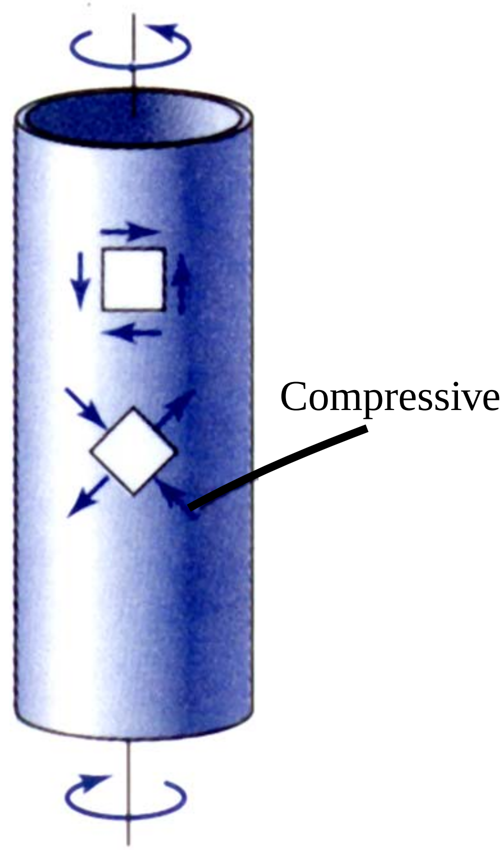


Local Buckling

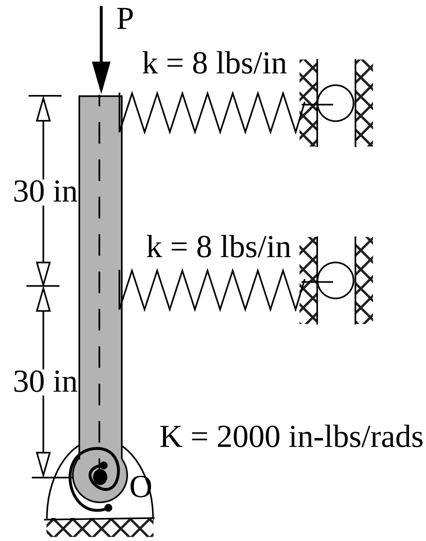
Axial Loads



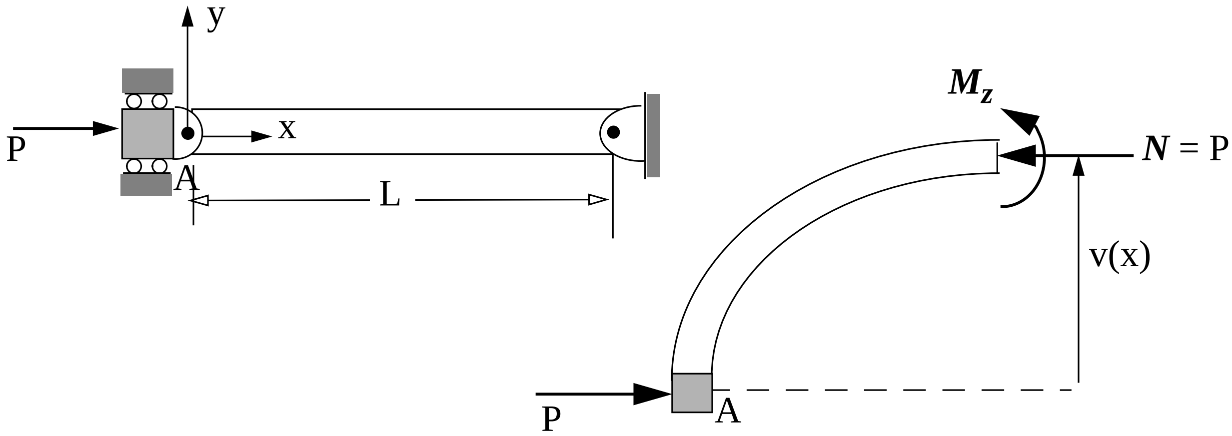
Torsional Loads



11.5 Linear deflection springs and torsional springs are attached to rigid bars as shown.The springs can act in tension or compression and resist rotation in either direction. Determine P_{cr}, the critical load value.



Euler Buckling



Boundary Value Problem

Differential Equation: $EI_{zz}\dfrac{d^2v}{dx^2} + Pv = 0$

Boundary conditions: $v(0) = 0 \qquad v(L) = 0$

Solution

Trivial Solution: $v = 0$

Non-Trivial Solution: $v(x) = A\cos\lambda x + B\sin\lambda x$

where: $\lambda = \sqrt{\dfrac{P}{EI_{zz}}}$

Characteristic Equation: $\sin\lambda L = 0$

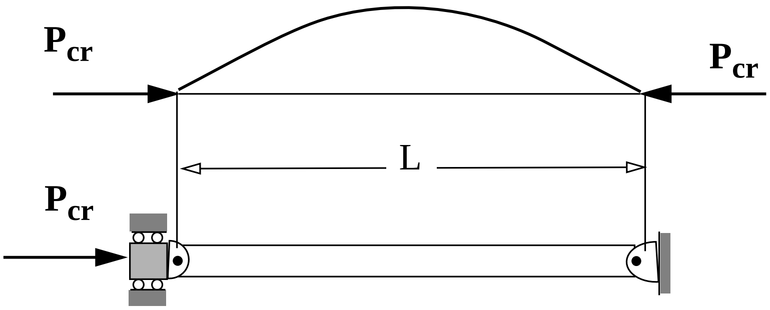
$$P_n = \frac{n^2\pi^2 EI_{zz}}{L^2} \qquad n = 1, 2, 3. \cdot \quad \cdot$$

Euler Buckling Load: $P_{cr} = \dfrac{\pi^2 EI}{L^2}$

- Buckling occurs about an axis that has a minimum value of I.

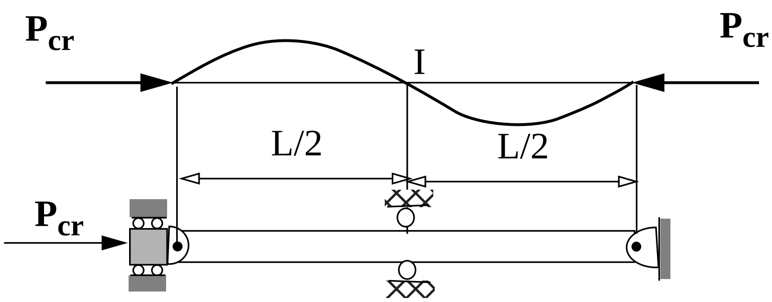
Buckling Mode: $v = B\sin\left(n\pi\frac{x}{L}\right)$

Mode shape 1



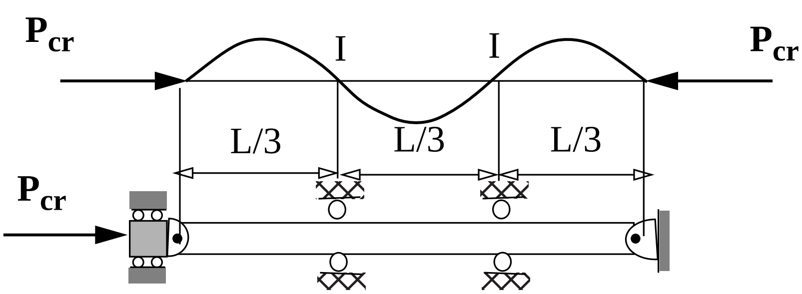
$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Mode shape 2



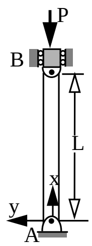
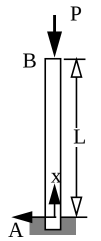
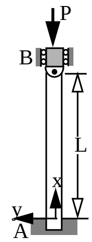
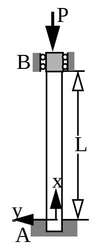
$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$

Mode shape 3



$$P_{cr} = \frac{9\pi^2 EI}{L^2}$$

Effects of End Conditions

Case	1.  Pinned at both Ends	2.  One end fixed, other end free	3.  One end fixed, other end pinned	4.  Fixed at both ends.
Differential Equation	$EI\frac{d^2v}{dx^2} + Pv = 0$	$EI\frac{d^2v}{dx^2} + Pv = Pv(L)$	$EI\frac{d^2v}{dx^2} + Pv = R_B(L-x)$	$EI\frac{d^2v}{dx^2} + Pv = R_B(L-x) + M_B$
Boundary Conditions	$v(0) = 0$ $v(L) = 0$	$v(0) = 0$ $\frac{dv}{dx}(0) = 0$	$v(0) = 0$ $\frac{dv}{dx}(0) = 0$ $v(L) = 0$	$v(0) = 0$ $\frac{dv}{dx}(0) = 0$ $v(L) = 0$ $\frac{dv}{dx}(L) = 0$
Characteristic Equation $\lambda = \sqrt{\frac{P}{EI}}$	$\sin\lambda L = 0$	$\cos\lambda L = 0$	$\tan\lambda L = \lambda L$	$2(1-\cos\lambda L)$ $-\lambda L\sin\lambda L = 0$
Critical Load P_{cr}	$\frac{\pi^2 EI}{L^2}$	$\frac{\pi^2 EI}{4L^2} = \frac{\pi^2 EI}{(2L)^2}$	$\frac{20.13EI}{L^2} = \frac{\pi^2 EI}{(0.7L)^2}$	$\frac{4\pi^2 EI}{L^2} = \frac{\pi^2 EI}{(0.5L)^2}$
Effective Length— L_{eff}	L	2L	0.7L	0.5L

$$P_{cr} = \frac{\pi^2 EI}{L_{eff}^2}$$

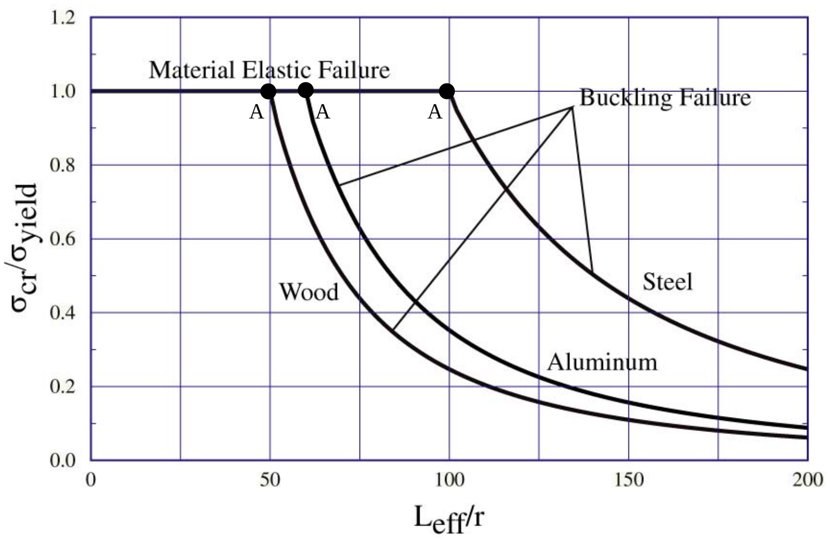
Axial Stress: $\sigma_{cr} = \frac{P_{cr}}{A}$

Radius of gyration: $r = \sqrt{\frac{I}{A}}$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L_{eff}/r)^2}$$

Slenderness ratio: (L_{eff}/r).

Failure Envelopes

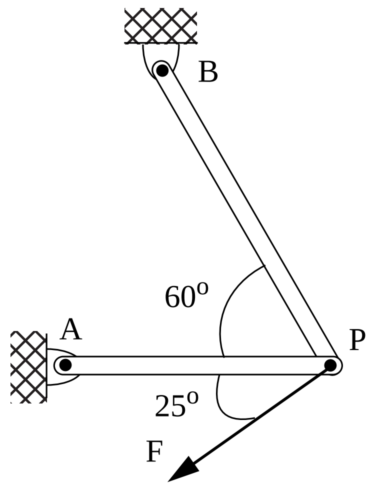


- Short columns: Designed to prevent material elastic failure.
- Long columns: Designed to prevent buckling failure.

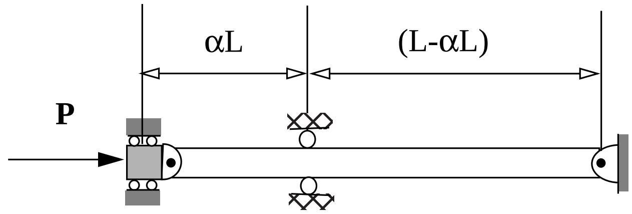
11.13 Columns made from an alloy will be used in a construction of a frame. The cross-section of the columns is a hollow-cylinder of thickness 10 mm and outer diameter of ‘d’ mm. The Modulus of Elasticity is E = 200 GPa and the yield stress is σ_{yield} = 300 MPa. Table below shows a list of the lengths ‘L’ and outer diameters ‘d’. Identify the long and the short columns. Assume the ends of the column are built in.

L (m)	d (mm)
1	60
2	80
3	100
4	150
5	200
6	225
7	250

11.16 A force F= 750 lb is applied to the two bars structure as shown. Both bars have a diameter of d = 1/4 inch, modulus of elasticity E = 30,000 ksi, and yield stress σ_{yield} = 30 ksi. Bar AP and BP have lengths of L_{AP}= 8 inches and L_{BP}= 10 inches, respectively. Determine the factor of safety for the two-bar structures.



11.26 Determine the critical buckling in terms of E, I, L, and α. (b) Determine the critical load when α = 0.5.



11.40 A hoist is constructed using two wooden bars to lift a weight of 5 kips. The Modulus of Elasticity for wood is E = 1,800 ksi and the allowable normal stress 3.0 ksi. Determine the maximum value of L to the nearest inch that can be used in constructing the hoist.