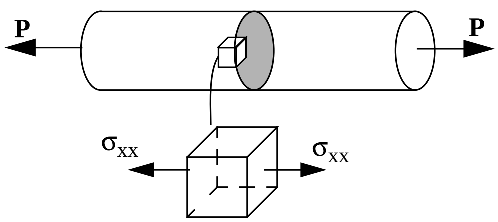
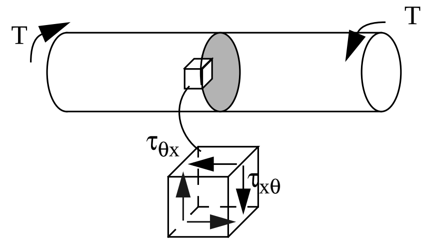
Stress Transformation

- Transforming stress components from one coordinate system to another at a given point.
- Relating stresses on different planes that pass through a point.





Cast Iron

Aluminum

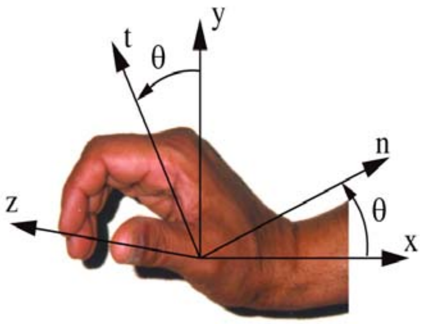
Learning Objectives

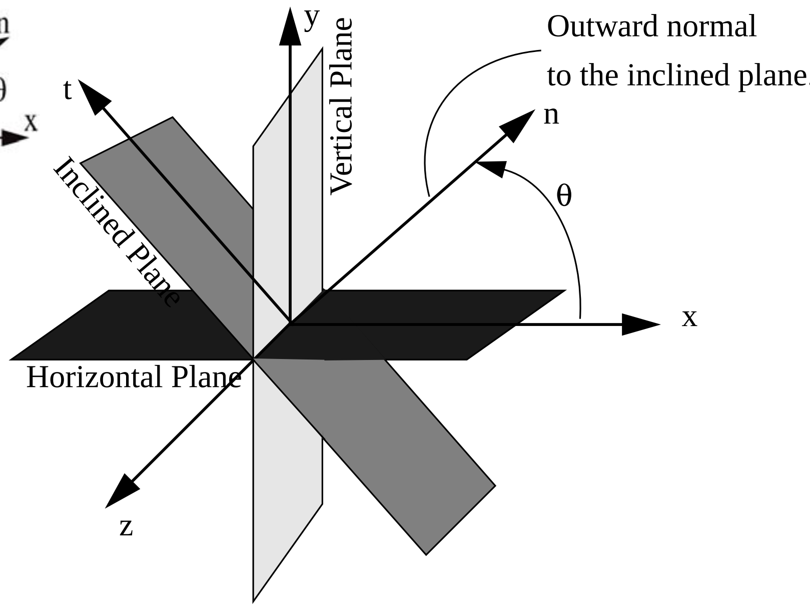
- Learn the equations and procedures of relating stresses (on different planes) in different coordinate system at a point.

- Develop the ability to visualize planes passing through a point on which stresses are given or are being found, particularly the planes of maximum normal stress and maximum shear stress.

Wedge Method

- The fixed reference coordinate system in which the entire problem is described is called the global coordinate system.
- A coordinate system that can be fixed at any point on the body and has an orientation that is defined with respect to the global coordinate system is called the local coordinate system.

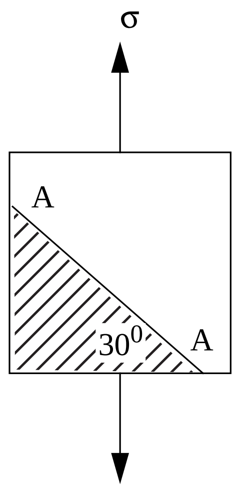




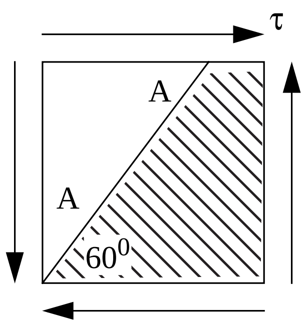
- Plane stress problem: We will consider only those inclined planes that can be obtained by rotation about the z-axis.

8.2 In the following problems one could say that the normal stress on the incline AA is in tension, compression or can't be determined by inspection. Similarly we could say that the shear stress on the incline AA is positive, negative or can't be determined by inspection. Choose the correct answers for normal and shear stress on the incline AA by inspection. Assume coordinate z is perpendicular to this page and towards you.

8.2

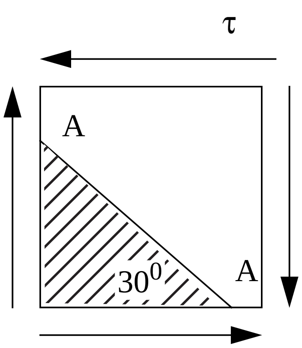


8.6

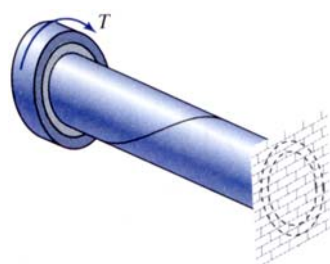


Class Problem 1

8-5



8-16



General Procedure for Wedge Method

Step 1: A stress cube with the plane on which stresses are to be found, or are given, is constructed.

Step 2: A wedge made from the following three planes is constructed:

- a vertical plane that has an outward normal in the x-direction,
- a horizontal plane that has an outward normal in the y-direction, and
- the specified inclined plane on which we either seek stresses or the stresses are given.

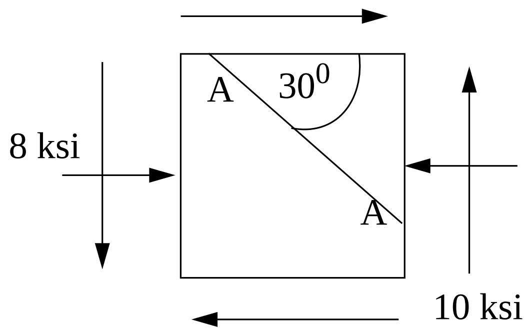
Establish a local n-t-z coordinate system using the outward normal of the inclined plane as the n-direction. All the known and the unknown stresses are shown on the wedge. The diagram so constructed will be called a stress wedge.

Step 3: Multiply the stress components by the area of the planes on which the stress components are acting, to obtain forces acting on that plane. The wedge with the forces drawn will be referred to as the force wedge.

Step 4: Balance forces in any two directions to determine the unknown stresses.

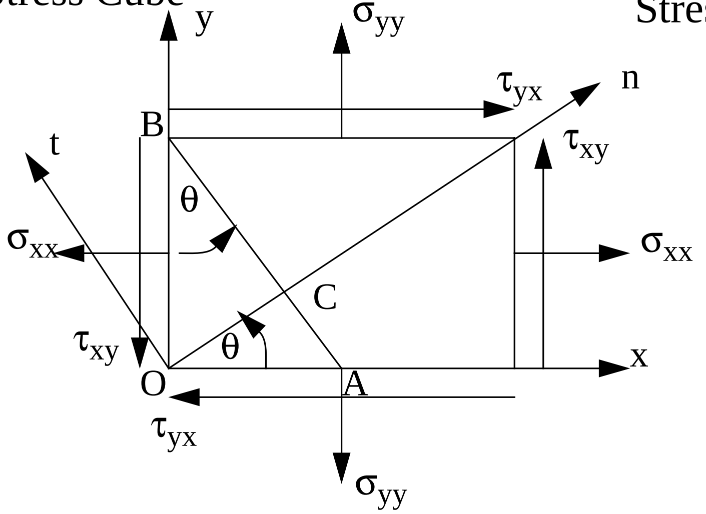
Step 5: Check the answer intuitively.

8.20 Determine the normal and shear stress on plane AA.

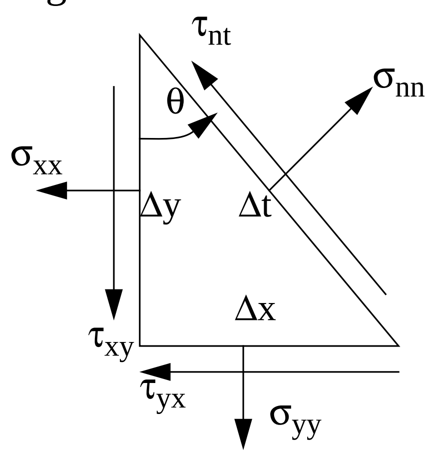


Stress Transformation By Method of Equations

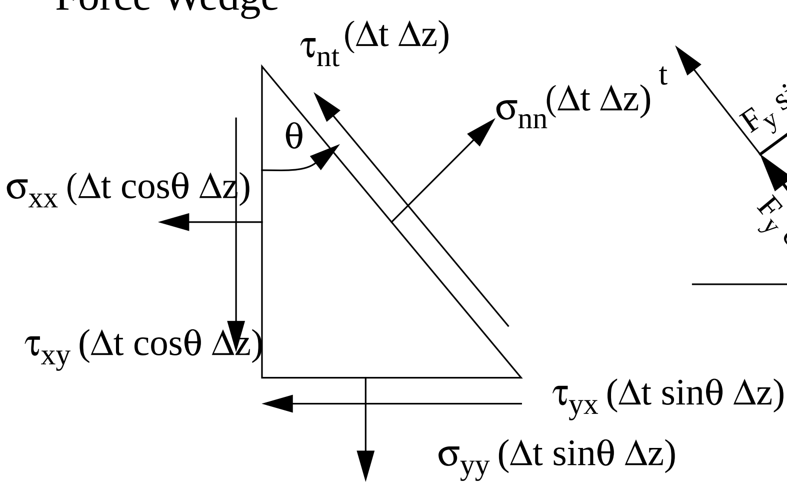
Stress Cube



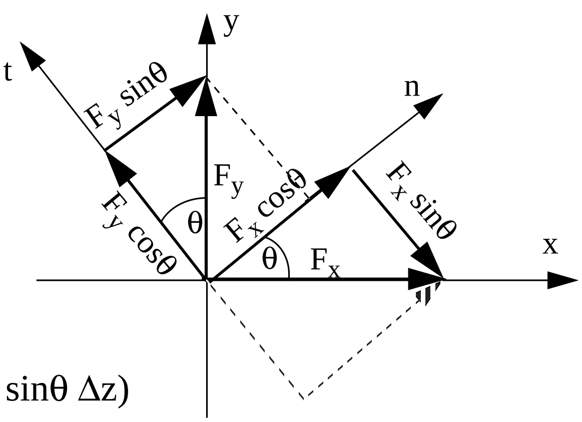
Stress Wedge



Force Wedge



Force Transformation



$$\sigma_{nn} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\tau_{nt} = -\sigma_{xx}\cos\theta\sin\theta + \sigma_{yy}\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

Trigonometric identities

$\cos^2\theta = (1 + \cos 2\theta)/2$ $\qquad$ $\sin^2\theta = (1 - \cos 2\theta)/2$

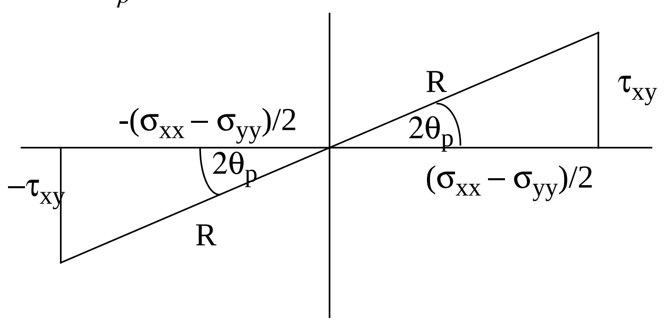
$\cos^2\theta - \sin^2\theta = \cos 2\theta$ $\qquad$ $\cos\theta\sin\theta = (\sin 2\theta)/2$

$$\sigma_{nn} = \frac{(\sigma_{xx} + \sigma_{yy})}{2} + \frac{(\sigma_{xx} - \sigma_{yy})}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\frac{(\sigma_{xx} - \sigma_{yy})}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Maximum normal stress

$$\left(\left.\frac{d\sigma_{nn}}{d\theta}\right|_{\theta = \theta_p} = 0\right) \Rightarrow \tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_{xx} - \sigma_{yy})}$$



$$\theta_1 = \theta_p$$

$$\theta_2 = 90 + \theta_p$$

$$\sigma_{1,2} = \frac{(\sigma_{xx} + \sigma_{yy})}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} \qquad \tau_{1,2} = 0$$

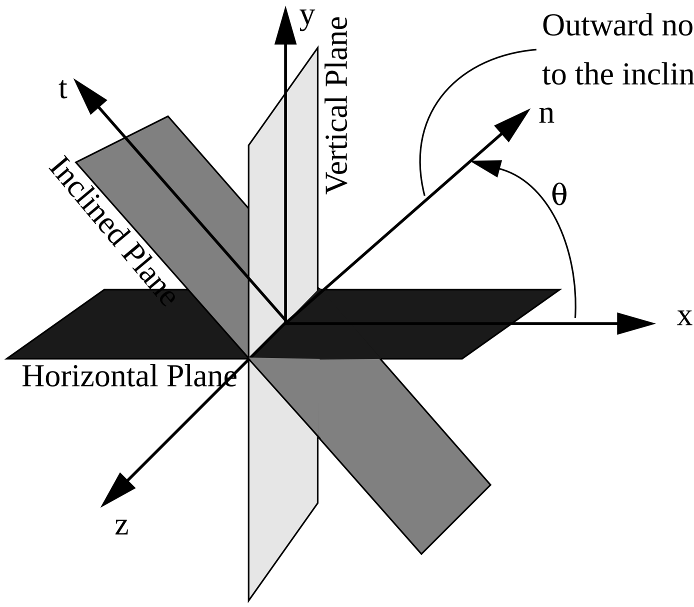
- Planes on which the shear stresses are zero are called the principal planes.
- The normal direction to the principal planes is referred to as the principal direction or the principal axis.
- The angles the principal axis makes with the global coordinate system are called the principal angles.
- Normal stress on a principal plane is called the principal stress.
- The greatest principal stress is called principal stress one.
- Only θ defining principal axis one is reported in describing the principal coordinate system in two-dimensional problems. Counterclockwise rotation from the x axis is defined as positive.

$$\sigma_{nn} + \sigma_{tt} = \sigma_{xx} + \sigma_{yy} = \sigma_1 + \sigma_2$$

- The sum of the normal stresses is invariant with the coordinate transformation.

$$\sigma_3 = \sigma_{zz} = \begin{cases} 0 & \text{Plane Stress} \\ \nu(\sigma_{xx} + \sigma_{yy}) = \nu(\sigma_1 + \sigma_2) & \text{Plane Strain} \end{cases}$$

In-Plane Maximum Shear Stress



- The maximum shear stress on a plane that can be obtained by rotating about the z axis is called the in-plane maximum shear stress.

$$\tau_{nt} = -\frac{(\sigma_{xx} - \sigma_{yy})}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

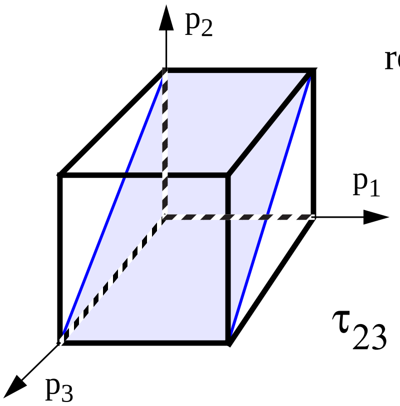
$$\left(\left.\frac{d\tau_{nt}}{d\theta}\right|_{\theta = \theta_s} = 0\right) \Rightarrow \left(\tan 2\theta_s = \frac{-(\sigma_{xx} - \sigma_{yy})}{2\tau_{xy}}\right) \qquad |\tau_p| = \left|\frac{\sigma_1 - \sigma_2}{2}\right|$$

- maximum in-plane shear stress exists on two planes, each of which are 45^{o} away from the principal planes.

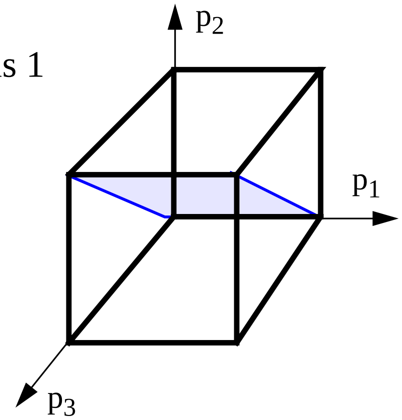
Maximum Shear Stress

- The maximum shear stress at a point is the absolute maximum shear stress that acts on any plane passing through the point.

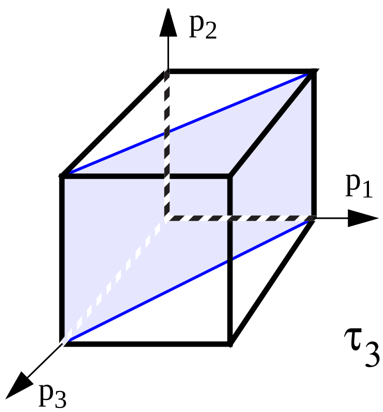
Planes of Maximum Shear Stress



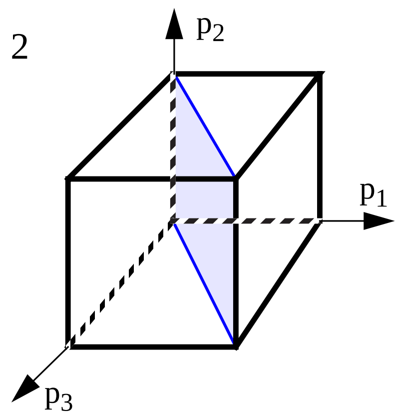
rotation about principal axis 1



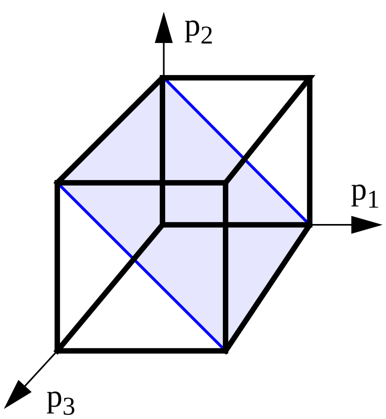
$$\tau_{23} = -\tau_{32} = \frac{\sigma_2 - \sigma_3}{2}$$



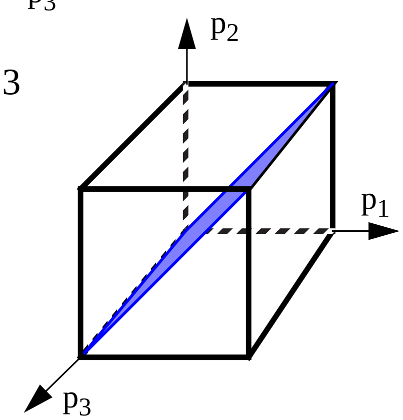
rotation about principal axis 2



$$\tau_{31} = -\tau_{13} = \frac{\sigma_3 - \sigma_1}{2}$$



rotation about principal axis 3
(In-plane)



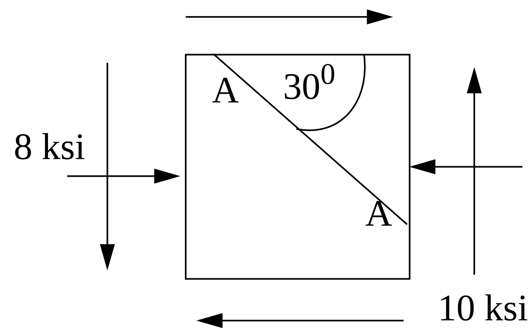
$$\tau_{21} = -\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{max} = \left| max\left(\frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_2 - \sigma_3}{2}, \frac{\sigma_3 - \sigma_1}{2}\right) \right|$$

$$\sigma_3 = \sigma_{zz} = \begin{cases} 0 & \text{Plane Stress} \\ \nu(\sigma_{xx} + \sigma_{yy}) = \nu(\sigma_1 + \sigma_2) & \text{Plane Strain} \end{cases}$$

- The maximum shear stress value may be different in plane stress and in plane strain.

8.38 Determine the normal and shear stress on plane AA using the method of equations (resolving problem 8-20).



Stress Transformation By Mohr’s Circle

$$\sigma_{nn} = \frac{(\sigma_{xx} + \sigma_{yy})}{2} + \frac{(\sigma_{xx} - \sigma_{yy})}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\frac{(\sigma_{xx} - \sigma_{yy})}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\left(\sigma_{nn} - \frac{\sigma_{xx} + \sigma_{yy}}{2}\right)^2 + \tau_{nt}^2 = \left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2 \qquad \textbf{8.1}$$

- Each point on the Mohr’s circle represents a unique plane that passes through the point at which the stresses are specified.
- The coordinates of the point on Mohr’s Circle are the normal and shear stress on the plane represented by the point.
- On Mohr’s circle, plane are separated by twice the actual angle between the planes.

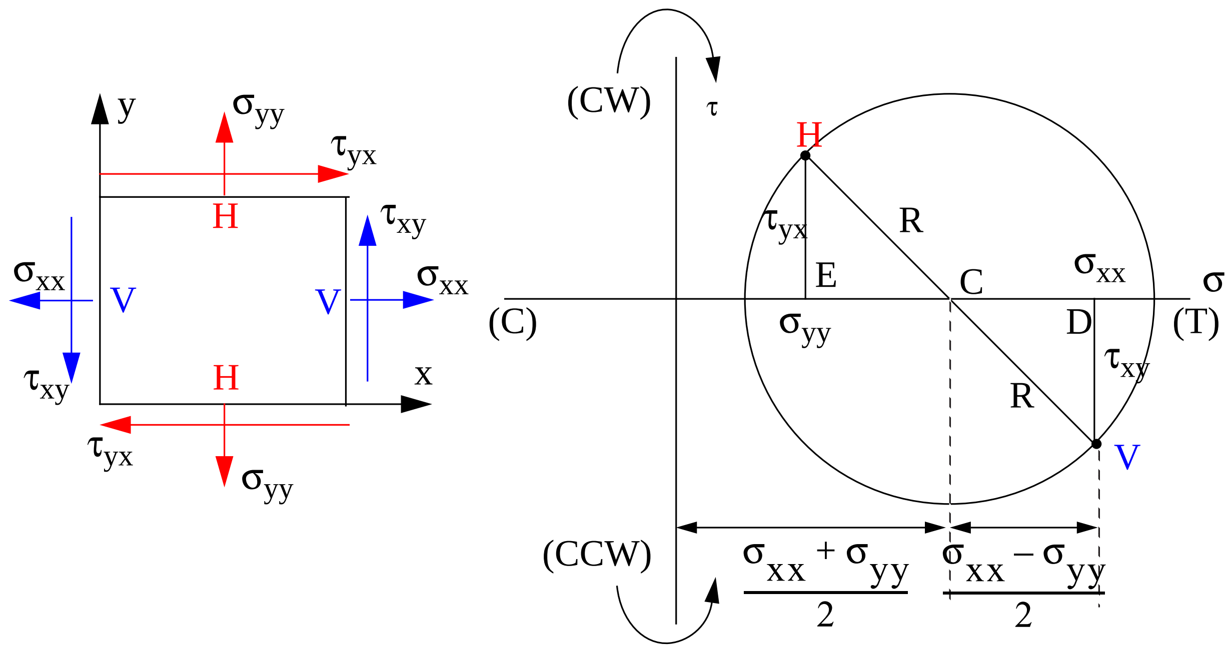
Construction of Mohr's Circle

Step 1. Show the stresses σ_{xx}, σ_{yy}, and τ_{xy} on a stress cube and label the vertical plane as V and the horizontal plane as H.

Step 2. Write the coordinates of points V and H as

$$V\,(\sigma_{xx}\,,\,\tau_{xy}\,)\ \text{and}\ \ H\,(\sigma_{yy}\,,\,\tau_{yx}\,)$$

The rotation arrow next to the shear stresses corresponds to the rotation of the cube caused by the set of shear stress on planes V and H.

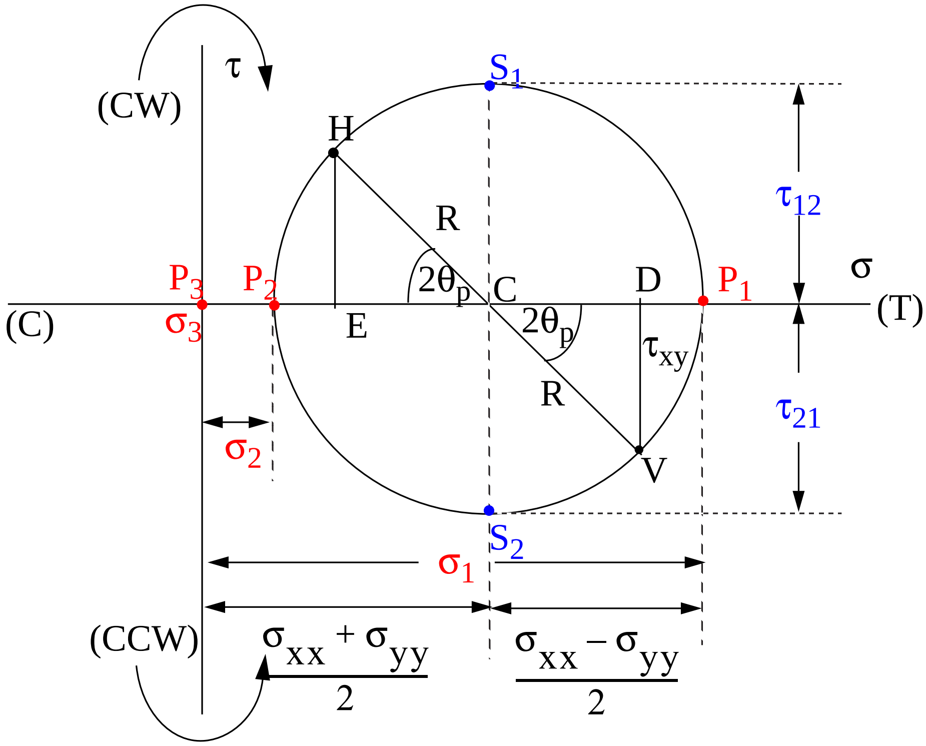


Step 3. Draw the horizontal axis with the tensile normal stress to the right and the compressive normal stress to the left. Draw the vertical axis with clockwise direction of shear stress up and counterclockwise direction of rotation down.

Step 4. Locate points V and H and join the points by drawing a line. Label the point at which the line VH intersects the horizontal axis as C.

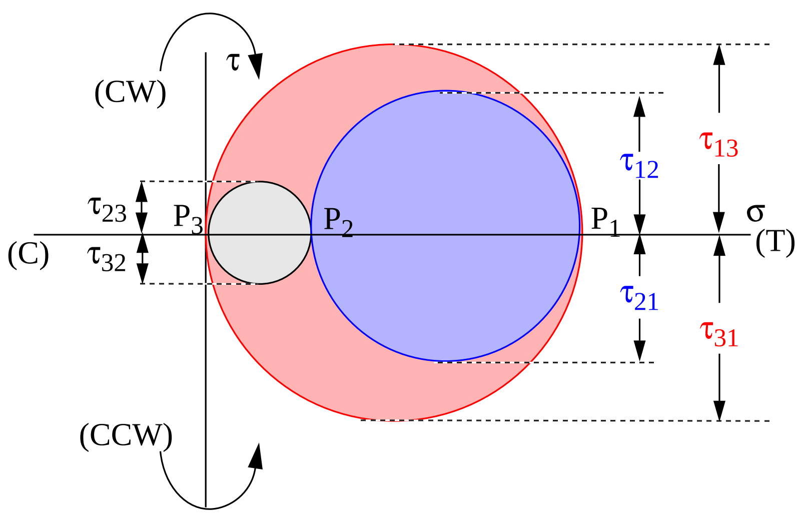
Step 5. With C as center and CV or CH as radius draw the Mohr's circle.

Principal Stresses & Maximum In-Plane Shear Stress from Mohr's Circle

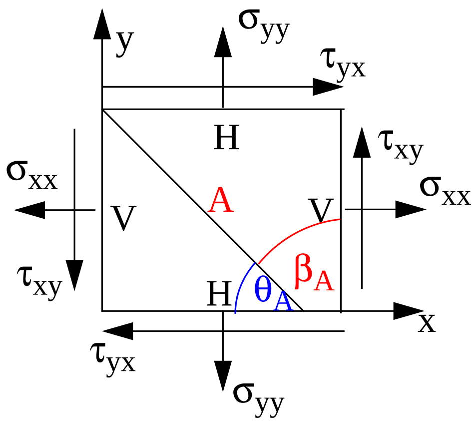


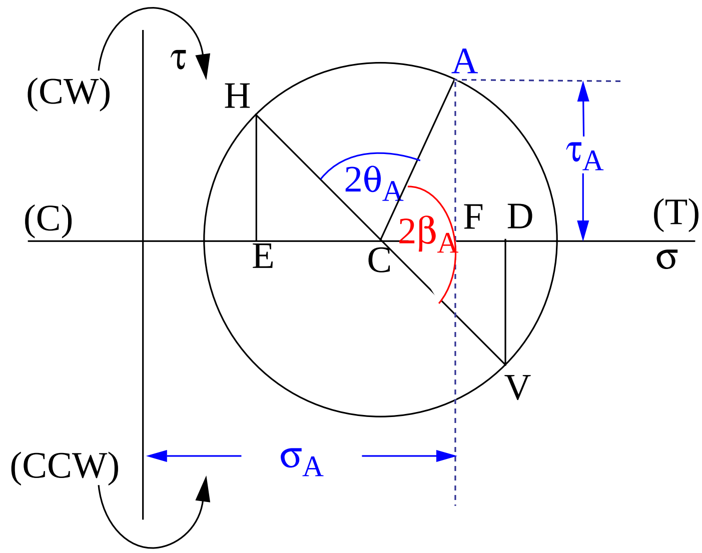
- The principal angle one θ_1 is the angle between line CV and CP_1. Depending upon the Mohr circle θ_1 may be equal to θ_p or equal to $(\theta_p \pm 90^o)$.

Maximum Shear Stress



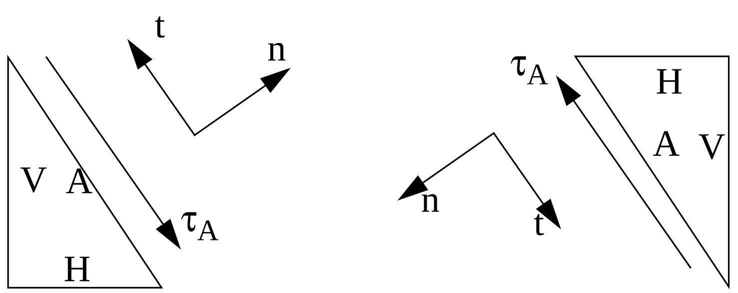
Stresses on an Inclined Plane



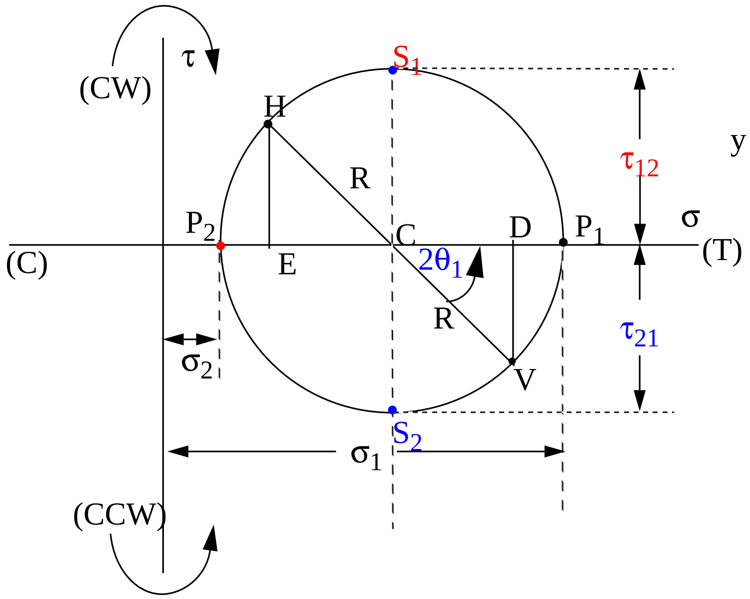


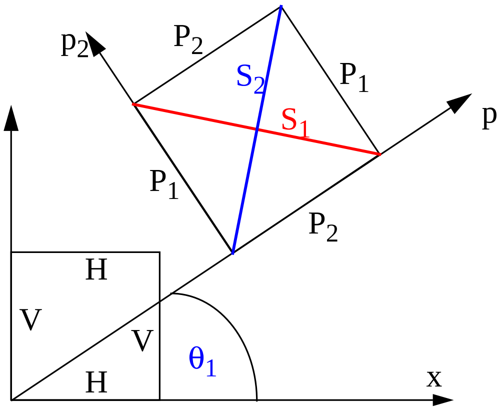
Sign of shear stress on incline:

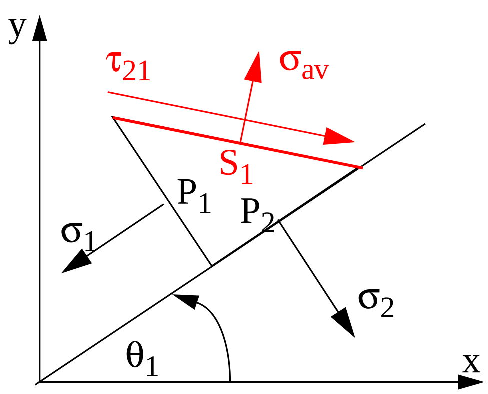
Coordinates of point A: (σ_A, τ_A)

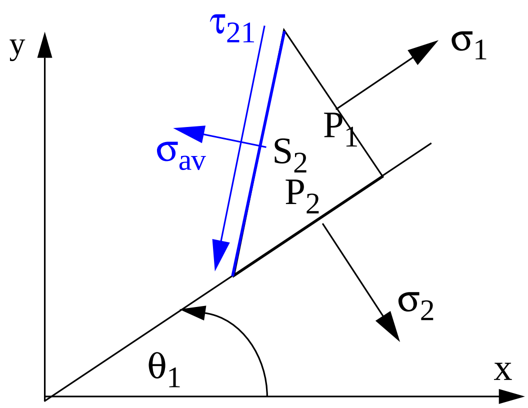


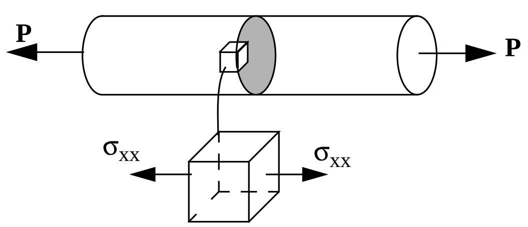
Principal Stress Element

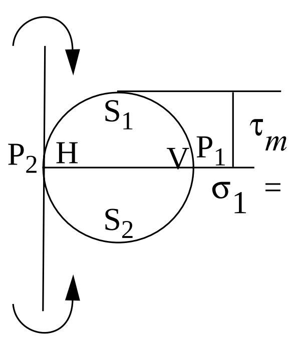






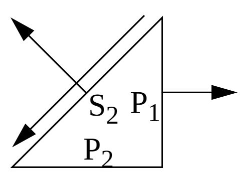






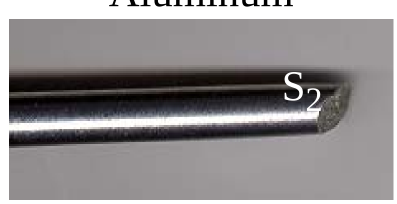
$$\tau_{max} = \frac{\sigma_{xx}}{2}$$

$$\sigma_1 = \sigma_{xx}$$

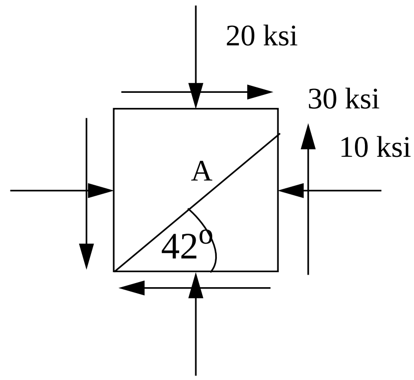


Cast Iron

Aluminum

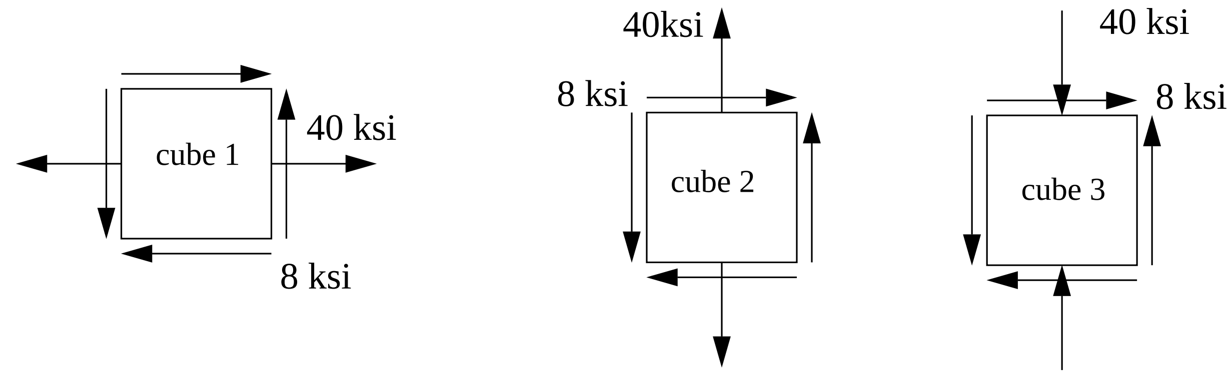


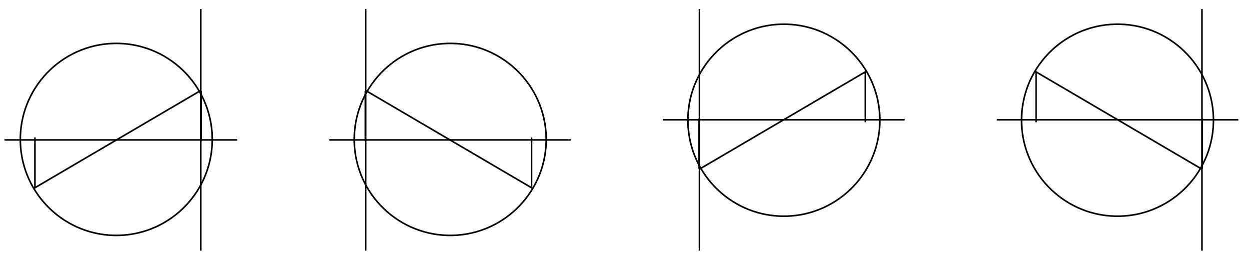
8.44 In a thin body (*plane stress*) the stresses in the x-y plane are as shown on each stress element. (a) Determine the normal and shear stresses on plane A. (b) Determine the principal stresses at the point. (c) Determine the maximum shear stress at the point. (d) Draw the principal element.



Class Problem 2

Associate the stress cubes with the appropriate Mohr's circle for stress.

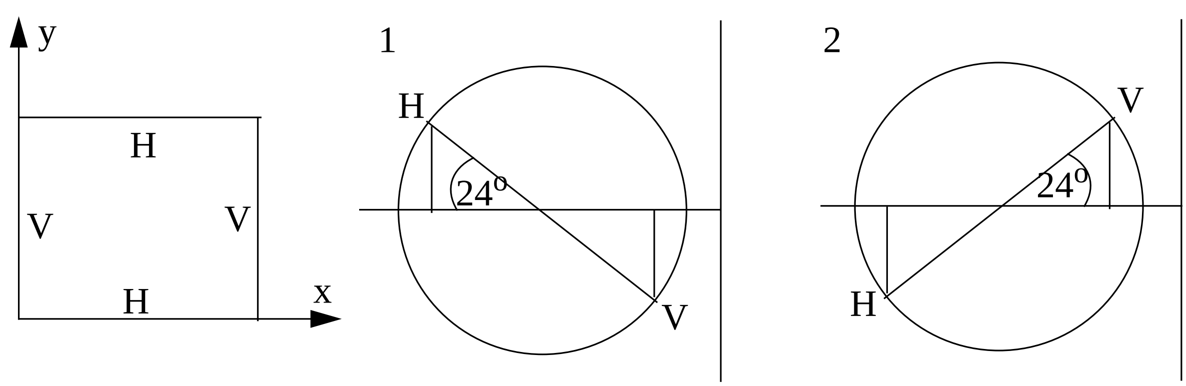




circle A circle B circle C circle D

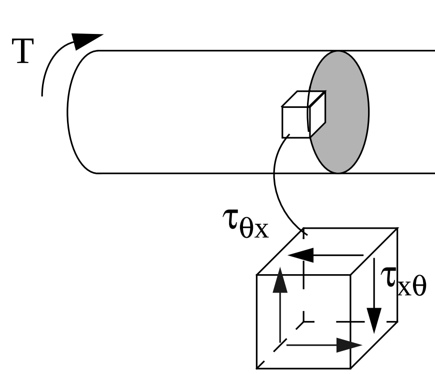
Class Problem 3

Determine the two possible values of principal angle one (θ_1) in each question.



Class Problem 4

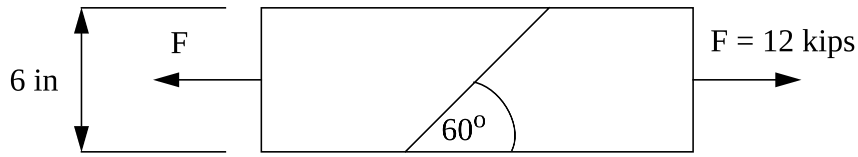
Explain the failure surfaces in cast iron and aluminum due to torsion by drawing the principal stress element



Cast Iron

Aluminum

8.54 A broken 2 in x 6 in wooden bar was glued together as shown. Determine the normal and shear stress in the glue.



8.56 If the applied force P = 1.8 kN, determine the principal stresses and maximum shear stress at points A, B, and C which are on the surface of the beam.