

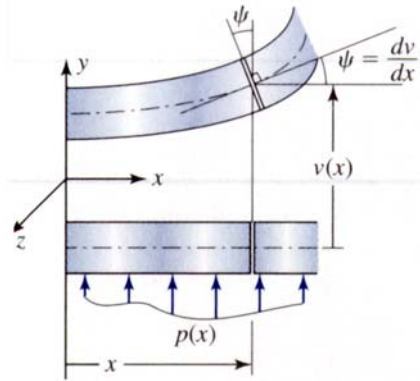
Deflection of Symmetric Beams



Learning objective

- To learn how to formulate and solve the boundary value problem for deflection of a beam at any point.

Second-Order Boundary Value Problem

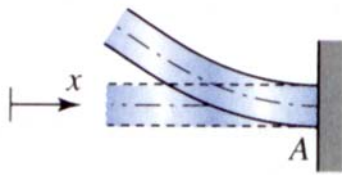


- The deflected curve represented by $v(x)$ is called the **Elastic Curve**.

Differential equation: $M_z = EI_{zz} \frac{d^2 v}{dx^2}$

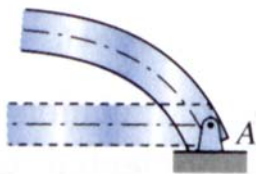
- The mathematical statement listing all the differential equations and all the conditions necessary for solving for $v(x)$ is called the **Boundary Value Problem** for the beam deflection.

Boundary Conditions

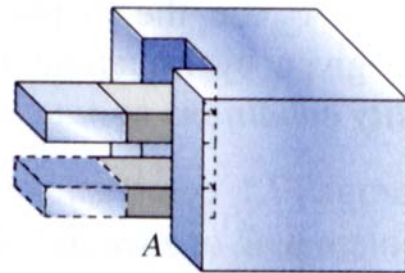


$$v(x_A) = 0$$

$$\frac{dv}{dx}(x_A) = 0$$

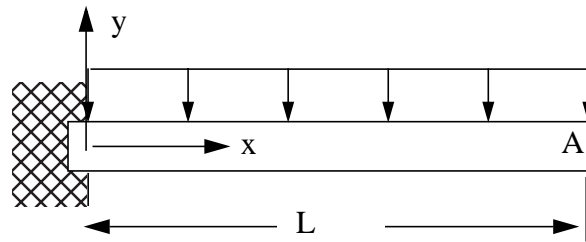


$$v(x_A) = 0$$



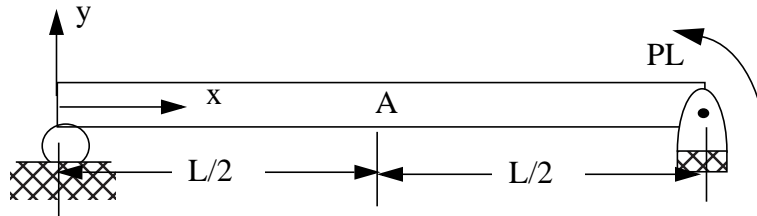
$$\frac{dv}{dx}(x_A) = 0$$

7.4 In terms of w , P , L , E , and I determine (a) equation of the elastic curve. (b) the deflection of the beam at point A.



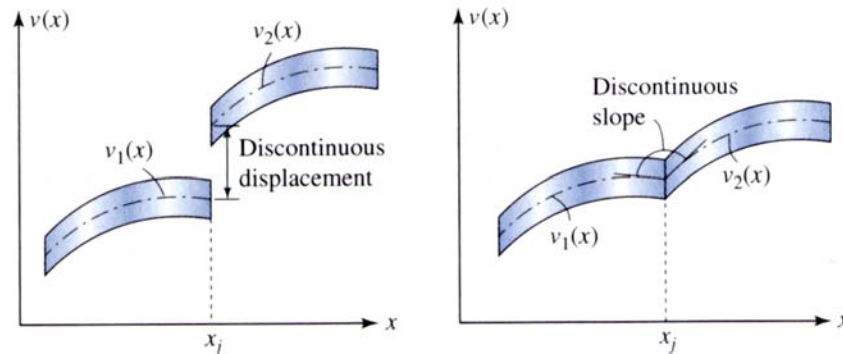
Class Problem 1

Write the boundary value problem to determine the elastic curve.



Continuity Conditions

- The internal moment M_z will change with change in applied loading.
- Each change in M_z represents a new differential equation, hence new integration constants.

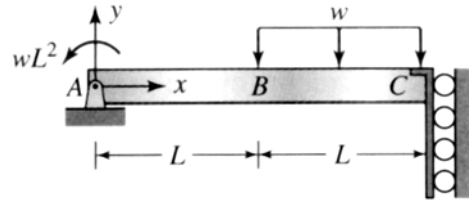


$$v_1(x_j) = v_2(x_j)$$

$$\frac{dv_1}{dx}(x_j) = \frac{dv_2}{dx}(x_j)$$

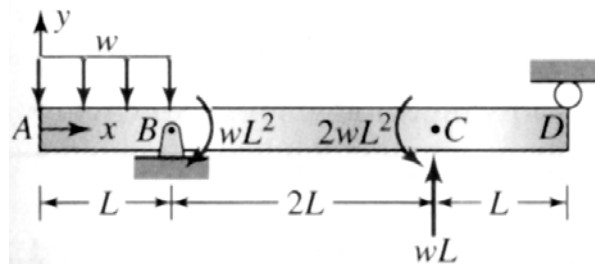
- ‘**continuity conditions**’, also known as ‘**compatibility conditions**’ or ‘**matching conditions**’.

7.5 In terms of w , L , E , and I , determine (a) the equation of the elastic curve. (b) the deflection at $x = L$.



Class Problem 2

7.14 Write the boundary value problem for determining the deflection of the beam at any point x . Assume EI is constant. Do not integrate or solve.



The internal moments are:

$$\text{In AB: } M_1 = -\frac{wx^2}{2} \qquad \text{In BC: } M_2 = \frac{wLx}{6} + \frac{wL^2}{3}$$

$$\text{In CD: } M_3 = \frac{7wLx}{6} - \frac{14wL^2}{3}$$