Axial Members

- Members with length significantly greater than the largest cross-sectional dimension and with loads applied along the longitudinal axis.

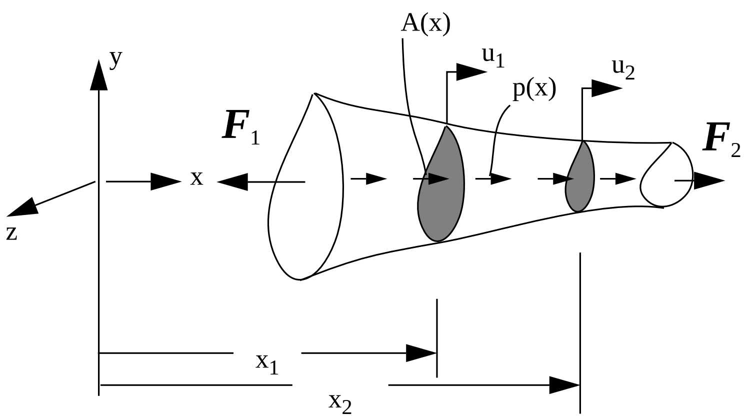
Learning objectives are:

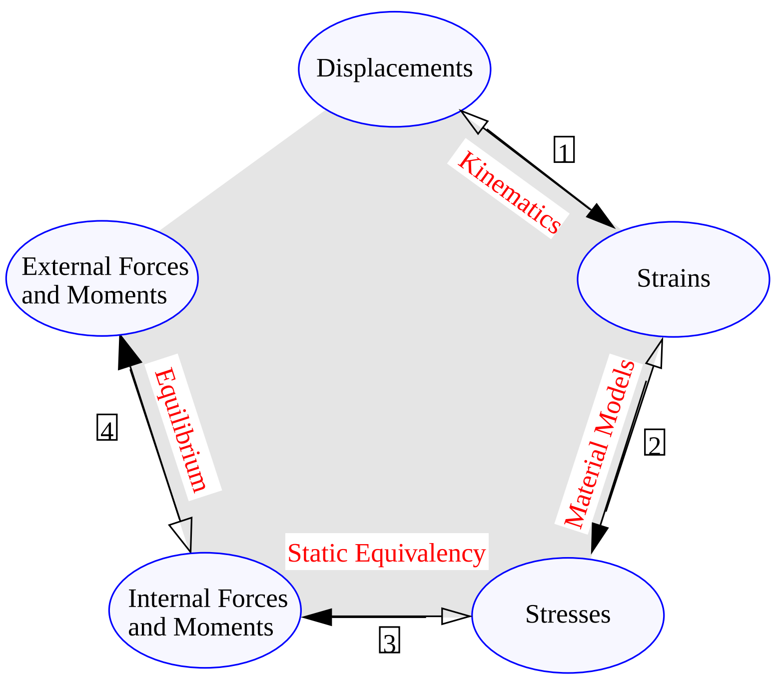
- Understand the theory, its limitations, and its applications for design and analysis of axial members.
- Develop the discipline to draw free body diagrams and approximate deformed shapes in the design and analysis of structures.

Theory

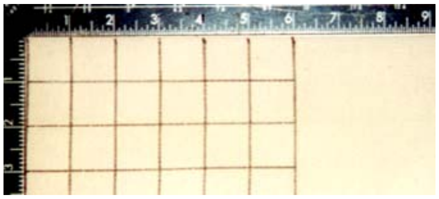
Theory Objective

- to obtain a formula for the relative displacements (u_2-u_1) in terms of the internal axial force ***N***.
- to obtain a formula for the axial stress σ_{xx} in terms of the internal axial force ***N***.

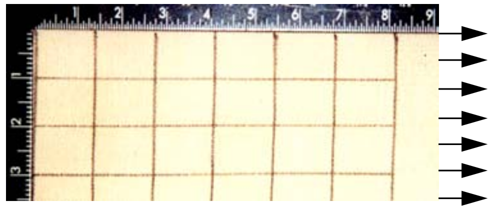




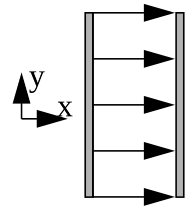
Kinematics



Original Grid



Deformed Grid



u is constant in y direction.

Assumption 1 Plane sections remain plane and parallel. $u = u(x)$

- The displacement u is considered positive in the positive x-direction.

Assumption 2 Strains are small. $\varepsilon_{xx} = \dfrac{du(x)}{dx}$

Material Model

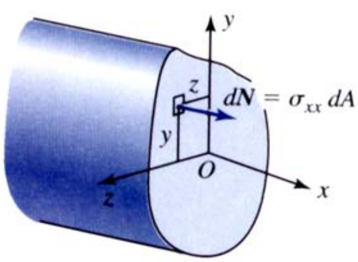
Assumption 3 Material is isotropic.

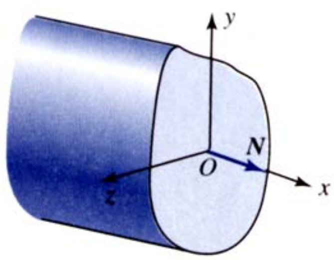
Assumption 4 Material is linearly elastic.

Assumption 5 There are no inelastic strains.

From Hooke's Law: $\sigma_{xx} = E\varepsilon_{xx}$, we obtain $\sigma_{xx} = E\dfrac{du}{dx}$

Internal Axial Force





$$N = \int_A \sigma_{xx}\, dA$$

$$N = \int_A E\frac{du}{dx}\, dA = \frac{du}{dx}\int_A E\, dA$$

- For pure axial problems the internal moments (bending) $\boldsymbol{M}_y$ and $\boldsymbol{M}_z$ must be zero.
- For homogenous materials all external and internal axial forces must pass through the centroids of the cross-section and all centroids must lie on a straight line.

Axial Formulas

Assumption 6 Material is homogenous across the cross-section.

$$\boldsymbol{N} = E\frac{du}{dx}\int_A dA = EA\frac{du}{dx} \qquad or \qquad \frac{du}{dx} = \frac{\boldsymbol{N}}{EA}$$

$$\sigma_{xx} = E\frac{du}{dx} = E\left(\frac{\boldsymbol{N}}{EA}\right) \qquad or \qquad \sigma_{xx} = \frac{\boldsymbol{N}}{A}$$

- The quantity EA is called the Axial rigidity.

Assumption 7 Material is homogenous between x_1 and x_2.

Assumption 8 The bar is not tapered between x_1 and x_2.

Assumption 9 The external (hence internal) axial force does not change with x between x_1 and x_2.

$$u_2 - u_1 = \frac{\boldsymbol{N}(x_2 - x_1)}{EA}$$

Two options for determining internal axial force ***N***

- ***N*** is always drawn in tension at the imaginary cut on the free body diagram.

  Positive value of σ_{xx} will be tension.

  Positive u_2-u_1 is extension.

  Positive u is in the positive x-direction.

- ***N*** is drawn at the imaginary cut in a direction to equilibrate the external forces on the free body diagram.

  Tension or compression for σ_{xx} has to be determined by inspection.

  Extension or contraction for $\delta=u_2-u_1$ has to be determined by inspection.

  Direction of displacement u has to be determined by inspection.

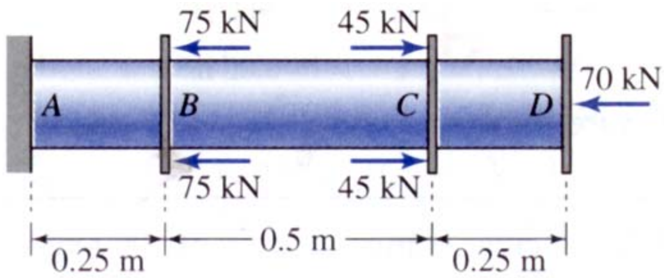
Axial stresses and strains

- all stress components except σ_{xx} can be assumed zero.

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E}$$

$$\varepsilon_{yy} = -\left(\frac{\nu\sigma_{xx}}{E}\right) = -\nu\varepsilon_{xx} \qquad \varepsilon_{zz} = -\left(\frac{\nu\sigma_{xx}}{E}\right) = -\nu\varepsilon_{xx}$$

4.8 Determine the internal axial forces in segments AB, BC, and CD by making imaginary cuts and drawing free body diagrams.



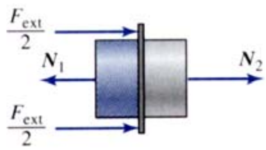
Axial Force Diagrams

- An axial force diagram is a plot of internal axial force ***N*** vs. x
- Internal axial force jumps by the value of the external force as one crosses the external force from left to right.
- An axial template is used to determine the direction of the jump in ***N*.**
- A template is a free body diagram of a small segment of an axial bar created by making an imaginary cut just before and just after the section where the external force is applied.

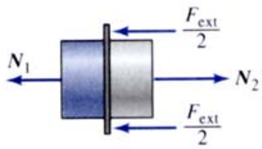
Template 1 Equation

$$N_2 = N_1 - F_{ext}$$

Template 1



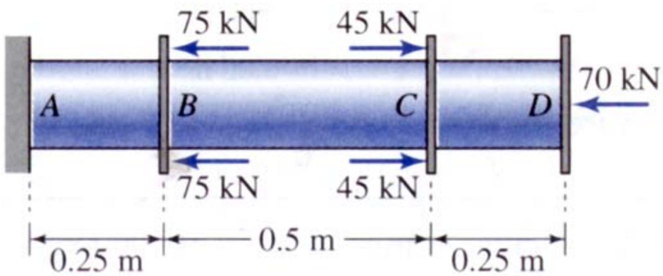
Template 2



Template 2 Equation

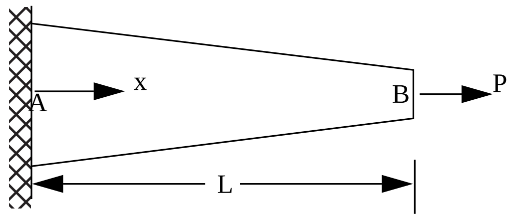
$$N_2 = N_1 + F_{ext}$$

4.8 Determine the internal axial forces in segments AB, BC, and CD by drawing axial force diagram.



4.12 The axial rigidity of the bar in problem 4.8 is EA = 80,000 kN. Determine the movement of section at C.

4.19 The tapered bar shown in Fig. P4.19 has a cross-sectional area that varies with x as given. Determine the elongation of the bar in terms of P, L, E and K.



$$A = K(4L - 3x)$$

Fig. P4.19

4.22 The columns shown has a length L, Modulus of elasticity E, specific weight γ, and length a as the side of an equilateral triangle. Determine the contraction of the column in terms of L, E, γ, and a.

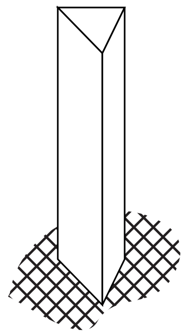
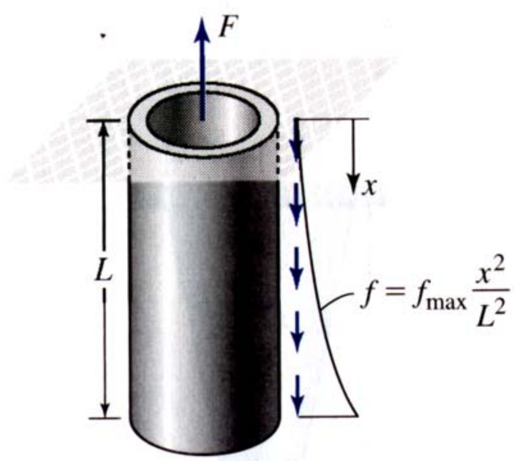


Fig. P4.22

4.28 The frictional force per unit length on a cast iron pipe being pulled from the ground varies as a quadratic function as shown. Determine the force F needed to pull the pipe out of ground and the elongation of the pipe before the pipe slips in terms of the modulus of elasticity E, area of cross-section A, length L and the maximum value of frictional force f_{max}.



4.32 A hitch for an automobile is to be designed for pulling a maximum load of 3,600 lbs. A solid-square-bar fits into a square-tube, and is held in place by a pin as shown. The allowable axial stress in the bar is 6 ksi, the allowable shear stress in the pin is 10 ksi, and the allowable axial stress in the steel tube is 12 ksi. To the nearest 1/16th of an inch, determine the minimum cross-sectional dimensions of the pin, the bar and the tube. Neglect stress concentration. (Note: Pin is in double shear)

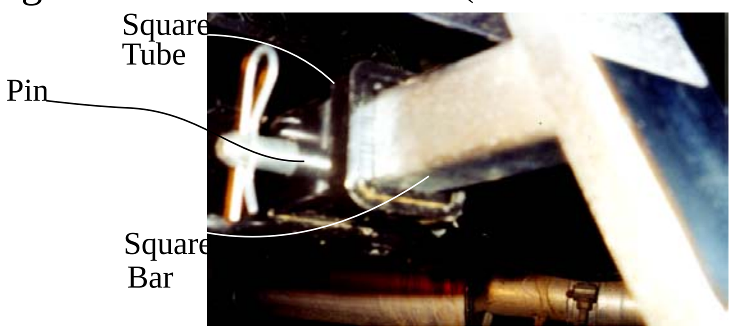


Fig. P4.32

4.41 Table below shows the value of distributed axial force at several point along the axis of a hollow steel (E = 30,000 ksi) rod. The rod has a length of 36 inches, an outside diameter of 1 inch, and an inside diameter of 0.5 inch. Determine (a) the displacement of the end A using numerical integration. (b) the maximum axial stress in the rod.

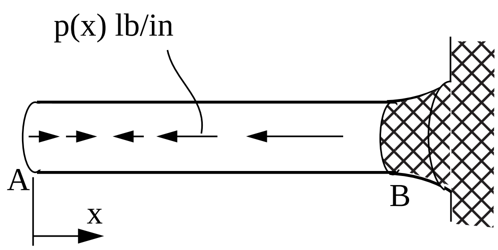
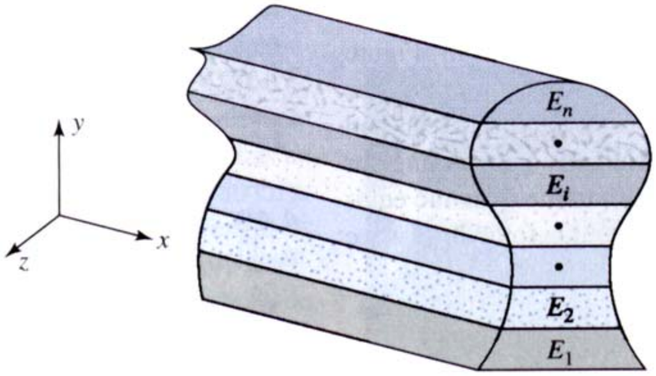


Fig. P4.41

x (inches)	p (x) lbs./in	x (inches)	p (x) lbs./in
0	260	21	-471
3	106	24	-598
6	32	27	-645
9	40	30	-880
12	-142	33	-1035
15	-243	36	-1108
18	-262		

Composite Bars



- Assumption 6 of material homogeneity across the cross-section is no longer valid.

$$N = \frac{du}{dx}\int_A E\,dA = \frac{du}{dx}\left[\int_{A_1} E_1\,dA + \int_{A_2} E_2\,dA + \cdot + \cdot + \int_{A_n} E_n\,dA\right]$$

- Rest of derivation is same as for homogenous cross-section.

Homogenous cross-section	Composite cross-section
$\frac{du}{dx} = N/(EA)$	$\frac{du}{dx} = N\Big/\left[\sum_{j=1}^{n} E_j A_j\right]$
$u_2 - u_1 = N(x_2 - x_1)/(EA)$	$u_2 - u_1 = N(x_2 - x_1)\Big/\left[\sum_{j=1}^{n} E_j A_j\right]$
$\sigma_{xx} = N/A$	$\left.\sigma_{xx}\right)_i = (NE_i)\Big/\left(\sum_{j=1}^{n} E_j A_j\right)$

4.47 A concrete column is reinforced using nine iron circular bars of diameter 1 inch. The modulus of elasticity for concrete and iron are E_c= 4,500 ksi and E_i = 25,000 ksi. Determine (a) the maximum axial stress in concrete and iron. (b) the contraction of the column.

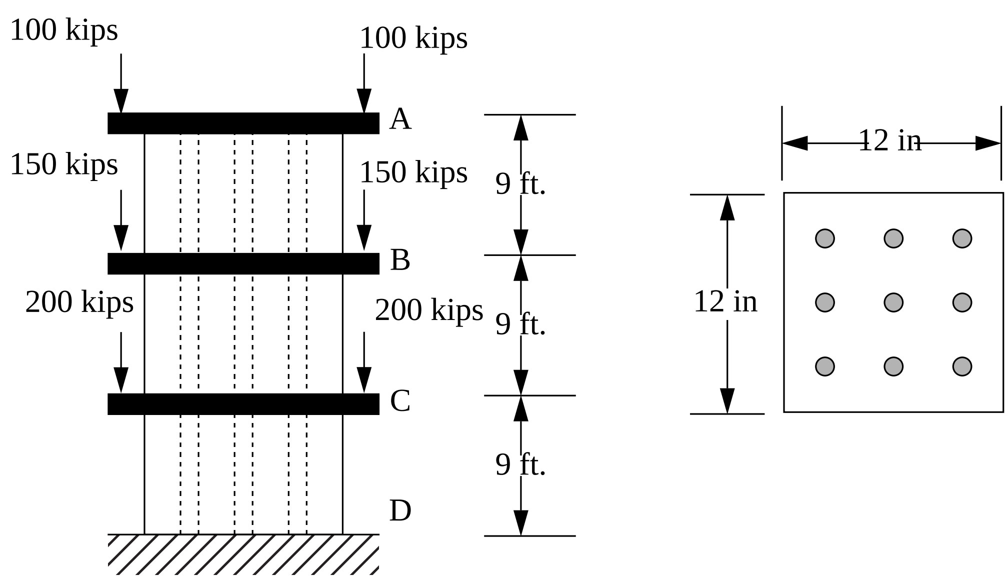


Fig. P4.47

4.49 A cross-section of a bar is made from two materials. Assume parallel sections remain parallel i.e., $\varepsilon_{xx} = \frac{du}{dx}(x)$. In terms of the variables P, E and h determine (a)the location (y_P) of force P on the cross-section so that there is only axial deformation and no bending. (b) the axial stress at point A.

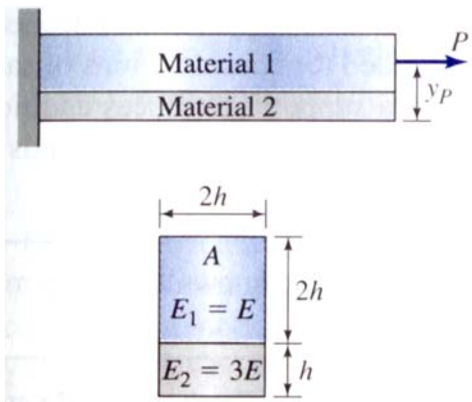


Fig. P4.49

Structural analysis

$$\delta = \frac{NL}{EA}$$

- δ is the deformation of the bar in the undeformed direction.
- If N is a tensile force then δ is elongation.
- If N is a compressive force then δ is contraction.
- Deformation of a member shown in the drawing of approximate deformed geometry must be consistent with the internal force in the member that is shown on the free body diagram.
- In statically indeterminate structures number of unknowns exceed the number of static equilibrium equations. The extra equations needed to solve the problem are relationships between deformations obtained from the deformed geometry.
- Force method----Internal forces or reaction forces are unknowns.
- Displacement method---Displacements of points are unknowns.

General Procedure for analysis of indeterminate structures.

- If there is a gap, assume it will close at equilibrium.
- Draw Free Body Diagrams, write equilibrium equations.
- Draw an exaggerated approximate deformed shape. Write compatibility equations.
- Write internal forces in terms of deformations for each member.
- Solve equations.
- Check if the assumption of gap closure is correct.

4.63 A force F= 20 kN is applied to the roller that slides inside a slot. Both bars have an area of cross-section of A = 100 mm^2 and a Modulus of Elasticity E = 200 GPa. Bar AP and BP have lengths of L_{AP}= 200 mm and L_{BP}= 250 mm respectively. Determine the displacement of the roller and axial stress in bar A.

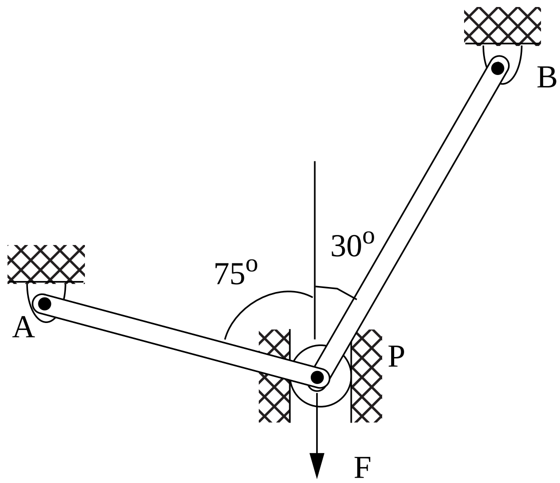


Fig. P4.63

4.68 In Fig. P4.68, a gap exists between the rigid bar and rod A before the force F=75 kN is applied. The rigid bar is hinged at point C. The lengths of bar A and B are 1 m and 1.5 m respectively and the diameters are 50 mm and 30 mm respectively. The bars are made of steel with a Modulus of Elasticity E = 200 GPa and Poisson's ratio is 0.28. Determine (a) the deformation of the two bars. (b) the change in the diameters of the two bars.

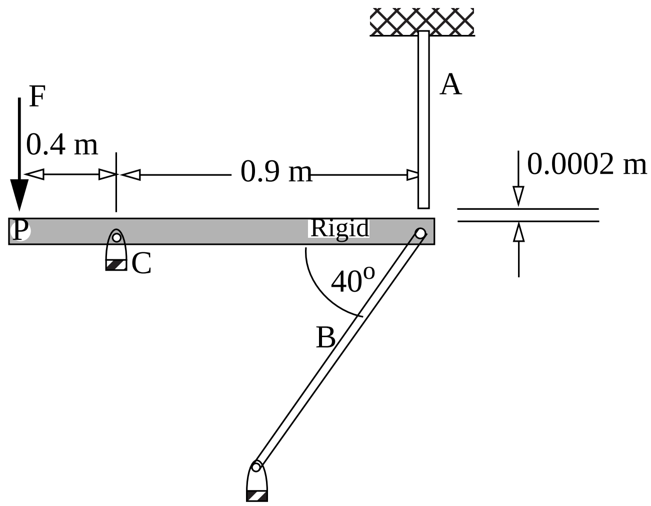
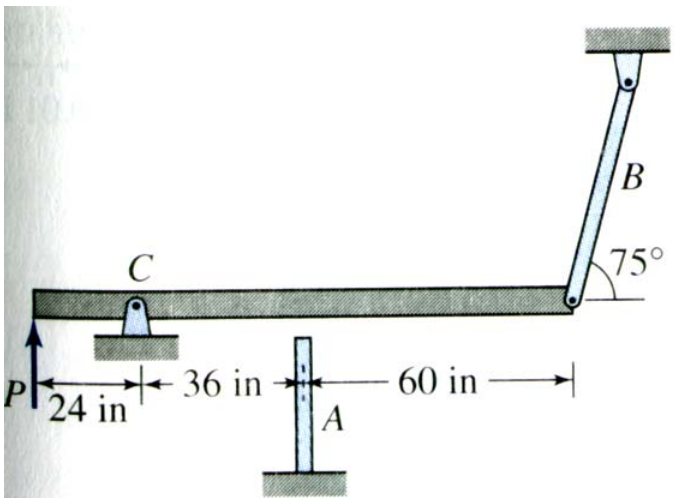
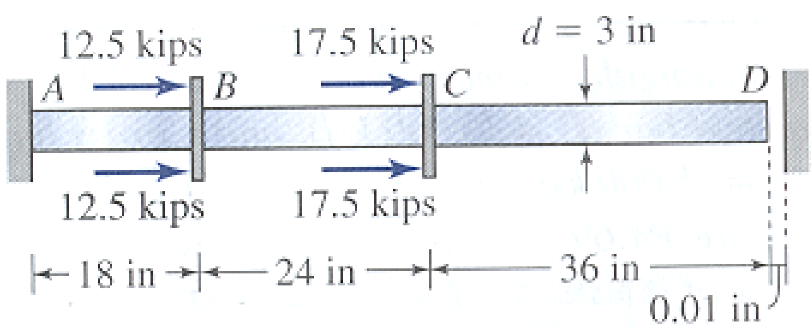


Fig. P4.68

Class Problem 1

Write equilibrium and compatibility equations for the following problems.





4.80 A rigid bar hinged at point O has a force P applied to it as shown. Bars A and B are made of steel (E = 30,000 ksi). The cross-sectional areas of the bars A and B are $A_A = 1\ \text{in}^2$ and $A_B = 2\ \text{in}^2$ respectively. If the allowable deflection at point C is 0.01 inch and the allowable stress in the bars is 25 ksi, determine the maximum force P that can be applied.

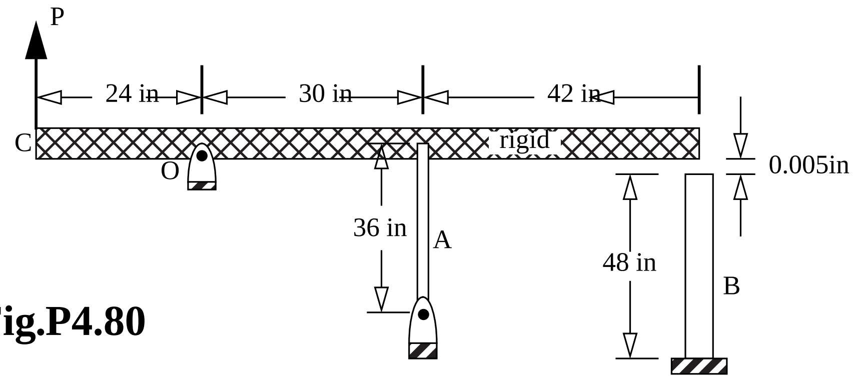
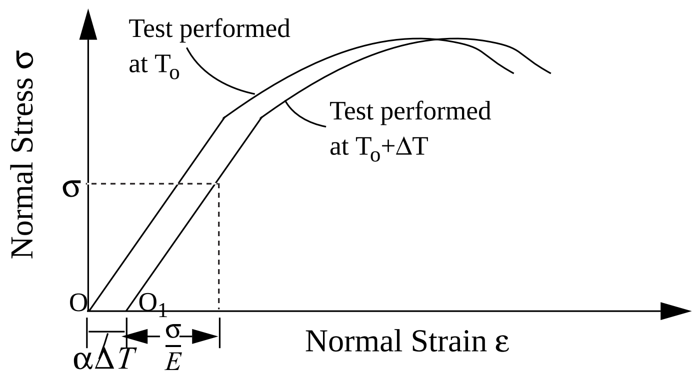


Fig.P4.80

Initial Stress/Strain and Temperature Effects

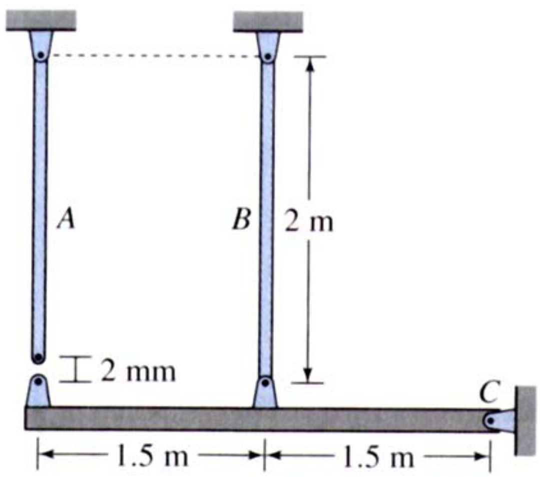
$$\delta = \frac{NL}{EA} + \varepsilon_o L$$

ε_o=Initial strain

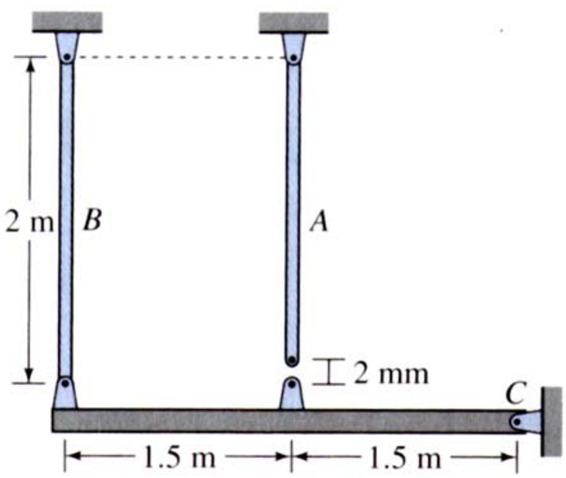


- $\varepsilon_o = \alpha\Delta T$ Thermal Strains.
- No thermal stresses are produced in a homogenous, isotropic, unconstrained body due to uniform temperature changes.
- Increase of temperature ---extension.
- Decrease of temperature---contraction.
- Sign of $\varepsilon_o L$ must be consistent with N shown on free body diagrams.

4.88 Bar A was manufactured 2 mm less than bar B due to an error. The attachment of these bars to the rigid bar would cause a misfit of 2 mm. Calculate the initial stress in each assembly. Which of the two assembly configuration you would recommend? Use modulus of elasticity of E = 70 GPa and diameter of the circular bars as 25 mm.



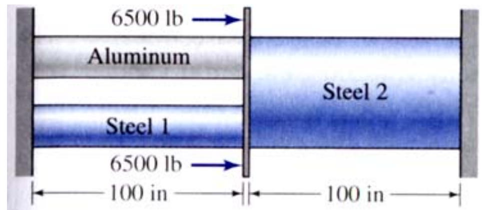
Assembly 1



Assembly 2

4.94 Three metallic rods are attached to a rigid plate as shown. The temperature of the rods is lowered by 100 ^{o}F after the forces are applied. Assuming the rigid plate does not rotate, determine the movement of the rigid plate

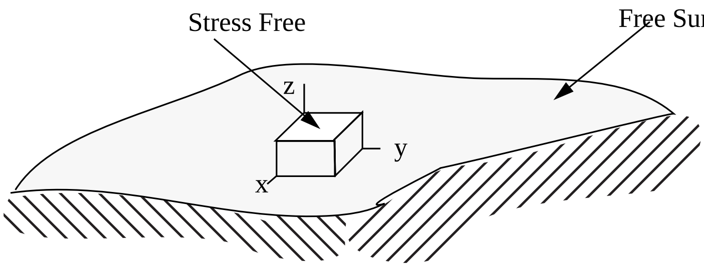
	Area in^2	E ksi	α $10^{-6}/\,^{o}$F
Aluminum	4	10,000	12.5
Steel-1	4	30,000	6.6
Steel-2	12	30,000	6.6

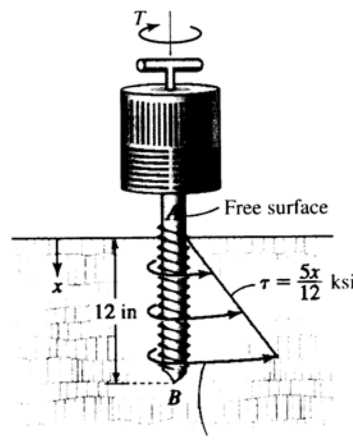


Stress Approximation

Free Surface

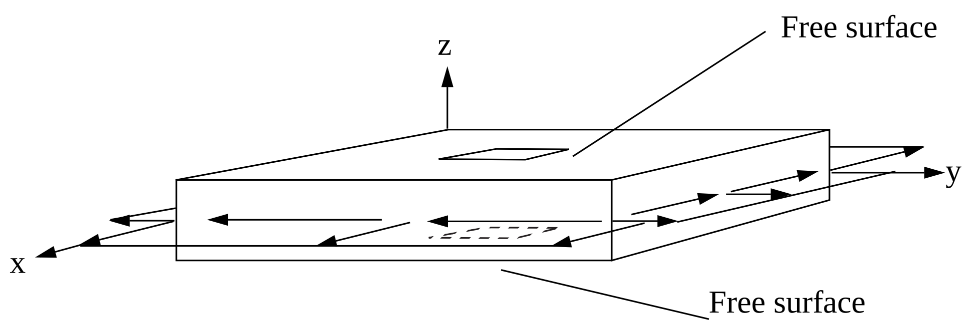
- A surface on which no external forces or moments are acting is called a *free surface.*





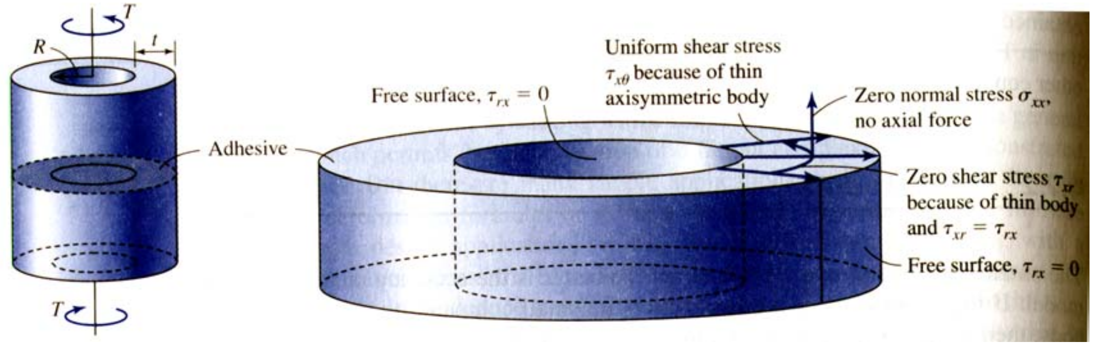
Thin Bodies

- The smaller the region of approximation, the better is the accuracy of the analytical model.



Axi-symmetric Bodies

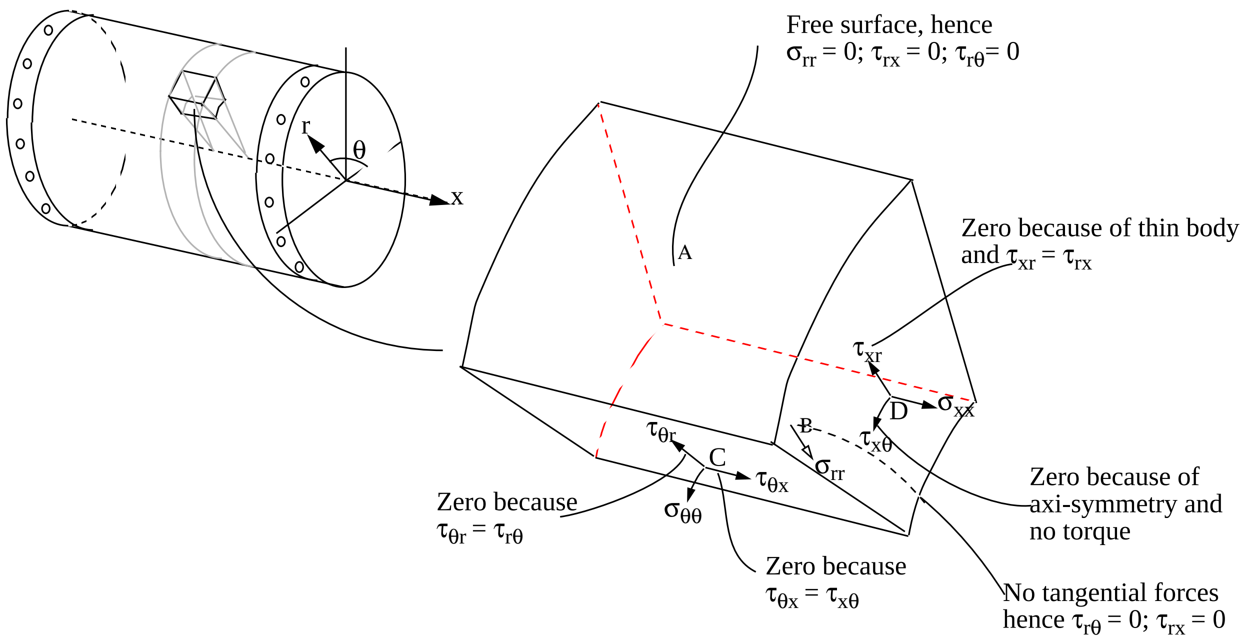
- If a body has a cross-section that is symmetric about an axis and if the applied external forces or moments are also symmetric about the same axis, then the stresses cannot depend upon the *angular* location of the point.



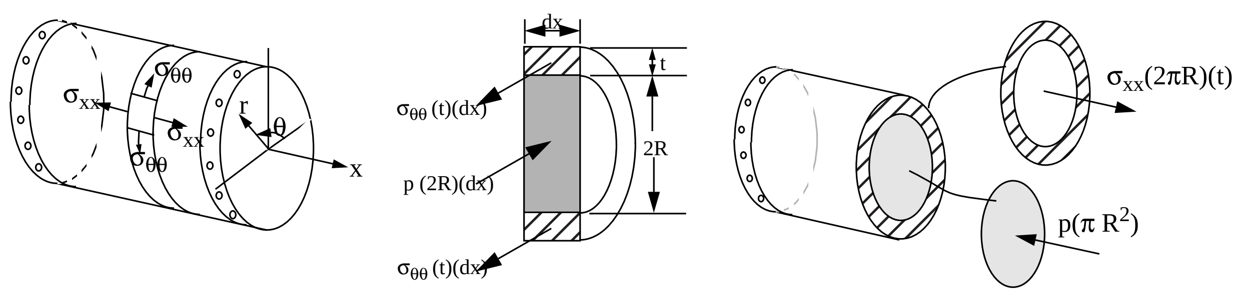
Thin Walled Pressure Vessels

- The *"thin wall"* limitation implies that the ratio of inner radius R to the wall thickness t is greater than 10.

Cylindrical Vessels



- All shear stresses are zero, the radial normal stress is neglected, the *axial stress* σ_{xx} and the *hoop stress* $\sigma_{\theta\theta}$ are assumed uniform across the thickness and across the circumference.



$$\sigma_{\theta\theta} = \frac{pR}{t} \qquad \sigma_{xx} = \frac{pR}{2t}$$

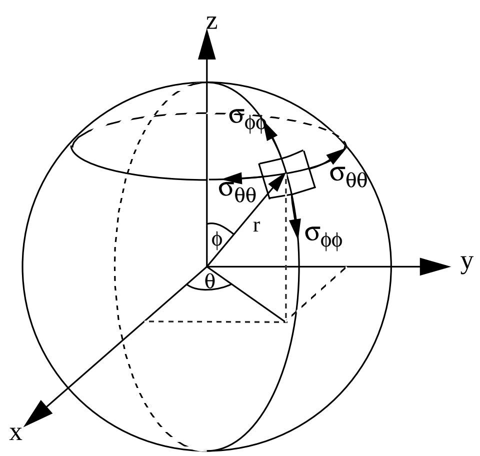
- With R/t > 10 the stresses σ_{xx} and $\sigma_{\theta\theta}$ are greater than the maximum value of radial stress σ_{rr} (=p) by a factor of at least 5 and 10, respectively.

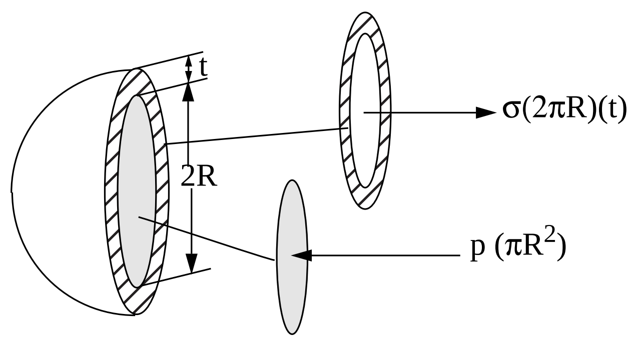
Spherical vessels

- *(i)* All shear stresses are zero. i.e.

  $\tau_{r\phi} = \tau_{\phi r} = 0 \qquad \tau_{r\theta} = \tau_{\theta r} = 0 \qquad \tau_{\theta\phi} = \tau_{\phi\theta} = 0$

- *(ii)* Normal radial stress σ_{rr} varies from a zero value on the outside to the value of the pressure on the inside. We will once more neglect the radial stress in our analysis and justify it post-priori.
- *(iii)* The normal stresses $\sigma_{\theta\theta}$ and $\sigma_{\phi\phi}$ are equal and are constant over the *entire vessel*. We set $\sigma_{\theta\theta} = \sigma_{\phi\phi} = \sigma$.





$$\sigma = \frac{pR}{2t}$$

4.105 A pressure tank 15 feet long and 40 inch diameter is to be fabricated from a 1/2 inch thick sheet. A 15 feet long, 8 inch wide and 1/2 inch thick plate is bonded onto the tank to seal the gap. What is the shear stress in the adhesive when the pressure in the tank is 75 psi? Assume uniform shear stress over the entire inner surface of the attaching plate.