

# Stress

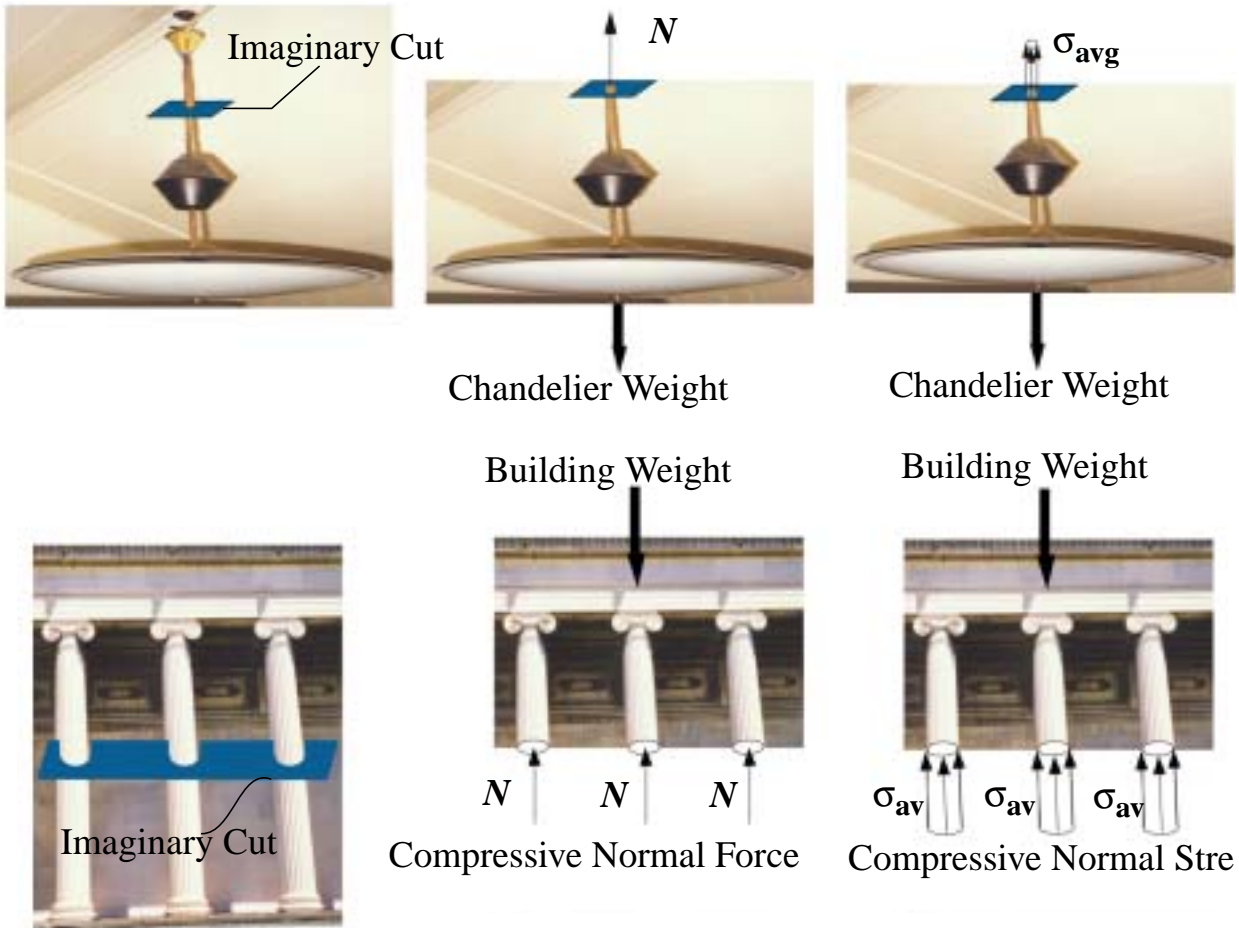
- A variable that can be used as a measure of strength of a structural member.

## Learning objectives

- Understanding the concept of stress.
- Understanding the two step analysis of relating stresses to external forces and moments.



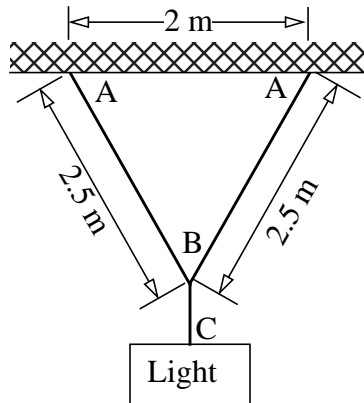
## Normal Stress



$$\sigma_{av} = N/A$$

- All internal forces (and moments) in the book are in ***bold italics***
- Normal stress that pulls the surface away from the body is called a **ten-**  
**sile stress**.
- Normal stress that pushes the surface into the body is called a **com-**  
**pressive stress**.
- The normal stress acting in the direction of the axis of a slender mem-  
ber (rods, cables, bars, columns, etc.) is called the **axial stress**.
- The compressive normal stress that is produced when one surface  
presses against other is called the **bearing stress**.

**1.5** A 6 kg light shown in Fig. P1.5 is hanging from the ceiling by wires of diameter of 0.75 mm. Determine the tensile stress in the wires AB and BC.



**Fig. P1.5**

# Shear Stress

Imaginary cut  
between the wall  
and the tape

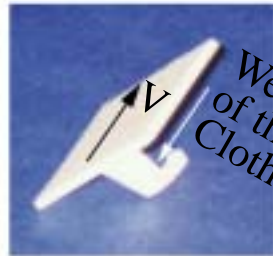


Weight  
of the  
Clothes

Imaginary cut  
along the possible path  
of the edge of the ring.



Pull  
of the  
hand



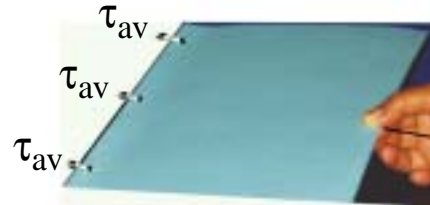
Weight  
of the  
Clothes



Pull  
of the  
hand



Weight  
of the  
Clothes

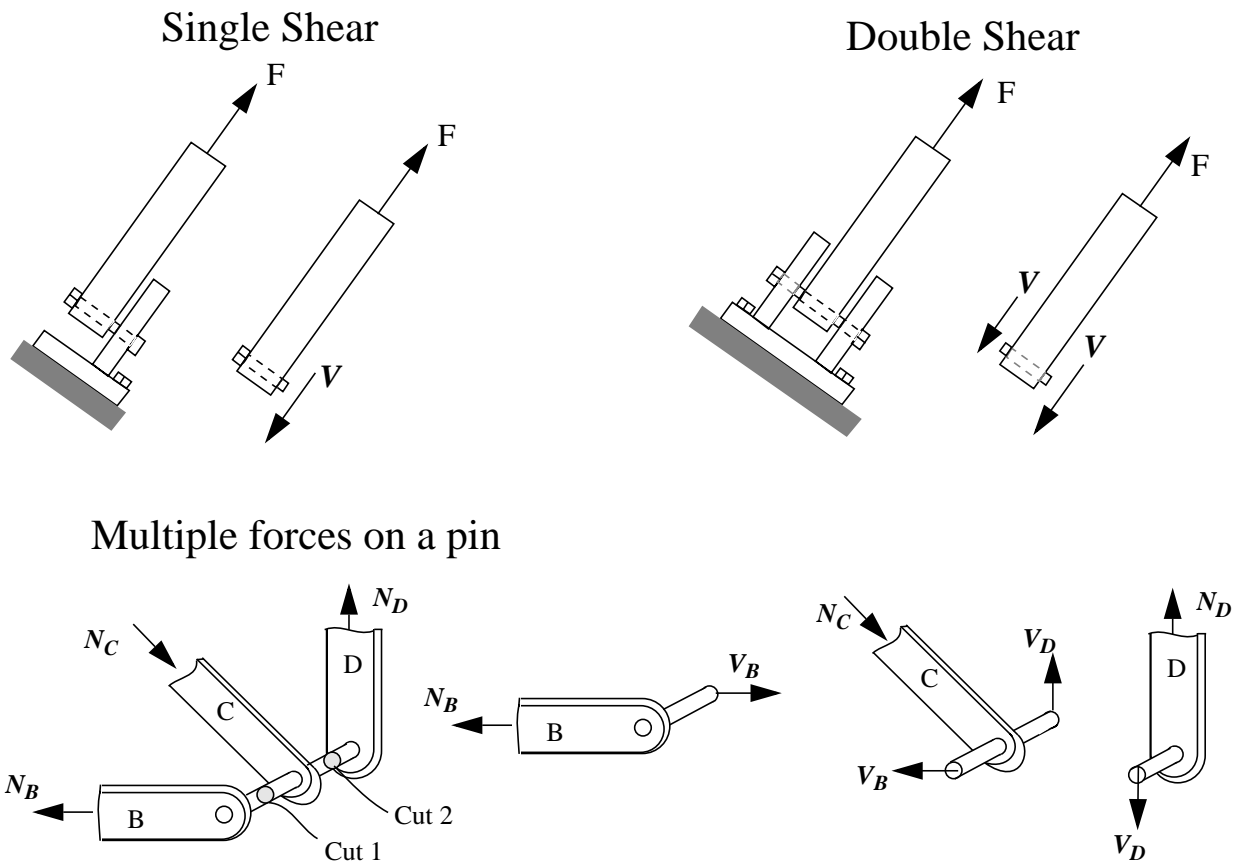


Pull  
of the  
hand

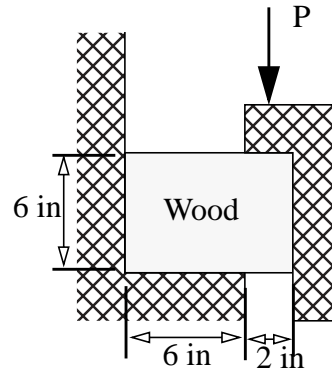
$$\tau_{av} = V/A$$

## Shear stress in pins

- Visualizing the surface on which stress acts is very important.

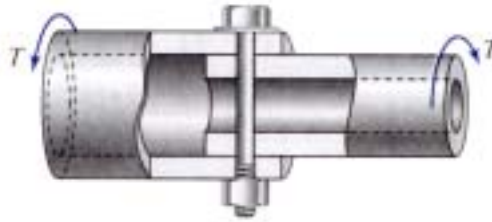


**1.24** The device shown in Fig. P1.24 is used for determining the shear strength of the wood. The dimensions of the wood block are 6 in x 8 in x 1.5 in. If the force required to break the wood block is 12 kips, determine the average shear strength of the wood.



**Fig. P1.24**

**1.31** Two cast iron pipes are held together by a bolt as shown. The outer diameters of the two pipes are 50 mm and 70 mm and wall thickness of each pipe is 10 mm. The diameter of the bolt is 15 mm. The bolt broke while transmitting a torque of 2 kN-m. On what surface(s) did the bolt break? What was the average shear stress in the bolt on the surface where it broke?

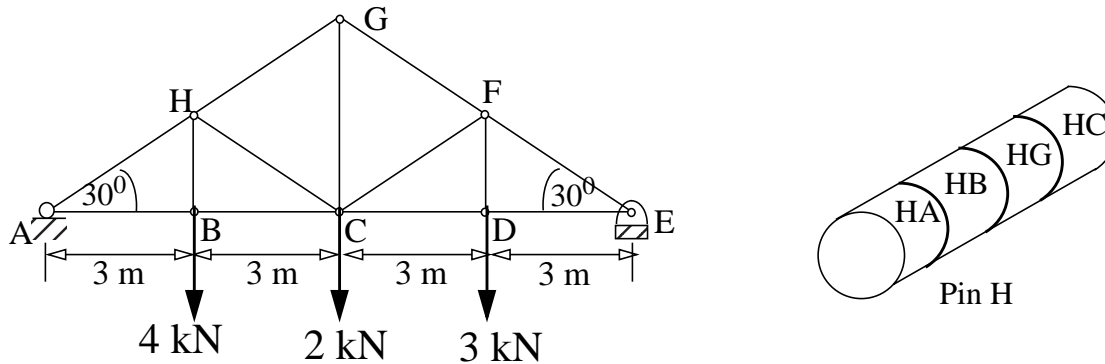


**Fig. P1.31**

Fig. P1.43 shows a truss and the sequence of assembly of members at pin H. All members of the truss have a cross-sectional area of  $250 \text{ mm}^2$  and all pins have a diameter of 15 mm.

**1.42** Determine the axial stresses in members HA, HB, HG and HC of the truss shown in Fig. P1.43.

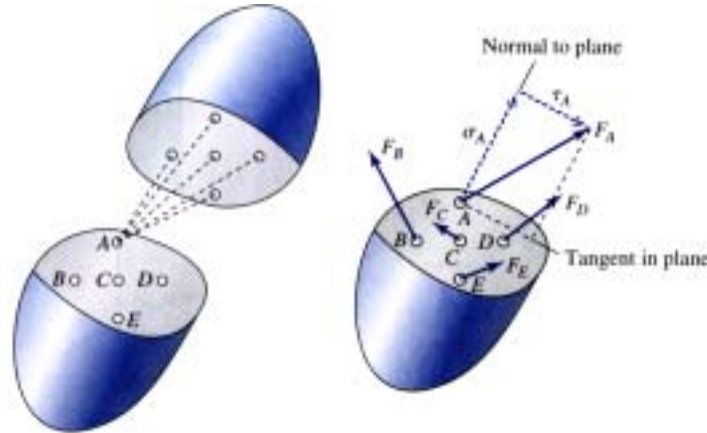
**1.43** Determine the maximum shear stress in pin H.



**Fig. P1.43**



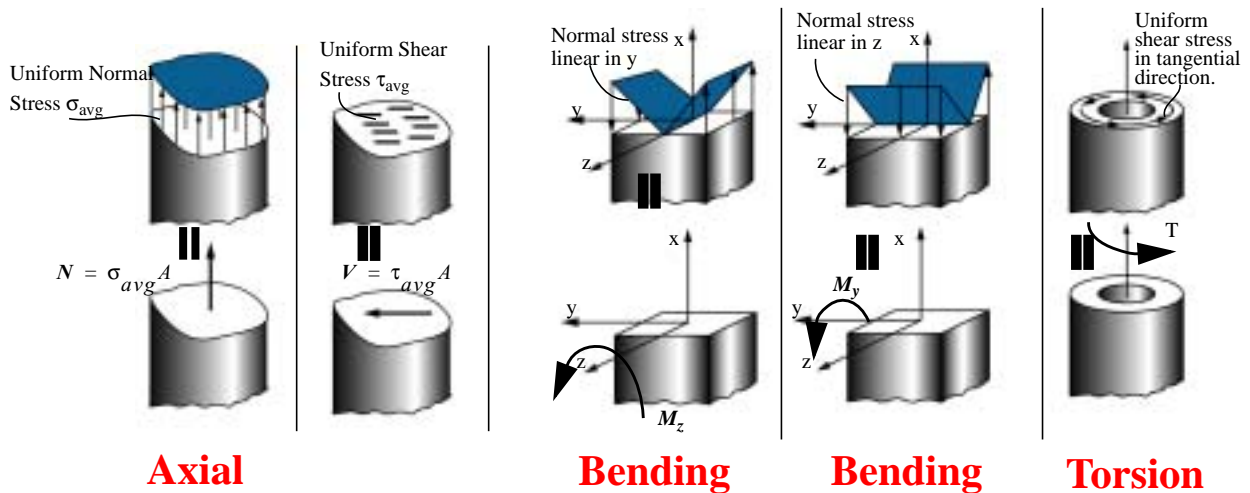
# Internally Distributed Force System



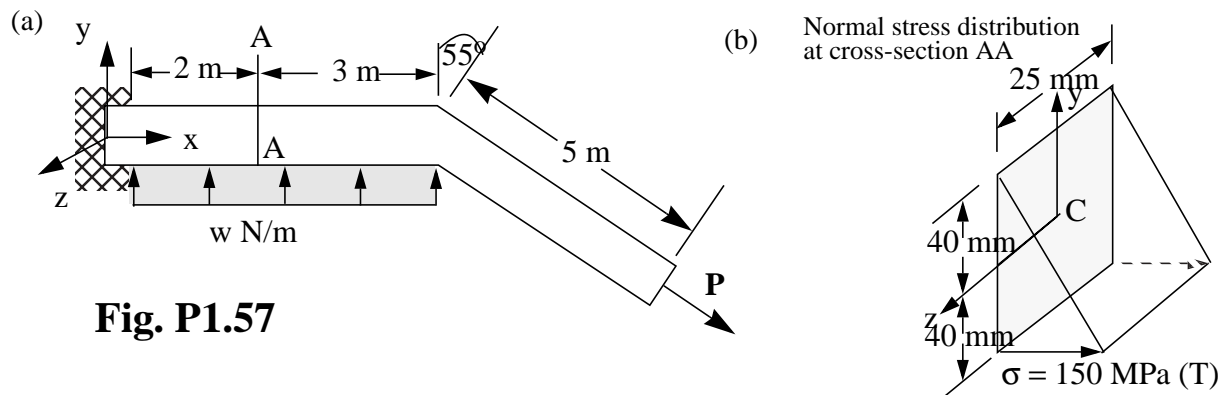
- The intensity of internal distributed forces on an imaginary cut surface of a body is called the *stress on a surface*.
- The intensity of internal distributed force that is normal to the surface of an imaginary cut is called the *normal stress* on a surface.
- The intensity of internal distributed force that is parallel to the surface of an imaginary cut surface is called the *shear stress* on the surface.
- Relating stresses to external forces and moments is a two step process.



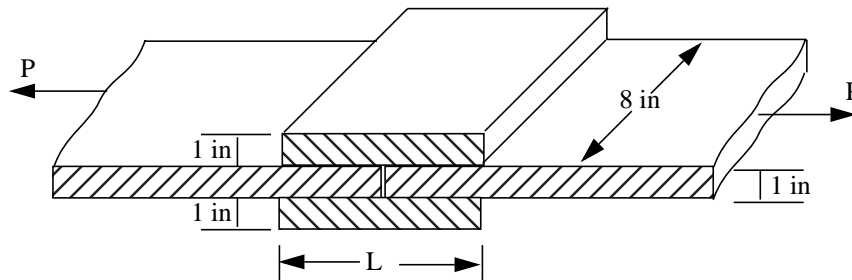
## Static equivalency



**1.57** The cross-section AA on the structure shown has an average shear stress  $\tau$  and a normal stress as shown in Fig. P1.57(b). (a) Replace the normal stress distribution by an equivalent internal normal force  $N$  and an internal moment  $M_z$  acting at the centroid  $C$  of the cross-section. (b) Determine the value of the average shear stress  $\tau$ , the intensity of the external distributed force  $w$ , and the force  $P$ .



**1.68** An adhesively bonded joint in wood is fabricated as shown in Fig. P1.68. The length of bonded region  $L = 5$  inches. Determine the maximum force  $P$  the joint can support if the shear strength of adhesive is 300 psi and the wood strength is 6 ksi in tension.

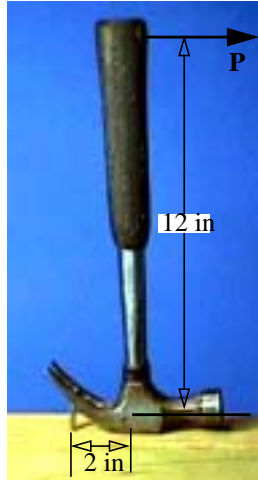


**Fig. P1.68**

**1.69** The joint in Fig. P1.68 is to support a force  $P = 25$  kips, what should be the length  $L$  of the bonded region if the adhesive strength in shear is 300 psi.

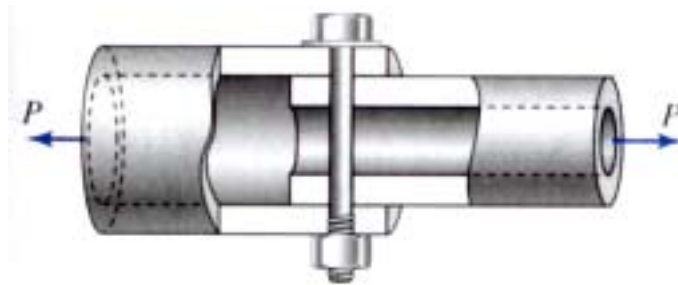
## Class Problem 1

A nail is being pulled out using a claw hammer. Draw the free body diagram that can be used for calculation of shear stress acting on the nail. Identify the surface and the direction of shear stress.



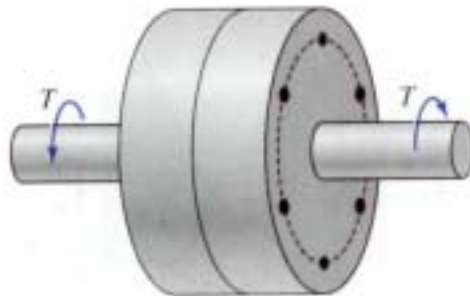
## Class Problem 2

The two pipes got separated while transmitting the force  $P$ . Draw the free body diagram that will permit the calculation of shear stress in the bolt at breaking time. Identify the surface and the direction of shear stress.

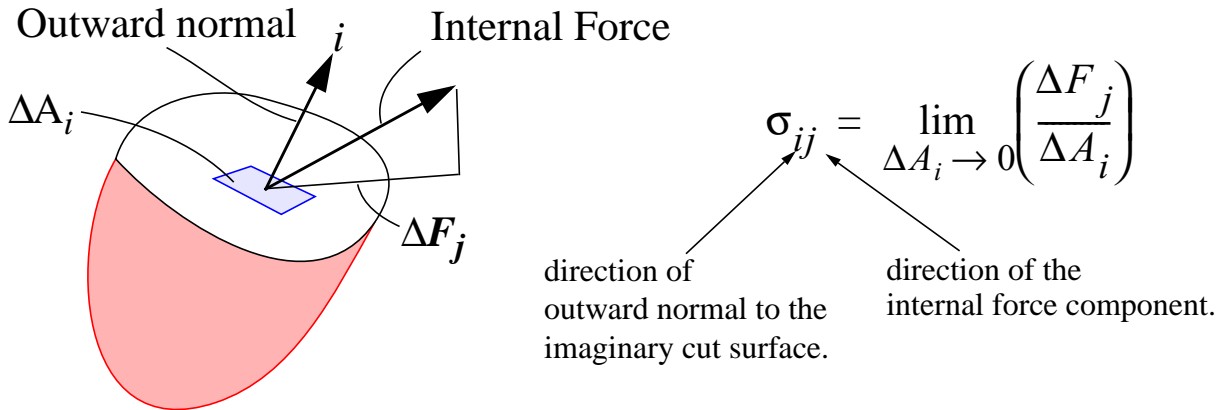


## Class Problem 3

Draw the free body diagram for calculating the shear stress in the bolts holding the coupling together. Identify the surface and the direction of shear stress on each bolt.



## Stress at a Point



- $\Delta A_i$  will be considered positive if the outward normal to the surface is in the positive  $i$  direction.
- A stress component is positive if numerator and denominator have the same sign. Thus  $\sigma_{ij}$  is positive if: (1)  $\Delta F_j$  and  $\Delta A_i$  are both positive. (2)  $\Delta F_j$  and  $\Delta A_i$  are both negative.

- **Stress Matrix in 3-D:**

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

**Table 1.1. Comparison of number of components**

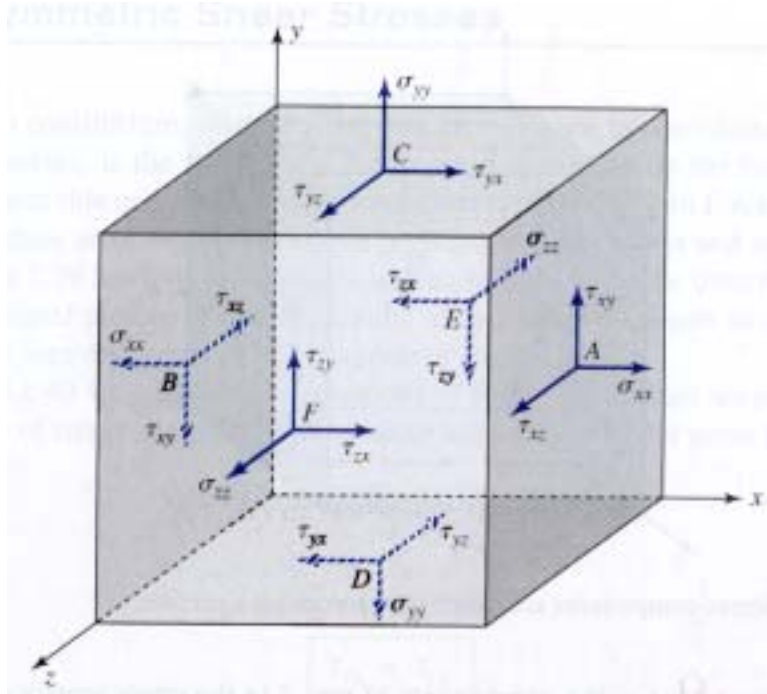
Quantity	1-D	2-D	3-D
Scaler	$1=1^0$	$1=2^0$	$1=3^0$
Vector	$1=1^1$	$2=2^1$	$3=3^1$
Stress	$1=1^2$	$4=2^2$	$9=3^2$

# Stress Element

- Stress element is an imaginary object that helps us visualize stress at a point by constructing surfaces that have outward normal in the coordinate directions.

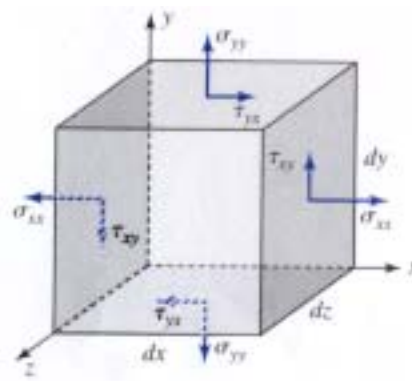
Stress cube showing all positive stress components

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$



**Plane Stress:** All stress components on a plane are zero.

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

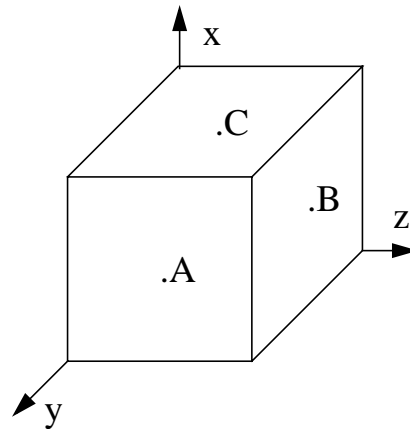


Symmetric Shear Stresses:  $\tau_{xy} = \tau_{yx}$        $\tau_{yz} = \tau_{zy}$        $\tau_{zx} = \tau_{xz}$

- A pair of symmetric shear stress points towards the corner or away from the corner.

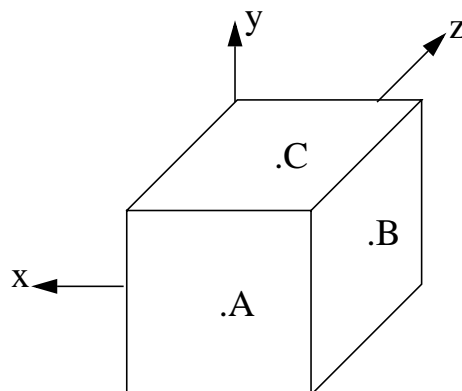
**1.78** Show the non-zero stress components on the A,B, and C faces of the cube.

$$\left[ \begin{array}{lll} \sigma_{xx} = 90 \text{ MPa}(C) & \tau_{xy} = -200 \text{ MPa} & \tau_{xz} = 0 \\ \tau_{yx} = -200 \text{ MPa} & \sigma_{yy} = 175 \text{ MPa}(C) & \tau_{yz} = 225 \text{ MPa} \\ \tau_{zx} = 0 & \tau_{zy} = 225 \text{ MPa} & \sigma_{zz} = 150 \text{ MPa}(C) \end{array} \right]$$



## Class Problem 4

Show the non-zero stress components of problem 1-78 on the A,B, and C faces of the cube below.





**1.87** Show the non-zero stress components in the  $r$ ,  $\theta$ , and  $x$  cylindrical coordinate system on the A,B, and C faces of the stress element shown.

$$\begin{bmatrix} \sigma_{rr} = 145 \text{ MPa}(C) & \tau_{r\theta} = 100 \text{ MPa} & \tau_{rx} = -125 \text{ MPa} \\ \tau_{\theta r} = 100 \text{ MPa} & \sigma_{\theta\theta} = 160 \text{ MPa}(T) & \tau_{\theta x} = 165 \text{ MPa} \\ \tau_{xr} = -125 \text{ MPa} & \tau_{x\theta} = 165 \text{ MPa} & \sigma_{xx} = 150 \text{ MPa}(T) \end{bmatrix}$$

