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Application of fuzzy time series models for forecasting the amount of Taiwan export

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ABSTRACT

The study proposed Traditional Time Series Method (ARIMA model and Vector ARMA model) and Fuzzy Time Series Method (Two-factor model, Heuristic model, and Markov model) for the forecasting problem. The real world case of Taiwan exports is employed for models' test to compare the forecasting ability among models and to examine the effects of different lengths of interval and increment information on the forecasting error of models. The results indicate that Fuzzy Time Series Method performs better forecasting ability in short-term period prediction, especially Heuristic model. The ARIMA model generates smaller forecasting errors in longer experiment time period. Nevertheless, introducing increment information is not necessarily in improving the forecasting ability of fuzzy time series. As a result, it is more convenient to use the fuzzy time series method in the limited information and urgent decision-making circumstance.

1. Introduction

Owing to the relatively small nature and the lack in resources of the domestic market in Taiwan, Taiwan's economic development relies heavily on international trade. In the past, Taiwan used to have a competitive edge in its low labor cost advantages as well as its labor-intensive production and processing of products. It could then expand its export and as a result promote the domestic economic growth. However, in recent years, wages are continuously rising in Taiwan. In fact, the standard wage in Taiwan is comparatively much higher than that of mainland China and countries in Southeast Asia. As the cost of labor gradually increases in Taiwan, the competitive nature of exports are subject to sever challenges. In addition, the economic growth recession and the unemployment boom over the past few years may be as result of the increased shrinking in the trade industry. Since Taiwan is highly dependant on foreign trade, the understanding of the future trend in export trade as well as the knowledge concerning economic policies, business strategies etc. becomes very important. Thus, being able to make forecasts and predictions about export trade becomes vital.

In recent years, due to the innovation of the forecasting technique and improvement of the forecasting accuracy, the forecasting methodology becomes indispensable for further decision making process in both industry and government. However, from the viewpoints of model base, traditionally, most of forecasting methods usually construct a statistical model as a tool for forecasting future value and data analysis. Within the forecasting process, it aims to find out a best model through the historical data of pretest groups. Nevertheless, due to the error, which may cause in data collection, time lag, and reciprocal effect between variables, that made the data itself may contained highly uncertainty. For instance, in forecasting currency exchange rate between NT dollars and US dollars, the data which may obtained from opening quotation, closing quotation, highest price, lowest price, or average price, that will cause to the difference of forecasting result.

The time series data may look like an exact number. However, it represents as a probable value within an interval. For example, the price fluctuation of the stock market, which may represent as linguistic terms, or asking someone's mood, which the data may contain highly uncertainty. If we insisted to analyze the data by using traditional method to match up a statistical model, and use the model to explained the trend of data that may cause to over-fit of the model and over-explained of the data.

Since Zadeh (1965) first defined "a fuzzy algorithm is an ordered set of fuzzy instructions which upon execution yield an approximate solution to a specified problem". The concept of fuzzy set was wildly applied to different fields and used to solve the problems of linguistic data. Recently, the idea of fuzzy logic has been successfully applied in dynamic analysis forecasting method. For instance, based on fuzzy theory, Song and Chissom (1993a, 1993b, 1994) construct the one-factor fuzzy time series. Sullivan and Woodall (1994) reviewed the first-order time-invariant fuzzy time series model and the first-variant model which proposed by Song and Chissom, where the models are compared with each

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other and with a time-invariant Markov model using linguistic labels with probability distributions.

Chen (1996) proposed a new concept of time series method which is more efficient than the one presented in Song and Chissom (1993a). Due to the fact that the proposed method uses simplified arithmetic operations rather than the complicated max-min composition operations presented in Song and Chissom (1993a). In further, Chen (1998) proposed the method which the variation of enrollment of this year is related to the trend of the enrollments of the past years. To define the degree of variations, he performs systematic calculations to calculate the relation between the variations of last year and the other past years. Then, he can get the forecasting enrollments from the derived relation.

Huarng (2001a, 2001b) proposed distribution and average-based length to approach this issue. Distribution-based length is the largest length smaller than at least half the first differences of data. Average-based length is set to one half the averages of the first differences of data. Empirical analysis shows that distribution and average-based lengths are simple to calculate and can greatly improve forecasting results. Huarng (2001a, 2001b) proposes heuristic models by integrating problem-specific heuristic knowledge with Chen's model to improve forecasting. This is because Chen's model was easy to calculate, was straightforward to integrate heuristic knowledge, and forecast better than the others.

Lee, Wang, and Chen (2006), based on the two-factor high-order fuzzy time series and historical data, proposed two-factor high-order fuzzy logical relationships to increase the forecasting accuracy rate of prediction. Lee, Wang, and Chen (2008) based on high-order fuzzy logical relationships and genetic simulated annealing techniques. The proposed method uses genetic simulated annealing techniques to adjust the length of each internal in the universe of discourse for increasing the forecasting accuracy rate.

The Two-factor model (Chen & Hwang, 2000), Heuristic model (Huarng, 2001a, 2001b), and Markov model (Wu, 1986) are most commonly used in the listing fuzzy time series method. However, less study pay attention to discuss the model forecasting ability issue. Besides, previous researches only compute the forecasting data using a certain length of interval, and did not consider if the difference of the length of interval may probably influence the outcome of the models. Moreover, is the conclusion that one variable modified to multi-variable or introducing increment information in fuzzy time series method could reduce the forecasting errors only for the specific patterns of data? In other words, if the stability and generality of fuzzy time series method for forecasting problem is needed to clarify.

The research takes the macroeconomic variable of Taiwan export as an example to compare the forecasting ability between traditional time series method (ARIMA and VARMA) and Fuzzy time series method (Two-factor, Heuristic, and Markov model). At the same time, the study will also investigate the effects of interval length and increment information on models' forecasting ability.

2. Research method

2.1. ARIMA model

The model was propose by Box–Jenkins in 1970, the model examine each variable by using auto-regression, AR (P) and Moving Average, MA (Q) to investigate the historical data and economic fluctuations. The algorithm presented as follows

(1) Data preparation.
(2) The first step in developing a Box–Jenkins model is to determine if the series is stationary and if there is any significant seasonality that needs to be modeled. The autocorrelation functions (ACF) which used for define the distribution of sample data.
(3) Differencing to achieve stationary.
(4) Identification: identify the phase of the series by using autocorrelation functions (ACF) and partial autocorrelation function (PACF).
(5) Estimation: The conditional likelihood and exact likelihood to estimate the parameters.
(6) Diagnostic Check: The process of diagnostic check involves testing the assumptions of the model to identify any areas where the model is inadequate. The statistical identification process which include: whether the parameters achieve statistical significant or multicollinearity: whether the residuals term was white noise or not. If the model is found to be inadequate, it is necessary to remedy and go back to Step 4 and try to identify a better model.

2.2. Vector ARMA model

The multi-variable time series method proposed by Box and Tiao (1977) was predicted future values by constrained to be linear functions of past observations, under the assumption that the data series is stationary. However, while there is conflict between variables, it will be better to use Vector ARMA model. The Vector ARMA model is the extension of the ARIMA model, which describes relationships among several time series variable. In this model, each variable not only depends not only on their past value, but also on the past value of other variables. That also overcomes the drawback of Box and Jenkins's method, which the limitation of the feedback between components. Due to the Vector ARMA model is the most flexibility time series model with minimum limitation and it not only considerate other variable and also explained the dynamic relations between variables which could effectively enhance its forecasting ability. The algorithm of Vector ARMA model as below:

The algorithm of Step (1) to step (6) was the same with ARIMA model

(7) Restrict on the parameter: restrict non-significant parameters, and used exact maximum likelihood re-estimated the parameter.
(8) Repeat Diagnostic Check the model.
(9) Define and analysis equation: while the results of estimated parameter pass the diagnostic check, establish a matrix for further analysis.

2.3. Two-factors time-variant fuzzy time series model

Two-factors time-variant fuzzy time series model was propose by Chen and Hwang (2000). The model assume that we want to forecast $F(t)$ and use $G(t)$ to aid the forecasting of $F(t)$, then $F(t)$ and $G(t)$ are called the main-factor fuzzy time series and second-factor fuzzy time series of the two-factor time-variant fuzzy series model. The model can be defined as follows:

Definition 2.3.1. Criterion vector $\mathbf{C(t)}$, $\mathbf{S(t)}$ and operation matrix $\mathbf{O}^w(t)$

$$\text{Criterionvector } C(t) = f(t-1) = [C_1, C_2, \cdots, C_m] \quad (1)$$

$$S(t) = g(t-1) = [S_1, S_2, \cdots, S_m] \quad (2)$$

Operationmatrix $O^w(t)$

$$=\begin{bmatrix} f(t-2)\\ f(t-3)\\ \vdots \\ f(t-w)\end{bmatrix}=\begin{bmatrix} O_{11} & O_{12} & \cdots & O_{1m}\\ O_{21} & O_{22} & \cdots & O_{2m}\\ \vdots & & & \vdots \\ O_{(w-1)1} & \cdots & \cdots & O_{(w-1)m}\end{bmatrix} \quad (3)$$

where $f(t-1)$ is the fuzzified variation of the main-factor fuzzy time series $F(t)$ between time $t-1$ and time $t-2$,m is the number of elements in the universe of discourse, C_j and O_{ij} are crisp values, $0\leqslant C_j\leqslant 1, 0\leqslant O_{ij}\leqslant 1, 1\leqslant i\leqslant w-1$ and $1\leqslant j\leqslant m$. $S(t)$ is the second-factor at time t,$g(t-1)$ is the fuzzified data of the second-factor fuzzy time series $G(t)$ at time $t-1$,m is the number of elements in the universe of discourse, $S_i\in[0,1]$ and $1\leqslant i\leqslant m$.w is the window basis.

Definition 2.3.2. Fuzzy relationship matrix $R(t)$

$$R(t)=O^w(t)\otimes S(t)\otimes C(t)$$
$$=\begin{bmatrix} O_{11}\times S_1\times C_1 & O_{12}\times S_2\times C_2 & \cdots & O_{1m}\times S_m\times C_m\\ O_{21}\times S_1\times C_1 & O_{22}\times S_2\times C_2 & \cdots & O_{2m}\times S_m\times C_m\\ \vdots & \vdots & \vdots & \vdots\\ O_{(w-1)1}\times S_1\times C_1 & O_{(w-1)2}\times S_2\times C_2 & \cdots & O_{(w-1)m}\times S_m\times C_m\end{bmatrix} \quad (4)$$

m is the number of elements in the universe of discourse, $R_{ij}=O_{ij}\times S_{ij}\times C_j, 1\leqslant i\leqslant w-1. \leqslant j\leqslant m$. and " $\times$ " is the multiplication operator.From the fuzzy relationship matrix $R(t)$ we can get the fuzzified forecasted variation $f(t)$ between times t and $t-1$ described by

$$f(t)=\left[\max\left(R_{11},R_{21},\ldots,R_{(w-1)1}\right),\max\left(R_{12},R_{22},\ldots,R_{(w-1)2}\right),\ldots,\max\left(R_{1m},R_{2m},\ldots,R_{(w-1)m}\right)\right] \quad (5)$$

where $f(t)$ is the fuzzified variation of fuzzy time series $F(t)$ between time t and $t-1$.

Two-factor time-variant fuzzy time series model consists of the following major steps:

Step 1. Partition the historical data into suitable groups and perform the following forecasting steps to each group.
Step 2. Compute the variations of the main-factor fuzzy time series between any two continuous data.
Step 3. Partition the universe of discourse U into several even length intervals $u_1,u_2,\ldots,u_m$.
Step 4. Define fuzzy sets on the universe of discourse U for the fuzzified variation of the main-factor fuzzy time series $F(t)$.
Step 5. Define fuzzy sets on the universe of discourse U for the second-factor fuzzy time series $G(t)$.
Step 6. Fuzzify the variation of the main-factor fuzzy time series and fuzzify the data of the second-factor fuzzy time series.

2.4. Heuristic fuzzy time series models

In the heuristic models, domain-specific knowledge is integrated with Chen's model (1996) to improve forecasting accuracy. Furthermore, in 2001, Huarng extended the method into multivariate model and also defined the algorithm for the heuristic models. In Huarng's method, that included three different methodologies.

(1) One-variable heuristic: compute the variations of the main-factor A($A_{i,t}$ and $A_{j,t+1}$) between two continuous data
(2) Two-variable heuristic: compute the variations of the second-factor B, and these two variables were used as ups or downs to aid the forecasting of main-factor.
(3) Three-variable heuristic: take the ups or downs of the second-factor B(a threshold to discriminate if the differences are significant)to aid the forecasting of main-factor.

Definition 2.4.1. Fuzzy logical relationship

If there exists a fuzzy relationship $R(t-1,t)$, such that $F(t)=F(t-1)\times R(t-1,t)$,where $\times$ is an operator, then $F(t)$ is said to be caused by $F(t-1)$. The relationship between $F(t)$ and $F(t-1)$ can be denoted by

$$F(t-1)\rightarrow F(t) \quad (6)$$

Suppose $F(t-1)=A_i$ and $F(t)=A_j$, a fuzzy logical relationship is defined as $A_i\rightarrow A_j$, where A_i is named as left-hand side of the fuzzy logical relationship and A_j the right-hand side.

Definition 2.4.2. Fuzzy logical relationships groups

Fuzzy logical relationship can be further grouped together into fuzzy logical relationships groups according to the same left-hand sides of the fuzzy logical relationships. For example, there are fuzzy logical relationships with the same left-hand sides (A_i):

$$A_i\rightarrow A_{j1}, A_i\rightarrow A_{j2}, A_i\rightarrow A_{j3}\ldots \quad (7)$$

These fuzzy logical relationships can be grouped into a fuzzy logical relationships group as follows:

$$A_i\rightarrow A_{j1}, A_{j2}, A_{j3},\ldots \quad (8)$$

The algorithm presented as follows:

Step 1. Defining the universe of discourse and the intervals.
Step 2. Defining the fuzzy sets A_i and fuzzifying the data.
Step 3. Establishing the fuzzy logical relationships and the fuzzy logical relationship groups.
Step 4. Introducing heuristic and establishing the heuristic fuzzy logical relationship groups.
Step 5. Forecasting.

2.5. Multivariate fuzzy time series Markov model

Wu (1986) proposed fuzzy Markov relation matrix as the base theory to constructing fuzzy time series model. Wu and Hsu (2004) extended the model into multivariate fuzzy time series Markov model, to forecast main-factor and second-factor at the same time, and using the fuzzy Markov correlation matrix square to calculate the forecasting outputs. The definition for the model as follows:

Definition 2.5.1. Fuzzy relation

Let U be the universe of discourse with $G=(\mu_1,\cdots,\mu_r)$ and $H=(v_1,\cdots,v_r)$, and $\{P_i, i=1,2,\cdots,r\}$, defined as an ordered partition set of U. Where μ_i and v_i are the membership function on universe of the fuzzy set U, and the fuzzy relations between G and H is defined as

$$R=G^T\circ H=[R_{ij}]_{r\times r} \quad (9)$$

where '$\circ$' is the max-min'operator', 'T' is the transpose, the R_{ij} defined as the membership function between G and H.

Definition 2.5.2. Fuzzy Markov relation Matrix

Suppose $\{F(X(t))\}$ is a Fuzzy autoregressive process of order one (FAR (1)). Then for any t, $F(X_t)$ is caused by $F(X_{t-1})$ and it is denoted as "$F(X_t)\rightarrow F(X_{t-1})$", where the membership function of $F(X_t)$ is $\mu_i(X_t)$, $i=1,2,\ldots,r$. The fuzzy Markov relation Matrix is represented by

$$\Re^*=\left[\Re^*_{ij}\right]_{r\times r}=\max_{2\leqslant t\leqslant n}\left[\min\left(\mu_i(X_{t-1}),\mu_j(X_t)\right)\right]_{r\times r} \quad (10)$$

Definition 2.5.3. *p*th order of fuzzy auto-regressive model

It the time series model was in the form as below, which we call (*p*th order of fuzzy auto-regressive model, where the $\Re^*$ is the fuzzy relation matrix between $F(t)$ and $F(t-1), F(t-2), \ldots, F(t-p)$.

$$F(t) = F(t-1) \circ F(t-2) \circ \cdots \circ F(t-p) \times \Re^* \tag{11}$$

Definition 2.5.4. First-order multi-variant fuzzy auto-regressive (FVAR (1))

Assume that the $\{(FX_{1,t}, FX_{2,t}, \ldots, FX_{k,t})\}$ was a first-order multi-variant fuzzy auto-regressive.

$$(FX_{1,t}, FX_{2,t}, \ldots, FX_{k,t}) = (FX_{1,t-1}, FX_{2,t-1}, \ldots, FX_{k,t-1}) \begin{bmatrix} \Re_{11} & \cdots & \Re_{1k} \\ \vdots & \ddots & \vdots \\ \Re_{k1} & \cdots & \Re_{kk} \end{bmatrix} \tag{12}$$

Suppose that $(FX_{1,t}, FX_{2,t}, \ldots, FX_{k,t})$ was cause by $(FX_{1,t-1}, FX_{2,t-1}, \ldots, FX_{k,t-1})$, for any t, the $\Re_{ij}$ was fuzzy Markov relation matrix between $\{FX_{i,j}\}$ and $\{FX_{j,t}\}$, where $i,j = 1,2,\ldots,k$. We defined this as the Multi-variant fuzzy Markov process.

The algorithm presented as follows:

Step 1. Define the universal discourse *U* for the historical data.
Step 2. Partition universal discourse *U* into several equal intervals.
Step 3. Define fuzzy sets on universal discourse *U*.
Step 4. Fuzzify the historical data.
Step 5. Determine fuzzy relation matrix *R*.
Step 6. Calculate the forecasted outputs.
Step 7. Interpret the forecasted outputs.

3. Empirical results

The data are obtained from AREMOS economic database, which including the amount of Taiwan exports and current exchange rate from January 1990 to April 2007, with total 208 data sets for both of it. Among these data, the Taiwan exports are the research object of model test. Taking use of current exchange rate as the increment information is according to international finance theory, indicating that there is closely relationship between the amount of exports and current exchange rate. The Mean Square Error (MSE) is selected to evaluate model's forecasting accuracy. In this paper, in order to investigate whether the length of interval will influence the forecasting ability of the models or not, we employ two-step algorithms to test the models with using the whole data and partition data. First, we will express the empirical results of the models using whole data as below.

3.1. The empirical results with whole data

The result of ARIMA model is obtained as:

$$X_{11t} = \varepsilon_t - 0.795\varepsilon_{t-1} + 0.308\varepsilon_{t-12} - 0.39\varepsilon_{t-13} \tag{13}$$

$X_{11_t} = \log X_1$. The MSE of ARIMA is shown in Table 1. It can be defined that the model is the seasonal moving average model; it means that the amount of exports is affected by the residual error of lag one $(t-1)$, lag 12 $(t-12)$ and lag 13 $(t-13)$.

The result of Vector ARMA model

$$X_{11t} = 0.965X_{11t-1} + 0.224 + 0.065Y_{11t-1} + \varepsilon_t \tag{14}$$

$$Y_{11t} = 0.965Y_{11t-1} - 0.06 + 0.02X_{11t-1} + \varepsilon_t + 0.11\varepsilon_{t-1} + 0.08\varepsilon_{t-2} \tag{15}$$

$$X_{ii} = \log X_i, \quad X_i \text{ is the amount of the export} \tag{16}$$

$$Y_{ii} = \log Y_i, \quad Y_i \text{ is the spot exchange rate} \tag{17}$$

The result of the experiment indicates that the amount of the export is influenced by the export value of lag one $(t-1)$ (96.5%), and also spot exchange rate in lag 1 $(t-1)$ for only 6.5%.

The MSE of fuzzy time series method as shown in Table 1: among the Two-factor model, in two months window basis ($w = 2$), the MSE of model only using the amount of exports (one-variable) is bigger than that of one simultaneously using the amount of exports and current exchange rate (two variable). However, in second order ($t = 2$) of Markov model, the forecasting error of one variable model is smaller than that of two variables.

For the problem whether to introduce increment information (current exchange rate) could enhance the forecasting ability or not, except Two-factor model, one variable model of both the traditional time series model and fuzzy time series generates the smaller forecasting error than that of two variable ones. Furthermore, for entirety forecasting ability, the ARIMA model performs best forecasting accuracy, next followed by Markov model. Is the finding resulted from the longer experiment period which makes the ARIMA model could easily capture the trend of the exports, and then got smaller forecasting errors.

3.2. The empirical results with partition data

For further examination of the effects of the length of interval on model forecasting accuracy, the study partition the data set into different periods which including: **(I)** August 1998 to April 2007, with total 104 data set, (**II**) December 2002 to April 2007, with total 52 data set, (**III**) February 2005 to April 2007, with total 26 data set.

The empirical result of ARIMA model as shown in Table 2: in the period of 1998.08–2007.04, the amount of exports was affected by the residual of lag one $(t-1)$, lag 12 $(t-12)$, and lag 13 $(t-13)$. Among all factors, lag one $(t-1)$ was the most significant factor. In the period of 2002.12–2007.04, the amount of exports was affected by the residual of lag one $(t-1)$ and lag 12 $(t-12)$, which lag one $(t-1)$ was the most significant factor, representing the highest weighted one. Moreover, in the period of 2005.02–2007.04, the amount of exports was affected by the residual of lag one $(t-1)$, lag 12 $(t-12)$ and lag 13 $(t-13)$, but the weighted was slightly reduced.

The empirical result of Vector ARMA model as shown in Table 3: except the period of 2002.12–2007.04, the amount of exports was all affected by lag one $(t-1)$. Moreover, in 2002.12–2007.04

Table 1
The MSE value of five forecasting methods.

Data sets	$n = 208$						
Period	1990.01–2007.04						
Method	ARIMA model	Vector ARMA model	Two-factor		Heuristic model	Markov model	
			One variable $w = 2$	Two variables $w = 2$	One variable	One variable $t = 2$	Two variables $t = 2$
MSE	6.14×10^8	1.14×10^9	1.65×10^9	1.58×10^9	1.01×10^9	8.83×10^8	2.76×10^9
Smaller MSE	****	*			**	***	

Table 2
The ARIMA model in different periods.

Macroeconomic variable	Time	ARIMA model
Amount of export	1998.08–2007.04	$X_{22t} = \varepsilon_t - 0.171\varepsilon_{t-1} + 0.533\varepsilon_{t-12} - 0.139\varepsilon_{t-13}$, $X_{22} = \log X_2$
Amount of export	2002.12–2007.04	$X_{33t} = \varepsilon_t - 0.707\varepsilon_{t-1} + 0.169\varepsilon_{t-12}$, $X_{33} = \log X_3$
Amount of export	2005.02–2007.04	$X_{44t} = \varepsilon_t - 0.719\varepsilon_{t-1} + 0.198\varepsilon_{t-12} - 0.128\varepsilon_{t-13}$, $X_{44} = \log X_4$

Table 3
The Vector ARMA model in different length of period.

Macroeconomic variable	Time	Vector ARMA model
Amount of export	1998.08–2007.04	$X_{22t} = 0.779X_{22t-1} + 2.799 + \varepsilon_t - 0.466\varepsilon_{t-1}$ $Y_{22t} = 0.97Y_{22t-1} + 0.09 + \varepsilon_t + 0.88\varepsilon_{t-1}$
Amount of export	2002.12–2007.04	$X_{33t} = 15.766 - 0.867Y_{33t-1} + \varepsilon_t + 0.252\varepsilon_{t-1}$ $Y_{33t} = 0.91Y_{33t-1} + 0.43 - 0.01X_{33t-1} + \varepsilon_t + 0.4\varepsilon_{t-1}$
Amount of export	2005.02–2007.04	$X_{44t} = 0.894X_{44t-1} + 1.363 + \varepsilon_t - 0.626\varepsilon_{t-1}$ $Y_{44t} = 0.93Y_{44t-1} + 0.24 + \varepsilon_t + 0.25\varepsilon_{t-1}$

$X_{11t} = \log X_t$, X_t: Amount of export; $Y_{11t} = \log Y_t$, Y_t: Spot exchange rate.

Table 4
The MSE of the five forecasting models.

Data set	Period	Method	ARIMA model	Vector ARMA model	Two-Factor		Heuristic model	Markov model	
					One variable $w = 2$	Two variable $w = 2$	One variable	One variable $t = 2$	Two variable $t = 2$
		Smaller MSE	****	*			**	***	
$n = 104$	1998.08–2007.04	MSE	9.28×10^8	1.21×10^9	2.25×10^9	2.18×10^9	1.16×10^9	1.16×10^9	3.19×10^9
		Smaller MSE	****	**		*	***	***	
$n = 52$	2002.12–2007.04	MSE	1.34×10^9	1.41×10^9	2.56×10^9	2.47×10^9	5.86×10^8	1.12×10^9	2.07×10^9
		Smaller MSE	**	*			****	***	
$n = 26$	2005.02–2007.04	MSE	1.66×10^9	1.19×10^9	2.46×10^9	2.44×10^9	5.47×10^8	2.41×10^9	2.41×10^9
		Smaller MSE	**	***			****	*	*

* Fourth smallest MSE.
** Third smallest MSE.
*** Second smallest MSE.
**** Smallest MSE.

period, the amount of exports was affected by the current exchange rate in lag one $(t-1)$, with a negative relationship for 86.7% respectively.

The forecasting result of five different models with three different interval lengths as shown in Table 4: for entirely forecasting accuracy comparison, while the experimental data was longer, the ARIMA model comparatively got smaller forecasting errors. However, the shortening of the experimental data is, the increasing of forecasting accuracy of fuzzy time series model is. In comparison of forecasting accuracy between one variable and two variable models, Markov model with one variable performs better forecasting ability than two variables model; However, while the experimental data reducing to 26 data sets, the forecasting errors of one variable and two variables model were nearly equal. In Two-factor model, there is slightly difference of forecasting errors between one variable model and two variables model. The result indicates that there is no significant effect of introducing increment information on fuzzy time series forecasting ability; On the contrary, in a shorter experiment period, to introduce increment information has significantly influenced the traditional time series forecasting ability. For instance, the Vector ARMA model was simultaneously using the amount of exports and current exchange rate to forecast. The model's forecasting error of using 26 data sets was smaller than that of 108 or 52 data sets.

While the experimental data contains longer time tendency, the manipulation of time series model could match up the empirical data and got better forecasting accuracy. However, if the time tendency is unapparent or even non-linear pattern, the use of time series method may cause calculation error. As a result, in shorter experiment period or uncertainty data pattern, the forecasting ability of fuzzy time series model performs comparatively better than traditional time series model, especially in the Heuristic model. As the computation procedure of Heuristic model was very easy and fast. As a result, under the situation of limited information and urgent decision-making circumstances, it is more convenient to use the fuzzy time series forecasting method.

4. Conclusion

The paper proposes the Multivariate Fuzzy Time Series models (Two-factor fuzzy model, Heuristic fuzzy model, and Markov fuzzy model) and the Traditional Time Series methods for the forecasting Taiwan export problem. The data are obtained from AREMOS economic database, including the amount of Taiwan exports and increment information current exchange rate from January 1990 to April 2007, with total 208 data sets for both of it.

The results indicate that (1). In time series model proposed for the amount of Taiwan exports: the amount of Taiwan exports was mainly influenced by external factors, not related to historical exports data. However, the overall trend of Taiwan exports is toward up, and affected by exports data in one period ahead, 12 periods ahead, and 13 periods ahead, in which one period ahead is the most significant one among experiment time period. (2). In the comparison of forecasting accuracy among five models: the longer experiment time is, the smaller forecasting errors of ARIMA model is. This means while the experiment time of prediction is longer, the ARIMA model seems more easily to capture the trend of exports movement; On the contrary, in a shorter time period, the ability of fuzzy time series to forecast becomes better. (3). In the comparison of forecasting accuracy between one variable and two variable fuzzy time series model: in Markov model, one variable model performs better forecasting accuracy than two-variable model. However, in Two-factor model, there is only slight difference of the forecasting ability between one variable and two variable model. Introducing increment information for fuzzy time series method may not be helpful in improving model's forecasting ability.

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