

Some words on the Analytic Hierarchy Process (AHP) and the provided ArcGIS extension 'ext_ahp'

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The extension ext_ahp has been tested with ArcGIS 9.1 running on a WindowsXP operating system.

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Short introduction to the principles of the Analytic Hierarchy Process (AHP)

The **A**lytic **H**ierarchy **P**rocess (AHP) was introduced by SAATY (1977) and is a very popular *means to calculate the needed weighting factors* by help of a preference matrix where all identified relevant criteria are compared against each other with reproducible preference factors.

AHP process description

All criteria/factors which are considered relevant for a decision are compared against each other in a pair-wise comparison matrix which is a measure to express the relative preference among the factors. Therefore numerical values expressing a judgement of the relative importance (or preference) of one factor against another have to be assigned to each factor. Since it is known from psychological studies that an individual cannot simultaneously compare more than 7 ± 2 elements, Saaty (1977) and Saaty & Vargas (1991) suggested a scale for comparison consisting of values ranging from 1 to 9 which describe the intensity of importance (preference/dominance). A value of 1 expresses "equal importance" and a value of 9 is given for those factors having an "extreme importance" over another factor (Table 0.1).

Table 0.1: Example scale for comparisons (Saaty & Vargas, 1991)

Intensity of importance	Description
1	Equal importance
3	Moderate importance of one factor over another
5	Strong or essential importance
7	Very strong importance
9	Extreme importance
2, 4, 6, 8	Intermediate values
Reciprocals	Values for inverse comparison

Example

The following table shows a simple comparison matrix of order 3 where 3 criteria C1, C2 and C3 are compared against each other. In the direct comparison of the criteria C1 and C3, for example, criterion C1 has been regarded strongly more important than C3, hence a value of 5 has been assigned to the corresponding matrix position. The transpose position automatically gets a value of the reciprocal value, in this case $1/5$ which equals 0.2.

Table 0.2: Pair-wise comparison matrix which holds the preference values.

Criteria	C1	C2	C3
C1	1	4	5
C2	0.25	1	0.5
C3	0.2	2	1

Next, the assigned preference values are synthesised to determine a ranking of the relevant factors in terms of a numerical value which is equivalent to the weights of the factors. Therefore the eigenvalues and eigenvectors of the square preference matrix revealing important details about patterns in the data matrix are calculated. The above square matrix of order three gives three eigenvalues with which three eigenvectors – each having three vector components - can be calculated. It is regarded sufficient to calculate only the eigenvector resulting from the largest eigenvalue since this eigenvector contains enough information to provide – by its eigenvector components - the relative priorities of the factors being considered (Saaty & Vargas 1991).

Of course, the values of the pair-wise comparison matrix will normally be well considered and not set arbitrarily. However, people's feelings and preferences remain inconsistent and intransitive and may then lead to perturbations in the eigenvector calculations. Such inconsistencies might be of the form that a factor A_i being preferred over another factor A_j with A_j being preferred over a factor A_k is not preferred over A_k (A_i must be preferred over A_k in this case). Therefore Saaty (1977) provided the consistency ratio CR which is a single numerical index to check for consistency of the pair-wise comparison matrix. It is defined as the ratio of the consistency index CI to an average consistency index RI, thus

$$CR = \frac{CI}{RI} \quad (1)$$

Table 0.3: Values for RI (Saaty & Vargas, 1991; with n = order of matrix)

n	2	3	4	5	6	7	8
RI	0.00	0.52	0.90	1.12	1.24	1.32	1.41

Following the illustrations of Saaty (1977) who calculated RI values up to a matrix order of 15, matrices with an order greater than 8 have an RI order of magnitude value of about 1.45.

The consistency index CI can be directly calculated from the preference matrix with

$$CI = \frac{\lambda_{\max} - n}{n - 1} \quad (2)$$

where

λ_{\max} : greatest eigenvalue of preference matrix

n : order of matrix

Based on expert knowledge and experience, Saaty & Vargas (1991) recommend a revision of the preference matrix if the consistency ratio CR exceeds a value of 0.1.

Prioritisation of alternatives

The AHP also allows a prioritisation of decision alternatives. Let us consider two alternatives A1 and A2 and three identified criteria C1, C2 and C3 whose preference relationships have been established in Table 0.2. The criteria weights calculated from the eigenvector of the largest eigenvalue of this criteria preference matrix are given in the following table.

Criterion	Weight
C1	0.687
C2	0.127
C3	0.186

In the following step, the two alternatives have to be compared with respect to the relevant criteria. The calculated weights are again calculated from the eigenvectors of the comparison matrices.

Criterion C1:			
Alternative	A1	A2	Weight
A1	1	5	0.83
A2	0.2	1	0.17
Criterion C2:			
Alternative	A1	A2	Weight
A1	1	3	0.75
A2	0.33	1	0.25
Criterion C3:			
Alternative	A1	A2	Weight
A1	1	0.166	0.14
A2	6	1	0.85

The utility of the available alternatives is then calculated as follows

Alternative A1: $0.687 \times 0.83 + 0.127 \times 0.75 + 0.186 \times 0.14 = 0.69$

Alternative A2: $0.687 \times 0.17 + 0.127 \times 0.25 + 0.186 \times 0.85 = 0.31$

Since alternative A1 gets a higher score, it should be preferred over alternative A2.

References

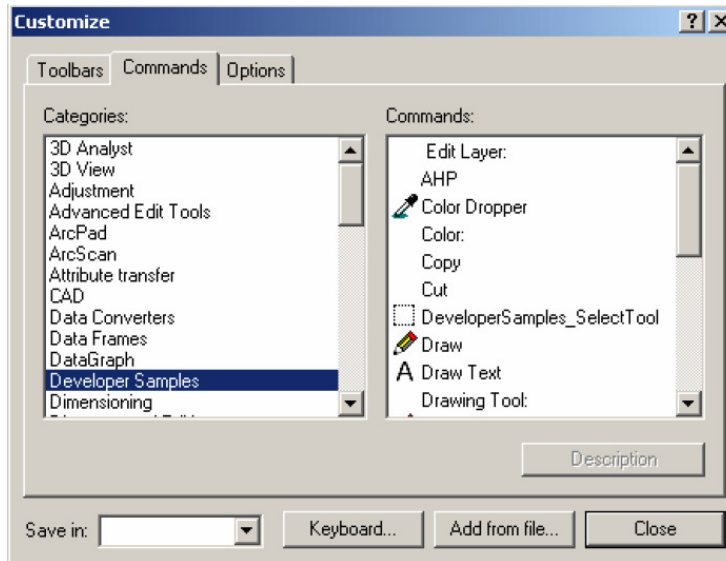
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- Pereira, J.M.C., Duckstein, L., 1993. A multiple criteria decision-making approach to GIS-based land suitability evaluation. *International Journal of Geographical Information Systems*, 7, 407-424.
- Saaty, T.L., 1977. A scaling method for priorities in hierarchical structures. *Journal of Mathematical Psychology*, 15, pp. 231-281.
- Saaty, T.L., Vargas, L.G., 1991. *Prediction, Projection and Forecasting*. Kluwer Academic Publishers, Dordrecht, 251 pp.

The AHP extension 'ext_ahp' in ArcGIS

Accessing the functionality of 'ext_ahp.dll'

To make the extension run it is necessary to copy the extension (extahp.dll). A good choice for a destination folder is the 'ArcGIS\Bin' folder, however other folders will do as well. Having copied this file perform the following steps:

1. launch ArcGIS



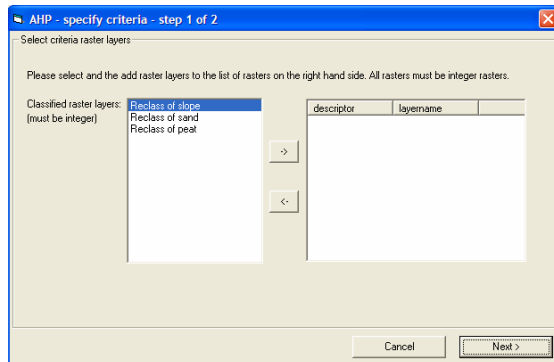
2. go to the Tools menu, choose Customize
3. push the 'Add from file' button
4. browse for the extension 'extahp.dll' and choose 'Ok'
5. go to the 'Commands' tab. In the 'Categories' list box choose 'Developer Samples'
6. Choose 'AHP' in the 'Commands' list box
7. Click on the list item which is labelled 'AHP' and move this item by dragging it somewhere onto the ArcMap environment.
8. Push the 'Close' button. The extension is ready for use.

If you have placed an additional button (point 7, see above) your ArcMap environment could look like this:

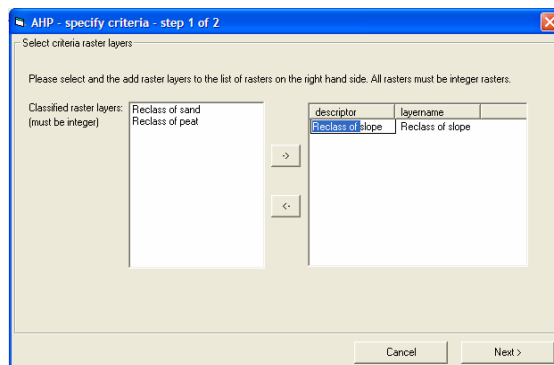


Using the extension:

Push the **AHP** button. A window appears which lets you define all criteria which you consider relevant for your decision. Please note: The extension **expects classified raster datasets**. Therefore **only integer rasters will appear as list items** in the classified map raster layers list box. The user must take care that all rasters are classified within the same class value range (no checking is performed if the classified criteria rasters have the same class range). Click on a list item and the push the **->** button to define it as a criterion which will be used in the subsequent analysis.



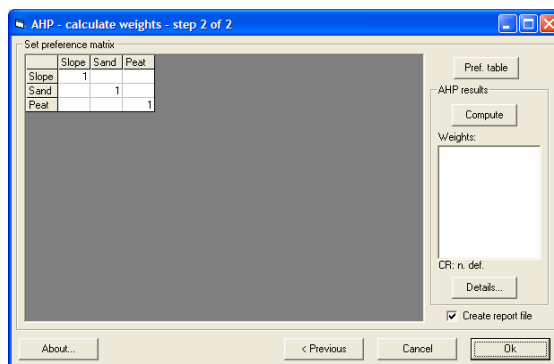
Please note that you can change the descriptor of the used layers (see the following figure).



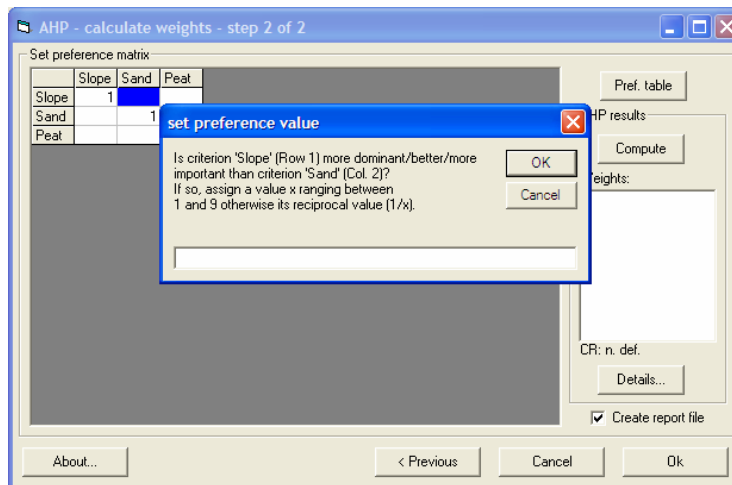
THE NUMBER OF CRITERIA WHICH CAN BE USED IS LIMITED TO 20! However a minimum of two rasters is required for the analysis.

If you are finished with your criteria definitions click on the **Next >** button.

The following window appears:



Enter your preference values in the preference matrix by double clicking on the corresponding matrix cell or by hitting the <Enter> key. A window will appear asking you to enter a value.



Having entered a valid value, the transpose position in the matrix is automatically inverted. Elements on the matrix' main diagonal will remain unchanged. Entering values higher than the maximum preference value (which is 9) will be reset to 9 values smaller or equal to 0 will be set to one. The 'pref. table' button displays a window which shows a 'legend' of the preference values.

To calculate the weights push the **Compute** button. The calculated weights are displayed in the text box which is underneath the compute button. The **Details...** button lets you view the details of the calculation in terms of the resulting eigenvalues and more. You can change your preference values at any time. Please note however, that when having changed a preference value the weights are not going to be changed automatically. The extension calculates with the former weights as long as the 'calculate' button has not been clicked. To actualize your weights the user must therefore again click the **Compute** button!

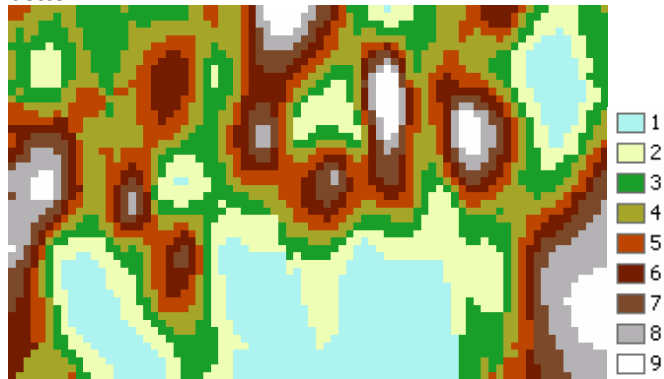
When you are done with the weight determination you click the 'Ok' button. When the 'create report file' check box is checked you will be prompted to enter a name for a text file where all the information of the AHP procedure is stored. If you do not want such a file to be created, de-check this checkbox.

ArcMap performs a raster operation and calculates the weighted sum of the previously defined criteria rasters. The result raster is being added to the current ArcMap session. The default name is 'ahpResultX' (where X stands for any integer value).

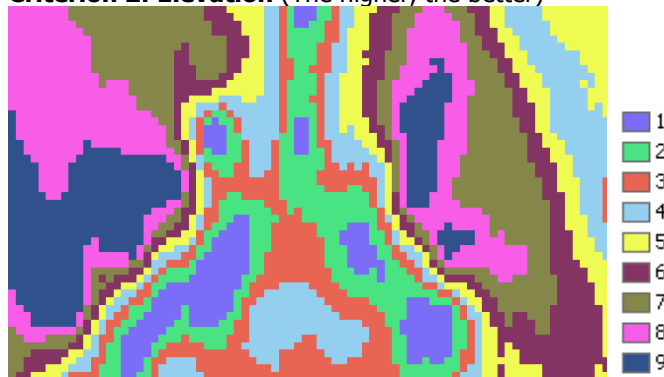
Example (the example files can be found in the example folder)

The suitability of a piece of land with respect to housing construction has to be found out. For reasons of simplicity only three criteria are being considered. The extension 'ext_ahp' expects that all relevant criteria are already classified as integer rasters. In the example these are:

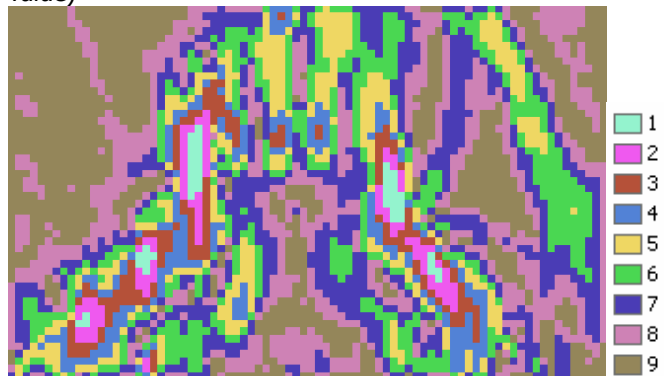
Criterion 1: Thickness of sand (classified into 9 classes). The higher the thickness of sand the better.



Criterion 2: Elevation (The higher, the better)



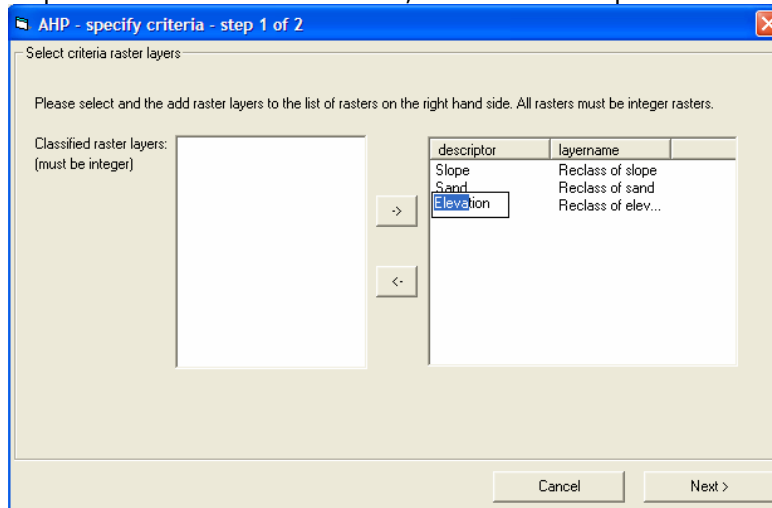
Criterion 3: Slope (the lower the better which means that high slope values receive a low class value)



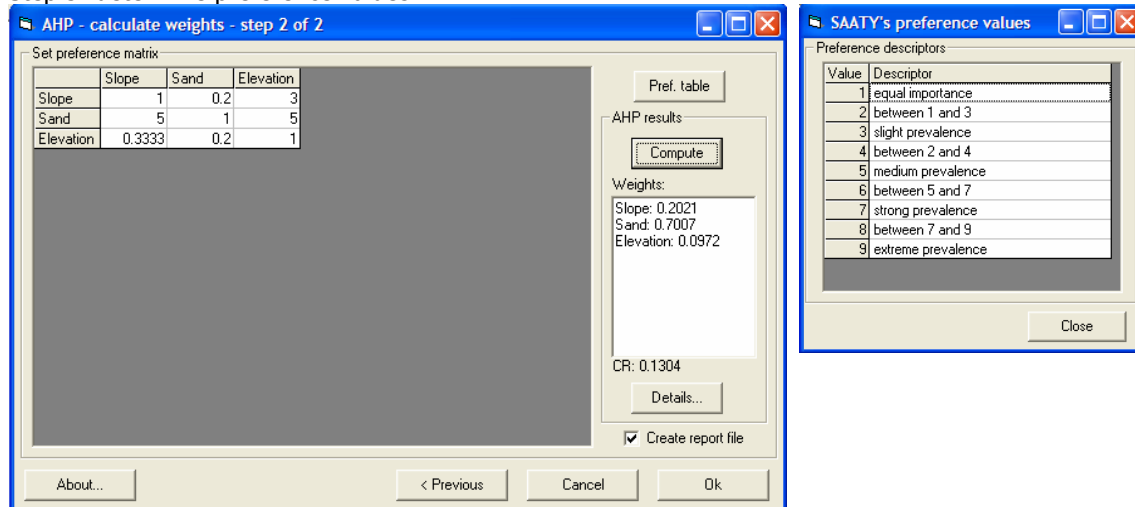
AHP evaluation

Step 1: Start the extension by clicking the AHP button.

Step 2: define the rasterized criteria, define new descriptors if desired



Step 3: determine preference values



The given preference values are a numerical way to express the importance or dominance of one criterion over another. Here, the criterion slope is regarded slightly more important than criterion elevation (→ value 3 in the first row); the thickness of sand has a medium prevalence over slope and elevation (→ values of 5).

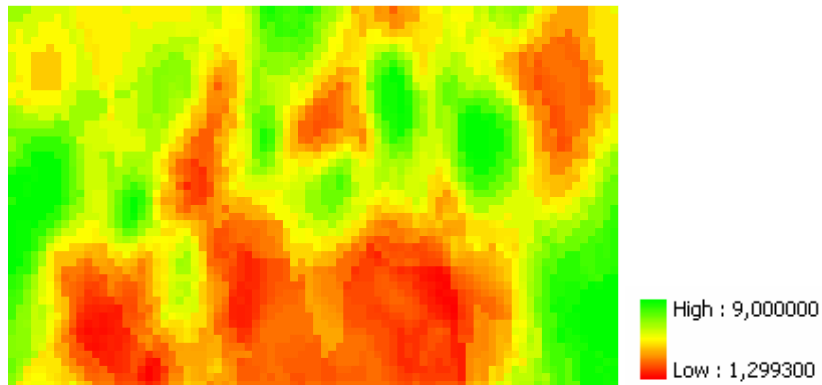
We can see that criterion sand receives by far the highest weight (~70%). Slope gets about 20%, elevation 9.7% (→ 10%).

Note: The consistency ratio 'CR' of 0.13 is above Saaty's recommended threshold of 0.1. However it is only slightly above 0.1 and we therefore accept the result.

Step 4: Push the Compute button. The results are displayed in the text box.

Step 5: Push the 'OK' button. If you want a report file to be created check 'create report file' (default); then specify a filename. A new raster is calculated as a weighted sum of the available rasters and added to the current ArcMap session.

Result raster



The most suitable locations are shown in green. One can clearly see that the reddish areas fall into areas of low sand thickness (see page 8). Since criterion sand received a weight of about 70% we should be expecting an impact of this criterion on the result map.

Example report file

The report file has the following structure:

```
[Criteria & LayerSource (clsfd.))
Slope      Reclass of slope
Sand       Reclass of sand
Elevation  Reclass of elevation

[Preference Matrix]
           SlopeSand Elevation
Slope      1         0.2      3
Sand       5         1        5
Elevation  0.333     0.2      1

[*****AHP results*****]
[Eigenvalues]
3.1356
-0.0678
-0.0678

[Eigenvector of largest Eigenvalue]
0.2747
0.9524
0.1321

[criteria weights]
0.2021 (Slope)
0.7007 (Sand)
0.0972 (Elevation)

[consistency ratio CR]
0.1304
(Revision of preference values is recommended if CR > 0.1)
```