Interference-Limited Analysis of the Convolutionally Coded Optical OOK/BPPM CDMA System

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Abstract: *The convolutional-coded Optical CDMA systems with On-Off Keying (OOK) and Binary Pulse Position Modulation (BPPM) schemes are studied in this paper. In this work, the employment of a single hard limiter is also considered. We evaluate the upper bound on the bit error probabilities of our proposed schemes under the interference-limited condition. For a convolutional coded optical OOK-CDMA system, we drive a more accurate upper bound than that obtained by Dale et.al in 1995. It is shown that the implementation of convolutional codes in an optical CDMA system provides significant improvement in system performance. Furthermore, for a certain bit error rate, our proposed schemes enable the use of Optical Orthogonal Codes (OOCs) with less weight and hence increase the channel bit rate compared to the one required in the uncoded system.*

Keywords: Optical CDMA system, OOK and BPPM modulation, Convolutional Codes

1 Introduction

Optical Code Division Multiple Access (OCDMA) has recently attracted great attention for optical communication systems. Optical CDMA is an asynchronous scheme, which does not require time synchronization and frequency management [1,2].
Multiple access interference (MAI) is one of the major factors for performance degradation in an optical CDMA system [1-4]. Recently, the use of channel coding with optical CDMA systems, for the performance improvement purpose, has been proposed by some research works [3], [4]. In [3], the employment of convolutional code in an optical OOK-CDMA system without hard limiter is suggested.
In this work we restrict our attention to the effect of MAI on the bit error performance of the convolutional coded optical OOK/BPPM-CDMA systems with and without hard limiter and evaluate the bit error probabilities. We come up with an upper bound on bit error rates that is more accurate than the one obtained in [3]. We show that the employment of the convolutional codes in optical CDMA systems can remarkably reduce the MAI effect. Particularly, our proposed schemes enable the use of OOCs with less weight in the CDMA system than that required in the comparable uncoded systems.
Section 2, reviews a convolutional coded OCDMA system. In Section 3, the channel models are presented for both OOK and BPPM schemes and then probabilities of bit error are evaluated. Furthermore, in this section we evaluate the tightness of the derived upper bounds by means of simulations. In Section 4, we compare the performance of the uncoded and the proposed coded optical systems. Finally, in Section 5, concluding remarks are presented.

2 Coded Optical CDMA System

Fig.1 shows a typical block diagram of a fiber optic CDMA system, which uses a convolutional encoder right after the information source and a hard Viterbi decoder at the receiving part.

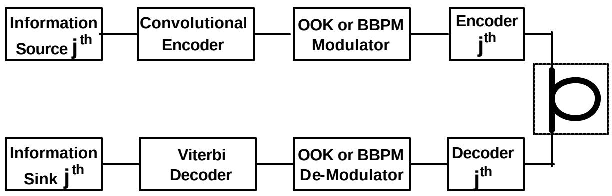

Fig. 1. Convolutional coded optical CDMA system.

Each convolutionally coded information bit modulates a laser source using either OOK or binary PPM (BPPM) modulation scheme. Then, each modulated signal is spread by a signature sequence assigned to each user's encoder. In this paper, OOC codes with minimum auto- and cross correlation constraints are used. Therefore, if N is the number of active users, it can be shown that [5]

$$N \le \left\lfloor \frac{F-1}{w(w-1)} \right\rfloor \tag{1}$$

where F and w are the length and weight of OOC codes, respectively and $\lfloor x \rfloor$ denotes the integer part of the real number x. The perfect optimal OOCs are also assumed in this paper.

At the receiving end, a copy of the desired signal along with N-1 interfering signals of all other users is received. We use the detector structure as Fig. 2. The output function of the ideal optical hard limiter (HL) is defined as follows

$$g(t) = \begin{cases} 1 & ; t \ge 1 \\ 0 & ; t < 1 \end{cases} \tag{2}$$

where t is the normalized input optical power. It is shown that the employment of hard limiter enhances the system performance significantly [2].

In an optical CDMA system, detection is performed by counting the number of photons. In an OOK system with correlator type receiver, if the collected photon counts over the entire chip time duration is less than a predefined threshold, 0 is detected, and otherwise 1 is declared. In a BPPM system with correlator type receiver, if the sum of the photon counts over the entire chip time duration of the first slot is greater than that of the second slot, 0 is declared and otherwise, 1 is detected. Afterward, a hard input Viterbi decoder decodes the recovered encoded information bits.

3 Performance Analysis

Now, we evaluate the probability of bit error under the quantum interference limited condition. We also consider a system with the worst-case

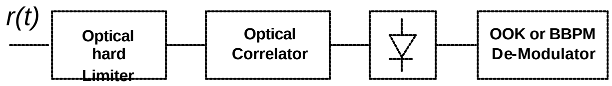

Fig. 2 Detector Structure.

interference among users, which is the chip synchronous case.

The optical CDMA channel can be regarded as a binary channel as shown in Fig.3.

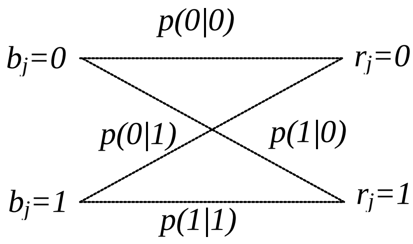

Fig. 3. Binary channel

In an OOK-CDMA system, $p(0|1)=0$ [1]. Hence, the OOK system can be characterized as a binary Z channel. Assuming equally likely input information bits, $p(1|0)$ will be

$$p(1|0) = p = 2P_e \tag{3}$$

where P_e denotes the probability of bit error for OOK system.

An alternative to OOK is PPM. A PPM system has the main advantage that there is no need to define a threshold level as it is required in the case of OOK. However, a PPM system suffers from the need for the extra bandwidth which can be considered as a drawback. Assuming equally likely input information bits, a BPPM system can be modelled as a binary symmetric channel (BSC) with the following transition probabilities

$$p(0|1) = p(1|0) = P_e \tag{4}$$

$$p(0|0) = p(1|1) = 1 - p_e \tag{5}$$

where P_e denotes the probability of bit error for the BPPM system. Upper bounds on bit error probabilities for both OOK/BPPM CDMA systems, with and without hard limiter, are presented in table 1 [1], [4]. The threshold level, th, should satisfy the following inequality [1]

$$0 < th \le w \tag{6}$$

In order to enhance the performance of an optical CDMA system, the employment of a convolutional code right after the binary source is proposed.

Table 1. The Upper Bounds on the Bit Error Probabilities, P_e

	Optical OOK-CDMA system	Optical BPPM-CDMA system
System without hard limiter (NHL)	$\frac{1}{2}\sum_{i=th}^{N-1}\left(\frac{w^2}{2F}\right)^i\left(1-\left(\frac{w^2}{2F}\right)\right)^{N-1-i}$	$\sum_{i=w}^{N-1}\sum_{m=0}^{N-1-i}\binom{N-1}{m}\binom{N-1}{m+i}\left(\frac{w^2}{2F}\right)^{2m+i}\left(1-\left(\frac{w^2}{2F}\right)\right)^{2(N-1-m)-i}$
System with hard limiter (HL)	$\frac{1}{2}\binom{w}{th}\prod_{i=0}^{th-1}\left(1-\left(1-\frac{w}{2F}\right)^{N-1-i}\right)$	$\prod_{i=1}^{w}\left(1-\left(1-\frac{w}{2F}\right)^{N-i}\right)$

The upper bound on the bit error probability of a convolutional code (k,n,m) is [6]

$$P_b \le \frac{1}{k}\sum_{d=d_{free}} B_d\, p_d \tag{7}$$

where B_d is the total numbers of nonzero information bits on all distance d paths and p_d is the event error probability.

3.1 Coded Optical OOK-CDMA System

A relation that incorporates B_d s to the bit error probability calculation of a convolutional coded optical OOK-CDMA system has been suggested in [3]. Here we present a closed form expression for the upper bound on the bit error probability in terms of the code generating function. To evaluate the event error probability we consider an incorrect path through the trellis, which differs from the correct path in d positions. We define the following parameters, which are used in our theoretical evaluations.

k: the number of 1's in the d bits of the correct sequence ($0 \le k \le d$)

n_{01}: The number of $0 \to 1$ transitions from the correct to the received path ($0 \le n_{01} \le d-k$)

n_{10}: The number of $1 \to 0$ transitions from the correct to the received path ($0 \le n_{10} \le k$)

Then, the event error probability can be written as

$$p_d = \sum_{k=0}^{d}\sum_{n_{01}=0}^{d-k}\sum_{n_{10}=0}^{k}\underbrace{(P(E|K=k, N_{01}=n_{01}, N_{10}=n_{10})}_{\le 1}. \\ P(N_{01}=n_{01}, N_{10}=n_{10}|K=k) \\ P(K=k)) \tag{8}$$

Where E denotes the event error. N_{01} and N_{10} are the binomial random variables with the parameters $(d-k,\, p(1|0))$ and $(k,\, p(0|1))$, respectively. Assuming that the input information bits are equally likely, K is also a binomial random variable with the parameter $(d, 0.5)$. Therefore the probability of the occurrence of k 1's in the d bits of the correct sequence $P(K=k)$, can be written as

$$P(K=k) = \binom{d}{k}\left(\frac{1}{2}\right)^d \tag{9}$$

Since N_{10} and N_{01} are statistically independent random variables, then

$$P(N_{10}=n_{10}, N_{01}=n_{01} \mid K=k) = \\ \binom{d-k}{n_{01}} p^{n_{01}}(1-p)^{d-k-n_{01}} . \binom{k}{n_{10}} q^{n_{10}}(1-q)^{k-n_{10}} \tag{10}$$

where p and q denote $p(1|0)$ and $p(0|1)$, respectively.

Therefore,

$$P_d \le \left(\frac{1}{2}\right)^d \sum_{k=0}^{d}\binom{d}{k}\underbrace{\sum_{n_{01}=0}^{d-k}\binom{d-k}{n_{01}} p^{n_{01}}(1-p)^{d-k-n_{01}}}_{\le 2^{d-k} p^{\frac{d-k}{2}}(1-p)^{\frac{d-k}{2}}} \\ \cdot \underbrace{\sum_{n_{10}=0}^{k}\binom{k}{n_{10}} q^{n_{10}}(1-q)^{k-n_{10}}}_{\le 2^k q^{\frac{k}{2}}(1-q)^{\frac{k}{2}}} \\ \le p^{\frac{d}{2}}(1-p)^{\frac{d}{2}}\sum_{k=0}^{d}\binom{d}{k}\left(\sqrt{\frac{q(1-q)}{p(1-p)}}\right)^k \\ = p^{\frac{d}{2}}(1-p)^{\frac{d}{2}}\left(1+\sqrt{\frac{q(1-q)}{p(1-p)}}\right)^d \tag{11}$$

As an optical OOK-CDMA system can be regarded as a Z channel, i.e. $q=0$, then

$$P_d \le p^{\frac{d}{2}}(1-p)^{\frac{d}{2}} \tag{12}$$

Denoting the generating function of the convolutional code by $T(D,I)$, it can be shown that [6]

$$\left.\frac{\partial T(D,I)}{\partial I}\right|_{I=1} = \sum_{d=d_{free}} B_d D^d \qquad (13)$$

Consequently,

$$P_{b-OOK} \le \frac{1}{k}\left.\frac{\partial T(D,I)}{\partial I}\right|_{\substack{I=1 \\ D=\sqrt{p(1-p)}}} \qquad (14)$$

where p is substituted with (3).

In order to verify the validity of the obtained upper bound, we simulate a convolutional coded system over a binary Z channel. We use optimal convolutional code (2,1,3) with generator matrix $G(D) = (1+D^2+D^3, 1+D+D^2+D^3)$ [6].

$T(D,I)$ of this convolutional code is [6]

$$T(D,I) = \frac{D^6 I^2 + D^7 I - D^8 I^2}{1 - 2DI - D^3 I} \qquad (15)$$

Fig.4 shows the performance of the convolutional code over a binary Z channel. It is observed that the simulation results tightly coincide with the obtained theoretical upper bounds. To compare our analytical approach with that of the previously reported one [3] we include the upper bound on the bit error probability using Dale's equation [3]. The required coefficients for evaluating the Dale's equation, i.e. B_d s, can be easily obtained by expanding of the code generating function. It is observed that our analytical approach predicts the performance of the codes more accurate than the Dale's approach.

3.2 Coded Optical BPPM-CDMA system

The optical channel of a BPPM-CDMA system is modelled as a BSC. The upper bound on the probability of bit error of an arbitrary convolutional code over BSC is recognized and expressed by [6]

$$P_{b-BPPM} \le \frac{1}{k}\left.\frac{\partial T(D,I)}{\partial I}\right|_{\substack{I=1 \\ D=2\sqrt{p\ (1-p)}}} \qquad (16)$$

where p is substituted with the appropriate relation given in Table 1.

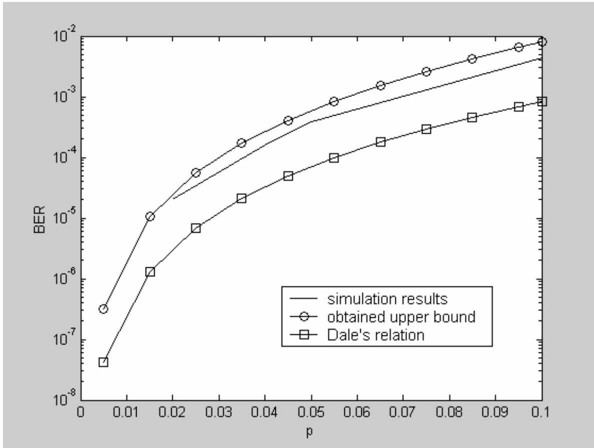

Fig.4 Performance of the convolutional code over the binary Z channel.

4 Numerical Results

Now, we calculate the upper bounds on the bit error probabilities versus the number of active users.

In an uncoded fiber optic CDMA system with OOCs *(F,w,1,1)* , the channel bit rate is

$$R_{uncoded} \cong \begin{cases} \dfrac{1}{Nw(w-1)T_c} & ; OOK \\ \dfrac{1}{2Nw(w-1)T_c} & ; BPPM \end{cases} \qquad (17)$$

It is observed that, for a certain OOC codes and chip time (T_c), the required bandwidth of BPPM is doubled. In order to have the same bit rate for the coded and uncoded systems the chip time and the number of users are held fixed. In this case the length and the weight of OOC codes, used in the coded systems, should satisfy the following equations [3]

$$\begin{cases} F_{coded} = \lfloor R_c F \rfloor \\ w_{coded} = \left\lfloor \dfrac{1+\sqrt{1+4R_c w(w-1)}}{2} \right\rfloor \end{cases} \qquad (18)$$

where F and w are the length and the weight of the OOC sequences used in the uncoded system, respectively, and R_c is the rate of the convolutional code.

We use the optimal convolutional code (2, 1, 3) with the given generator matrix in Section 3.1, in all numerical evaluations.

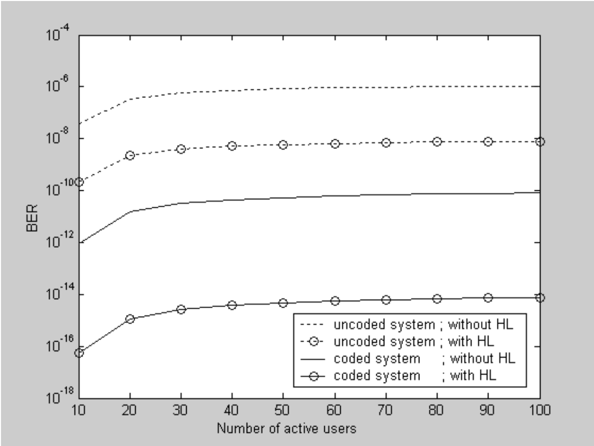

Fig. 5. Performance of the optical OOK-CDMA system for a fixed channel bit rate.

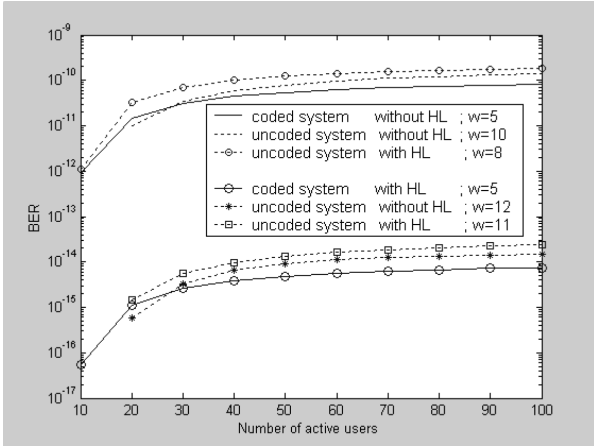

Fig. 6. Performance of various optical OOK-CDMA systems.

4.1 Optical OOK-CDMA System

Figs.5 shows the BER of various systems with OOK scheme. In order to have the same channel bit rate we consider different values for the OOC code weights. The threshold values for these systems are set to their corresponding OOCs code weights. It is observed that the employment of the convolutinal code in an optical CDMA system provides a remarkable performance improvements compared to the uncoded system. Furthermore, the performance of coded system with hard limiter is superior with respect to the other systems.

In addition, for a certain bit error rate, our numerical results (Fig.6 and Table 2) show that by the use of a convolutional code, it is possible to implement an optical CDMA system with less OOC code weight. Consequently, the coded system can operate at higher channel bit rates. Furthermore, having a smaller *w* value is beneficial from the hardware system implementation point of view [4].

4.2 Optical BPPM-CDMA System

Fig. 7 shows the BER of various BPPM systems for a constant channel bit rate. Again, it is observed that the coded systems outperform the uncoded ones. As expected, the system performance improves significantly in a convolutional coded system with hard limiter.

Furthermore, for a certain bit error rate, as shown in Fig. 8 and table 3, smaller *w* values can be used to implement the coded optical CDMA system. Hence, for a given bit error rate and the number of active users, the employment of a convolutional code reduces the required OOC code length

Table 2. Weights of OOCs which is required in the uncoded OOK system for a certain bit error rate, number of users and chip time.

Coded system (*w*=5)	P_e	*w* used in the uncoded system	$\frac{R_b}{R_{uncoded}}$
Without HL(NHL)	5×10^{-11}	$w_{NHL}=10$	2.25
		$w_{HL}=8$	1.40
HL	5×10^{-15}	$w_{NHL}=12$	3.30
		$w_{HL}=11$	2.75

(According to (1)) and increases the available band width. A convolutional coded optical CDMA system permits higher number of active users with less hardware requirements than the ones required in the uncoded system.

5 Conclusions

In this paper, we evaluated the upper bounds on the bit error probability of the convolutional coded fiber-optic CDMA system assuming both OOK and binary PPM schemes. Systems with and without hard limiter were considered. It was shown that the presented upper bound on the bit error probability of an OOK-CDMA system is more accurate than that obtained by Dale et.al. Our numerical results showed that the convolutional codes significantly improves the performance of the optical systems; particularly in those ones that employ hard limiter. In addition, for a certain bit error rate, the employment of convolutional codes, whereas accommodating higher number of users, enables the use of OOCs with less weight and hence providing higher channel bit rates.

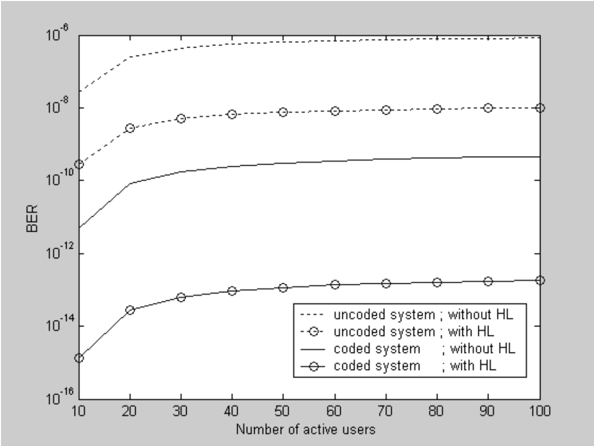

Fig. 7. Performance of the optical BPPM-CDMA system for a fixed channel bit rate.

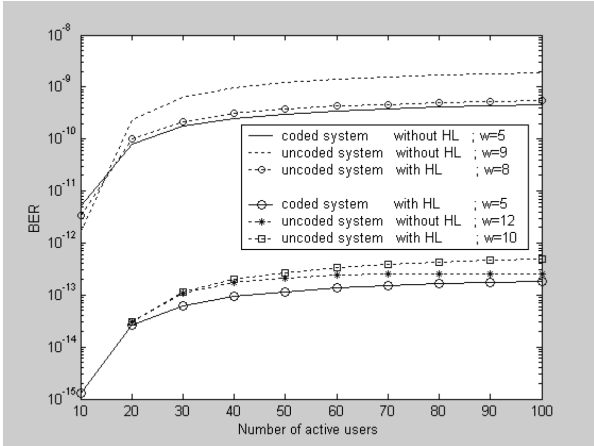

Fig. 8. Performance of various optical BPPM-CDMA systems.

Table 3. Weights of OOCs required in the uncoded BPPM system for a certain bit error rate, number of users and chip time.

Coded system (*w*=5)	P_e	*w* used in the uncoded system	$\frac{R_b}{R_{uncoded}}$
Without HL(NHL)	3×10^{-10}	$w_{NHL} = 9$	1.80
		$w_{HL} = 8$	1.40
HL	10^{-13}	$w_{NHL} = 12$	3.30
		$w_{HL} = 10$	2.25

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